Measuring the Performance of Market-Based Credit Risk Models

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Agenda

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II. A Brief History of Credit Risk Models

III. Credit Model Performance Measures
   i. Independent Obligors
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IV. Relationship Between Price and Credit Risk

V. CDS Spread-based Models for Credit Transitions
Disclaimer

• The models and analyses presented here are exclusively part of a research effort intended to better understand the strengths and weaknesses of various approaches to evaluating model performance and interpreting credit market pricing data.

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Introduction
Themes

• Questions:
  – What credit risk information is embedded in the market price of a credit-risky instrument?
  – How does the market price-derived credit risk measure complement fundamental credit analysis?

• Related Questions:
  – How do you measure the effectiveness of a model for determining credit risk?
  – What credit market price information is available now to drive models?
Definition and Focus of Credit Risk

• Credit Risk – The risk of not receiving timely principal and interest payments set forth according to financial contracts

• Primary focus of credit models:
  – Single Obligor Default Risk (Probability of Default – PD)
  – Single Obligor Credit Quality Transition/Deterioration Risk (Transition Matrices)
  – Portfolio Credit Risk (Correlation, Default Dependencies)
  – Recovery Estimation (Loss Given Default – LGD)
  – Exposure at Default (Bank Loan Portfolios)
Evolution of Credit Models - A Brief Incomplete History
Evolution of Credit Models – 1980’s and earlier

• The credit business was mostly ‘buy and hold’. Investment grade corporate bond portfolios for institutional investors and retail bond funds, investment grade portfolios of corporate and prime consumer loans. Trading was often driven by interest rate risk.

• Late 80’s saw rise of high yield bonds and early CDOs – first credit derivatives.

• Shorting or hedging credit risk was difficult – limited ability to sell loans (some syndications of large loans to highly rated companies).

• Analysis and Modeling: Fundamental, qualitative assessment of individual obligors – ‘loan to those you know’. Capital charges typically fixed. Large concentration risk and inefficient use of capital in bank portfolios. Illiquid debt market – not much to calibrate models to. Merton Structural Models for Probability of Default and bond pricing based on equity markets had not found practical application.
1990s - The Expansion of Credit Markets

• Credit Derivatives begin to grow – total return swaps on bonds, credit default swaps, CBOs, synthetic CDOs referencing bank loan portfolios – first opportunities to efficiently short, hedge and securitize credit risk to create customized credit risk profiles.

• Corporations tend to more leverage, lower credit quality – fewer extremely high grade bond issuers. Investors attracted to highly rated securitized debt. Securitization of consumer loans (mortgages, auto, credit cards, student loans, …) grows.

• Concept of ‘Active Credit Portfolio Management’ forms – Credit Value-at-Risk, risk-adjusted capital allocation, marginal capital for investment decisions, measures of concentration risk and diversification benefits. Banks measure economic capital.
1990s – The Rise of Credit Default Models

- Merton-style Structural Models for Probability of Default prove effective and commercially viable with KMV as an industry leader. This provides a more dynamic, equity market-based view of credit quality to compliment fundamental analysis.

- Reduced form default intensity models to price bonds and options are introduced (Jarrow and Turnbull).

- Regression-based PD models incorporating firm-specific financial ratios and macro-economic variables prove effective, particularly for private firms when sufficiently large default databases are collected.

- Mortgage foreclosure frequency and loss severity models appear based on consumer characteristics and loan properties.
1990s – The Rise of Credit Portfolio Models

• KMV and RiskMetrics develop credit portfolio models in structural model framework with joint default dependencies derived from equity market correlations. KMV model captures changes in portfolio value due to both credit quality transitions and default and becomes a benchmark economic capital model for large banks.

• Default Time models with a Gaussian copula used to create joint default dependencies are introduced (Li) and widely adopted for pricing portfolio credit derivatives.

