Introduction

• Contagions make understanding network structure of financial interactions critical

• Need tools to help evaluate potential risk of contagion

• Need to understand effects of integration and diversification
Our Contributions:

• Develop network measures related to cascades

• Illuminate *and distinguish* the effects of diversification and integration

• Examine policies to avoid initial failures and some associated moral hazard issues
Outline

• Model

• Cascades: Diversification and Integration

• Endogenous Values and Moral Hazard

• An Illustration with European Debt Data
Basics of the Model:

• Organizations (firms, banks, countries, etc.) hold
  • Assets
  • Shares in each other

• Dropping below some level of value, an organization experiences a discontinuous drop:
  • Cash flow problems disrupt production...
  • Bankruptcy costs...

• Drop in value of one organization leads to drop in values of others they have financial arrangements with, cascades...
Model

• \( \{1, \ldots, n\} \) Organizations (countries, firms, banks...)

• \( \{1, \ldots, m\} \) Assets (primitive investments)

• \( p_k \) price of asset \( k \)

• \( D_{ik} \) holdings of asset \( k \) by organization \( i \)
Cross Holdings:

• $C_{ij}$ cross holdings: fraction of org j owned by org i

• $C_{ii} = 0$ (don’t own yourself)

• $\hat{C}_{ii} = 1 - \sum_j C_{ji}$ fraction of org i privately held
Value of an Organization

\[ V_i = \sum_k D_{ik} p_k + \sum_k C_{ij} V_j \]

Book Value

Direct Asset Holdings

Cross Holdings
Value of an Organization

\[ V_i = \sum_k D_{ik} p_k + \sum_k C_{ij} V_j \]

\[ V = Dp + CV \]

\[ V = (I - C)^{-1} Dp \quad \text{Book Value} \]
Value of an Organization

\[ V = (I - C)^{-1} Dp \]  

\text{Book Value}

Market Value: value to final (private) investors

\[ v = \hat{C}V = \hat{C} (I - C)^{-1} Dp \]  

\text{Market Value}

\[ v = A \ Dp \]  

\text{Market Value}

(c.f. Brioschi et al 89, Fedenia et al 96)
Example

- Two organizations $n=2$
- Each own half of each other

$$C = \begin{bmatrix} 0 & .5 \\ .5 & 0 \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix}$$

holdings by private investors
Example

- Two organizations $n=2$
- Each own half of each other

\[
C = \begin{bmatrix}
0 & 0.5 \\
0.5 & 0
\end{bmatrix}
\]

\[
\hat{C} = \begin{bmatrix}
0.5 & 0 \\
0 & 0.5
\end{bmatrix}
\]

\[
A = \hat{C} (I-C)^{-1} = \begin{bmatrix}
2/3 & 1/3 \\
1/3 & 2/3
\end{bmatrix}
\]

private investors’ claims on values
Example

\[ \hat{C}_{11} = .5 \]

\[ C_{21} = .5 \]

\[ \hat{C}_{22} = .5 \]

\[ C_{12} = .5 \]
Example

What happens to 1$ of investment income to 1?
What happens to 1$ of investment income to 1?
Example

What happens to 1$ of investment income to 1?
Example

What happens to 1$ of investment income to 1?
What happens to 1$ of investment income to 1?
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What happens to 1$ of investment income to 1?
What happens to 1$ of investment income to 1?
Example

What happens to 1$ of investment income to 1?
Example

What happens to 1$ of investment income to 1?

\[
\begin{align*}
&\text{2/3} &\rightarrow &\text{1} &\rightarrow &\text{2} &\rightarrow &\text{1/3} \\
&0.5 &\rightarrow &0.5 &\rightarrow &0.5 &\rightarrow &0.5 \\
&\text{1} &\rightarrow &\text{2} &\rightarrow &\text{1} \\
&0.5 &\rightarrow &0.5 &\rightarrow &0.5 &\rightarrow &0.5
\end{align*}
\]
Drops in Values

• If an organization’s value drops below some $v_i$, it incurs a cost $b_i$

• Cash flow problems:
  • E.g., Spanair unable to pay for fuel, forced to cancel flights

• Bankruptcy costs
  • Legal costs, costs of reorganization

• Changes in production
  • discontinuous changes in production decisions
Bankruptcy/Liquidity Costs:

\[ b(v) \text{ bankruptcy costs} = \begin{cases} b_i & \text{if } v_i < v_i \\ 0 & \text{otherwise} \end{cases} \]

\[ v = A (Dp - b(v)) \]
Example

- Each organization starts with an asset worth $p_i$

- Bankrupt if $v_i$ drops below 50, incurs cost of 50

$$v = A(Dp - b(v)) = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \begin{bmatrix} p_1 - b_1(v) \\ p_2 - b_2(v) \end{bmatrix}$$
Equilibria

- The equilibria form a complete lattice

- We focus on the unique "best-case" equilibrium where the fewest organizations fail

- Easy Algorithm to find it:
  - Identify organizations that fail even if no others do
  - Identify those that fail due to the failures identified above
  - Iterate...
Outline

• Model

• Cascades: Diversification and Integration

• Endogenous Values and Moral Hazard

• An Illustration with European Debt Data
Three Necessary Components of a Cascade:

- **A first failure**: some organization needs to fail
- **Initial Contagion**: some neighbors need to be sufficiently exposed to fail too
- **Interconnection**: to continue to cascade, the network must have sufficiently large components
What Affects Cascades:

- **Diversification**: How many other organizations does a typical organization cross hold?