• Basel I is adopted to bring uniformity to capital measures in the banking industry and Basel II development begins.
2000 - 2007 – Active Credit Markets Grow Rapidly

- Structured Finance markets experience a huge growth in securitization of mortgages, including new mortgage products (subprime, Alt-A, ARMs, etc.).
- CLOs market grows fueled by increasing leveraged loan lending and private equity.
- CDS market explodes and overtakes cash bond market in notional traded.
- CDS indices introduced creating a liquid index market, as well as a liquid index securitization market for tranches.
- Numerous credit derivative products are introduced or grow in popularity including ABS CDOs, SIVs, CDPCs, CPDOs, etc.
- By mid-decade, credit spreads are extremely tight and investors turn to new products for higher yield.
2007 – 2010: Credit Crisis Stresses Financial System

• In 2007 the housing bubble bursts, property values collapse sharply, and mortgage default rates begin to rise. RMBS bonds and CDOs back by RMBS bonds deteriorate sharply in credit quality, leading to many defaults and great loss in value.

• In 2008, financial institutions with large mortgage exposure either fail (Lehman, Bear Stearns, Countrywide), are subject to distressed take-overs (Merrill Lynch, WAMU, Wachovia), or received extraordinary government support (AIG, Fannie, Freddie).

• For part of this period, credit markets freeze with little lending and extremely high credit spreads.

• Private mortgage securitization market mostly disappears, along with new-issuance in CLOs.
2000s – Wide Spread Adoption of Quantitative Credit Models

• KMV’s EDFs become widely accepted as predictors of default (KMV acquired by Moody’s in 2002).

• Other PD models are developed commercially (S&P, Kamakura, etc.)

• Credit Portfolio models are increasingly used for active portfolio management

• Default Time/Gaussian copula model becomes industry standard for pricing and hedging synthetic CDOs and index tranches with the introduction of ‘base correlation’ idea

• Semi-Analytic numerical methods speed index tranche calculations

• Top-Down portfolio models are introduced for pricing index tranches to address Gaussian copula calibration issues

• Credit valuation models are introduced that price illiquid loans and bonds based on PDs and estimates of market price of risk

• Consumer asset credit models further developed
Credit Modeling Today

- **Studying Probability of Default and Credit Transition Models**
  - e.g. applying information decay theory for PDs at longer horizons

- **Developing new or updated models for a range of assets:**
  - Residential mortgages, commercial mortgages, municipal bonds, SMEs/private firms, consumer assets

- **Incorporating credit marketing pricing data in models:**
  - CDS spreads, Bond OAS, House Price Appreciation indices, etc.

- **Understanding fair credit value vs market prices that incorporate liquidity risk, counterparty risk and supply and demand**

- **Expanding credit portfolio models to cover more asset classes, better dependence modeling, credit cycle effects, correlated recoveries, changes in value due to credit migration, etc.**

- **Improving measures of counterparty risk, systemic risk and contagion.**
Credit Model Performance Measures
Credit Model Performance Measures

• Credit model performance is often determined by a model’s ability to:

  – Rank obligors by default/downgrade risk to discriminate between potential defaulters and non-defaulters

  – Anticipate realized default/downgrade rates: compare probabilities to observed rates

• Performance evaluated with statistical measures on a validation data set

• Method 1: Cumulative Accuracy Profile (CAP)

  – Sort obligors from riskiest to safest as predicted by the credit model (x-axis) and plot against fraction of all defaulted obligors. Accuracy Ratio = B / (B + A)
Credit Model Performance Measures

- **Method 2: Receiver Operating Characteristic (ROC)**
  - Plot the distribution of model scores, R, for defaulters and non-defaulters
    - Note: For a perfect model, no overlap in the distributions whereas for a random / uninformative model, 100% overlap (i.e., identical distributions)
  - Specify a cutoff value C such that scores less than C are potential defaulters and rank scores higher than C are potential survivors
  - Given C, 4 outcomes are possible:
    - Incorrect decisions: R < C and survive (Type II error) or R > C and default (Type I error)
    - Correct decisions: R < C and default or R > C and survive
Credit Model Performance Measures

- **Method 2: ROC (cont.)**
  - Define Hit Rate (given C), \( H_C = \frac{\text{# of predicted defaulters}}{\text{total defaulters}} \)
  - Define False Alarm Rate (given C), \( F_C = \frac{\text{# of non-defaulters predicted to default}}{\text{total non-defaulters}} \)
  - Plot \( H_C \) vs. \( F_C \) for all values of C to generate the ROC curve