- **Integration**: How much of a typical organization is cross held?
Diversification/Integration

- n=100 organizations

- Random network $g$ with $\Pr(g_{ij} = 1) = d/(n-1)$

- $d = \text{expected } \# \text{ other organizations that an organization cross holds} \quad (d = \text{level of diversification})$

- Fraction $c$ of org cross-held (evenly split among those holding it), $1-c$ held privately \quad (c = \text{level of integration})

- So, $C_{ij} = cg_{ij}/d_j \quad \hat{C}_{ii} = 1-c$
Diversification/Integration

• One asset per organization (their investments), starts at value 1

• Pick one asset to devalue to 0

• Threshold is $v_i = \theta v_i$ for all $i$

• Look at resulting cascade
Diversification Preview: Dangerous Middle Levels

• Low diversification:
  • fragmented network, no widespread contagion

• Medium diversification
  • Connected network, contagion is possible
  • Exposure to only a few others makes it easy to spread

• High diversification
  • Little exposure to any single other organization
  • Failures do not spread
Diversification and Contagion: 93% threshold, c=.5

Percent of Orgs that Fail

Degree: Expected # of cross-holdings
Diversification and Contagion: various thresholds

Percent of Orgs that Fail

Degree: Expected # of cross-holdings
Proposition 1: Diversification

Consider a regular random network where organizations have in and out degree (an integer adjacent to) $d$, and common threshold $v$ and integration $c$, and asset values 1. Drop an asset to 0.

If $c(1-c) < 1-v$, then there is no contagion.

Otherwise,

If $d < 1$, then there is no limit contagion.

If $1 \leq d < \frac{c(1-c)}{(1-v)}$, there is limit contagion.

If $\frac{c}{(1-v)} < d$, then there is no limit contagion.
Diversification

- *non*monotonicity: middle ranges of connections maximize contagion

- Competing forces:
  - Increased diversification increases component size
  - Increased diversification decreases spread from one organization to neighbor

- Degree that maximizes contagion is increasing with threshold
Integration

• Low integration: little exposure to others, failures don’t trigger others

• Middle integration: exposure to others substantial enough to trigger contagion

• High integration: difficult to get a first failure – failure of own assets does not trigger failure
Integration: .93 threshold

Percent of Orgs that Fail

Degree: Expected # of cross-holdings
Integration: .93 threshold

Percent of Orgs that Fail

Degree: Expected # of cross-holdings
High Integration - First Failures, threshold .8

Frequency of first failures

Degree: Expected # of cross-holdings
Integration

- Increases exposure to others
- Decreases exposure to own idiosyncratic risks
- Increases contagion, but can decrease first failures
Proposition 3: Integration

Conditional upon a first failure, trades at `fair’ prices (determined by values relative to initial p’s) that weakly increase $A_{ij}$ for all $i$ and $j$ (i.e., integrations) weakly increase the number of organizations that fail in any cascade.

Integration that decreases $A_{ii}$ decreases circumstances of own-asset induced first failures.
Proposition 3: Integration

This is a general result - intuition:

increasing integration increases the number of organizations exposed to bankruptcy costs of failing organizations

trades at fair prices do not change any of their initial market values
Summary on Cascades: Integration and Diversification:

- **A first failure:** some organization needs to fail own risk with integration

- **Initial Contagion:** some neighbors need to be sufficiently exposed to fail too
  exposure w integration exposure w diversification

- **Interconnection:** to continue to cascade widely, the network must have sufficiently large components
  connectedness w diversification
• Model

• Cascades: Diversification and Integration

• Endogenous Values and Moral Hazard (skip)

• An Illustration with European Debt Data
Illustrating Application

• Consider 6 key countries in Europe that have substantial cross holdings of each other’s debt

• Treat them as an isolated system (illustrating exercise, not for policy...)