- The larger the ROC (i.e. the area under the ROC curve - shaded region), implying higher hit rate to false alarm rate, the better the ranking model
  - Note: Hit rate corresponds to 1 – Type I Error
Credit Model Performance Measures

- Relationship between ROC and CAP
  - If the same weight is attributed to Type I vs. Type II errors, it can be shown that ROC and CAP communicate the same information
    \[ AR = 2 \times ROC - 1 \]
  - ROC is however a more general measure as different weights may be given to Type I and II errors.
    - Typically, more weight may be given to Type I vs. Type II error
      - Example: Giving a loan to a defaulting firm (I) vs. losing potential interest income by not extending credit to a non-defaulting firm (II)

We assume equivalence of ROC and CAP for this study (i.e. equal weights to I vs II errors)
Performance Measures: Independent Obligors
Rankings based on PD vs. Default Prediction

• Typical measures: focus on correct identification of defaulters vs. non-defaulters (binary)

• Credit models: rankings may be based on relative risk (probability or likelihood)

Implication: A perfect (PD based) ranking model cannot have a perfect Accuracy Ratio.

– Even the highly ranked “buckets” can have defaults (albeit low)
– The lowest ranked buckets can have survivors (1-PD)

• Example

– Very large pool categorized into 10 equally weighted Risk Category (RC) buckets on the basis of PD
– Safest bucket (RC1) has PD 2bp, RC2 has PD 4bp, RC3 has PD 8bp, and so on i.e., each subsequent bucket twice as risky in PD terms
– Large pool (theoretically infinite) ensures realized defaults in each RC bucket equal expected defaults
### Rankings based on PD vs. Default Prediction (Large Pools)

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<thead>
<tr>
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<th>PD (%)</th>
<th>% Defaulters</th>
<th>% Obligors</th>
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<td>10.24%</td>
<td>50.05%</td>
<td>10.00%</td>
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<td>RC9</td>
<td>5.12%</td>
<td>75.07%</td>
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<td>2.56%</td>
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<td>30.00%</td>
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<td>1.28%</td>
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<td>40.00%</td>
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<td>RC6</td>
<td>0.64%</td>
<td>96.97%</td>
<td>50.00%</td>
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<td>RC5</td>
<td>0.32%</td>
<td>98.53%</td>
<td>60.00%</td>
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<td>RC4</td>
<td>0.16%</td>
<td>99.32%</td>
<td>70.00%</td>
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<td>RC3</td>
<td>0.08%</td>
<td>99.71%</td>
<td>80.00%</td>
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<tr>
<td>RC2</td>
<td>0.04%</td>
<td>99.90%</td>
<td>90.00%</td>
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<tr>
<td>RC1</td>
<td>0.02%</td>
<td>100.00%</td>
<td>100.00%</td>
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</table>

Even with perfect risk categorization on a PD basis, accuracy ratio is only 71.66%.

Default rate matters: If the PDs for all buckets are 5 x larger, CAP curve remains the same, but the AR = 78.19%.
Additional Observations for Large Pools

• Even if obligors are categorized on the basis of "noisy" PD estimates, all results hold provided expected PD for each bucket is monotonic
  • Large pools of uncorrelated obligors diversify away "idiosyncratic" defaults perfectly making realized defaults always equal to expected defaults

• Buckets defined in terms of PD ranges – both overlapping and non overlapping will yield similar results
  • PDs range can be viewed equivalent to a noisy PD
    • Small "noise" component can be made to correspond to distinct PD ranges
    • Large "noise" component can be made to correspond to overlapping PD ranges
Finite Sample Size: Impact of Smaller Pools

- Size of the pool has most impact on AR values.
- Similar results hold for smaller pools as large pools with regard to PD distribution within buckets, both non-overlapping and overlapping.
Finite Sample Size: Impact of PD levels

• Study impact on AR of reducing absolute PD levels (and PD differences) across buckets
  – Base case: Safest bucket 2 Bps PD; each subsequent bucket PD 2x preceding one
  – Other cases: Safest bucket 2 Bps; subsequent buckets 2.5x, 1.5x, 1.25x, 1.1x