• See what happens if one of them fails
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<th></th>
<th>France</th>
<th>Germany</th>
<th>Greece</th>
<th>Italy</th>
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<td>0.13</td>
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<td>0.07</td>
<td>0.11</td>
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</table>
Cascades

• Set \( v_i \) to be a fraction \( \theta \) of 2008 GDP

• Look at 2011 GDP as initial \( p_i \)

• Calculate \( v_i \)

• Calculate cascades in best equilibrium

• Have \( b_i = v_i / 2 \) (could rescale everything to debt levels – here set to GDP levels)
## Normalized GDPs

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<th>2011</th>
<th>Drop %</th>
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<td>11.62</td>
<td>3</td>
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<td>14.88</td>
<td>3</td>
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<td>5</td>
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<td>Portugal</td>
<td>1.06</td>
<td>1.00</td>
<td>6</td>
</tr>
<tr>
<td>Spain</td>
<td>6.70</td>
<td>6.25</td>
<td>7</td>
</tr>
<tr>
<td>θ fraction</td>
<td>.90</td>
<td>.93</td>
<td>.935</td>
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<tr>
<td>------------</td>
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<tr>
<td>First Failure</td>
<td>Greece</td>
<td>Greece</td>
<td>Greece</td>
</tr>
<tr>
<td>Second Failure</td>
<td></td>
<td>Portugal</td>
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</tr>
<tr>
<td>Fifth Failure</td>
<td></td>
<td></td>
<td>Italy</td>
</tr>
</tbody>
</table>
European Debt

- Portugal fragile: little exposure, but close to threshold
- Portugal triggers Spain, triggers France, Germany
- Italy is last to *cascade*: held by others, but much less exposed to Spain than France, Germany (but exposed to Fr, G)
Conclusions

• Values need to be derived from cross holdings carefully

• Diversification and Integration both face (different) competing effects, nonmonotonicities

• Model can serve as a foundation for studying bailouts and incentives...

• Can be taken to data...
Summary on Cascades: Integration and Diversification:

• **A first failure:** some organization needs to fail
  ↓ own risk with integration

• **Initial Contagion:** some neighbors need to be sufficiently exposed to fail too
  ↑ exposure w integration  ↓ exposure w diversification

• **Interconnection:** to continue to cascade widely, the network must have sufficiently large components
  ↑ connectedness w diversification
Extra Slides
Outline

• Model

• Cascades: Diversification and Integration

• Endogenous Values and Moral Hazard

• An Illustration with European Debt Data
Endogenous Values:

\[ C = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix} \quad \text{and} \quad A = \hat{C} (I-C)^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \]

- Let \( p_1 = p_2 = 10 \) (D=I) and so \( v_1 = v_2 = 10 \)

- What if \( v_1 = 8 \) and \( v_2 = 11 \) and \( b_2 = 6 \) ?

- Without any intervention, 2 loses 6
Bailout:

\[
C = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix} \quad A = \hat{C} (I-C)^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}
\]

- Without any intervention, incur a cost 6 and so the values become: \( v_1 = 8 \) and \( v_2 = 6 \)
  
  \((2 \text{ bears } 2/3 \text{ of the cost and } 1 \text{ bears } 1/3)\)
Bailout:

\[ C = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}, \quad A = \hat{C} (I - C)^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \]

- Without any intervention, incur a cost 6 and so the values become: \( v_1 = 8 \) and \( v_2 = 6 \)
  
  (2 bears 2/3 of the cost and 1 bears 1/3)

- If instead 1 gave a \$ to 2, then values are
  
  \( v_1 = 9 \) and \( v_2 = 11 ! \)
• Suppose that 2 can choose:
  \[ b_2 \] either 2 or 6
  \[ v_2 \] either 10 or 11

Only gets a transfer if \( b_2 = 6 \) and \( v_2 = 11 \)

Best off by choosing highest bankruptcy cost and threshold
Bailouts: No free lunch
Proposition 4

Suppose at prices $p$, org $i$ is closest to first failure and would just fail at prices $\lambda p$.

For any fair trades of assets or cross holdings that change $A_i$, there exists some $p'$ in any neighborhood of $\lambda p$ such that $i$ now fails at $p'$ but did not before.

So, avoiding $i$’s failure will require unfair trades if it covers any neighborhood of prices.
• L
Integration: .96 threshold

Percent of Orgs that Fail

Degree: Expected # of cross-holdings
Proposition 2: Integration

Suppose that 1-c of every organization is owned privately for some $c \leq 1/2$. Then:

$A_{ji}$ is decreasing in $c$, and

$A_{ij}$ is nondecreasing in $c$, and increasing if $i$ and $j$ are path connected.
``First Failure Frontier of organization 1''
First Failure Frontier
``Second Failure Frontier'':
If org 2 fails, 1 loses $50/3$.
More fragile
Prices for which cascades occur due to organizations’ interdependencies.
1 and 2 fail for sure

Cascades in the best case equilibrium
Multiple Equilibria
Example

- Two organizations n=2

- Each own half of each other

\[ \hat{C} = \begin{bmatrix} .5 & 0 \\ 0 & .5 \end{bmatrix} \quad \text{C} = \begin{bmatrix} 0 & .5 \\ .5 & 0 \end{bmatrix} \]

\[ A = \hat{C} (I-C)^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \]

How much of j’s asset value ‘belongs’ to investors of i
Multiple Equilibria

\[ v = A \ (Dp - b(v)) \]

Multiple solutions:  multiple equilibria
Outline

• Model

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Discontinuities:

- Costs need not be large:
  - Small changes in one organization's value can trigger discontinuities in others’ values
  - Cascades occur with small changes in values, when margins are low
Let us now examine the equilibria.

Examine when it is that failure of one organization leads to a cascade.
Example

\[
C = \begin{bmatrix}
0 & 0.75 & 0.75 \\
0.85 & 0 & 0.10 \\
0.10 & 0 & 0
\end{bmatrix}
\]

\[
\hat{C} = \begin{bmatrix}
0.05 & 0 & 0 \\
0 & 0.25 & 0 \\
0 & 0 & 0.15
\end{bmatrix}
\]

\[
A = \hat{C} (I - C)^{-1} = \begin{bmatrix}
0.18 & 0.13 & 0.15 \\
0.77 & 0.83 & 0.66 \\
0.05 & 0.04 & 0.19
\end{bmatrix}
\]
Example

- Two organizations \( n=2 \)
- Each own half of each other

\[
C = \begin{bmatrix}
0 & 0.5 \\
0.5 & 0
\end{bmatrix}
\]
Example

Holdings of 1 by 2: \( C_{21} = .85 \)

\( \hat{C}_{11} = .05 \) private investors

\( C_{31} = .10 \)
Example

\[ C + \hat{C} \]
Example

1 dollar flows into 1
Example

Follow the .85 from 2
Example

Iterating:
\[ A = \hat{C} (I-C)^{-1} \]

\[ A_{11} = 0.18 \]
\[ A_{12} = 0.13 \]
\[ A_{13} = 0.15 \]

Diagram:
- Node 1 connected to 2 with weight 0.77, to 3 with weight 0.05.
- Node 2 connected to 3 with weight 0.04, back to 1 with weight 0.83.
- Node 3 connected to 1 with weight 0.66, to 2 with weight 0.19.
Example

\[ v = \hat{C} (I - C)^{-1} Dp \]

unit assets

\[ 2.26 = v_2 \]

\[ 0.46 = v_1 \]

\[ 0.28 = v_3 \]
survives

fails

$p_1 < v_1$

$p_1 > v_1$

1 fails

1 survives
\[ p_2 > v_2 \]

2 survives

\[ p_2 < v_2 \]

2 fails
First Failure Frontier
1 and 2 fail for sure

No cascades and no multiple equilibria
1 and 2 fail for sure
Cascades in the best case equilibrium

1 and 2 fail for sure
1 and 2 fail for sure

Cascades in the best case equilibrium
After Trade

Multiple equilibria

1 and 2 fail for sure
Current factor values
Current factor values
\(A_{11} p_1 + A_{12} p_2 < v_1\) 

1 fails

\(A_{11} p_1 + A_{12} p_2 > v_1\)

1 survives
\[ A_{21} p_1 + A_{22} p_2 > v_2 \]

2 survives

\[ A_{21} p_1 + A_{22} p_2 < v_2 \]

2 fails
First Failure Frontier
First Failure Frontier
New First Failure Frontier
The diagram illustrates the relationship between two price variables, $p_1$ and $p_2$. The lines labeled $FF_1$, $FF_1'$, $FF_2$, and $FF_2'$ represent different equilibrium points in the market. The arrows indicate the direction of change in these variables, suggesting adjustments in response to market conditions.
Before Trade

Multiple equilibria

1 and 2 fail for sure
Multiple equilibria

1 and 2 fail for sure

After Trade
After Trade

Multiple equilibria

1 and 2 fail for sure
Comparison

New Multiple Failure Regions

Old Multiple Failure Regions
Price tomorrow with probability 0.25

Key

- Price Today
- Price tomorrow with probability 0.25
Trades at fair prices cannot save B from bankruptcy with the bad price realization.
But A may be prepared to engage in trade favorable to B.
After this trade B looks fairly safe again.
However, if the bad price realization occurs, we are back in the same situation as before.
Suppose A bails out B again.
B then looks fairly safe again.
But if the bad price realization occurs, both organizations are now at risk.
In the counterfactual of no trade, B would have gone bankrupt but A would be safe.
\[ C = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix} \quad \hat{C} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \]

\[ A = \hat{C} (I-C)^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \]
$T = \begin{bmatrix}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{2} & \frac{1}{2} & 0 \\
\frac{1}{2} & 0 & \frac{1}{2}
\end{bmatrix}$