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<th>2x (Base Case)</th>
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<td>AR</td>
<td>S.E</td>
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<td>S.E</td>
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<td>63.43%</td>
<td>0.097%</td>
<td>11.08%</td>
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• Absolute levels of PD determine the level of heterogeneity in the pool
  – Small PDs cause larger variability in number of realized defaults, including frequent instances of low or no defaults
  – Small PD differences across buckets imply more homogenous pools, making default rates similar across buckets
  – Both factors impact AR negatively, even though the categorization is "perfect" in PD terms
Performance Measures: Correlated Obligors
Correlation

Fact: - Correlation does not affect a pool’s expected defaults, …
    - … but correlation introduces more variability in realized defaults

Recall: AR being a non-linear function of defaults, i.e., $E_D(AR) \neq AR(E_D)$, any change in shape of the default distribution introduced by correlation can be expected to impact AR

Correlation modeled in the context of a Systemic Risk factor model

- For a pool of N obligors, most generic model would be to specify a joint distribution of $[u_1, u_2, \ldots, u_N]$, $u_i \sim U[0,1]$ through a copula function and define obligor $i$ as a default if $u_i < PD_i$

- Factor model representation allows for systemic/idiosyncratic separation

$$\varepsilon_i = \sqrt{\rho_i} Z + \sqrt{1-\rho_i} \phi_i$$

Note: Z may be dependent on several independent factors. We assume a single factor model for the following analysis.
Correlation (continued)

- Key feature of model: Conditional independence i.e., conditioned on systemic latent factors:
  - Unconditional PDs for correlated obligors become conditional PDs for independent obligors
    - A given period’s observation corresponds to a specific systemic latent factor realization and can be viewed as a description of a given state of the economy
  - Unconditional results can be viewed as an average (over systemic factors) of conditional, independent results
Correlation (continued)

Base Case Example: $Z \sim N(0,1)$, $\phi_i \sim N(0,1)$

Unconditional PDs

<table>
<thead>
<tr>
<th>Bucket 1</th>
<th>Bucket 2</th>
<th>Bucket 3</th>
<th>Bucket 4</th>
<th>Bucket 5</th>
<th>Bucket 6</th>
<th>Bucket 7</th>
<th>Bucket 8</th>
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<th>Bucket 10</th>
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<tr>
<td>PD (%)</td>
<td>10.24%</td>
<td>5.12%</td>
<td>2.56%</td>
<td>1.28%</td>
<td>0.64%</td>
<td>0.32%</td>
<td>0.16%</td>
<td>0.08%</td>
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Conditional PDs:  
$$PD_i(Z) = \Phi\left(\frac{\Phi^{-1}(PD_i) - \sqrt{\rho_i}Z}{\sqrt{1-\rho_i}}\right)$$

$\rho = 20\%$  \hspace{1cm} $\rho = 80\%$
Correlation: Large Pool

- AR can be calculated using semi-analytic methods
  - Given Z, determine conditional PDs.
    - PDs determine exact default rates for each bucket
    - AR|Z can be calculated analytically
  - Integrate AR|Z over Z to obtain unconditional ARs for different values of ρ.

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<th>2x</th>
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- Main result: Increasing correlation improves AR for large pools, except for maximum correlation of 100%
  - Large pool eliminates variance around conditional default rates
  - Increasing correlation introduces more heterogeneity in conditional PDs for negative shocks
Correlation: Finite pools – 1000 observations per Bucket

Simulation method: Estimate mean AR over 100,000 trials

- Conditioned on Z, finite pool buckets have a default rate distribution instead of a point mass for large pools, bringing into play the effect of the non-linear relationship between AR and default rates, $E_D(AR) \neq AR(E_D)$

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<tr>
<th>Correlation</th>
<th>AR</th>
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<th>AR</th>
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- Main observation: Correlation improves mean AR for heterogeneous (in PD) portfolios but is detrimental to homogenous portfolios
Recap

• Typical performance measures such as AR/CAP or ROC may not be very informative in measuring performance of credit rankings based on relative risk (probability or likelihood of default)
  – Perfect (PD-based) ranking models do not have perfect ARs.
  – ARs may vary over time simply due to variations in population default rates

• Typical measures are more meaningful when used for comparing different categorization models over the same data set

• Correlated changes in credit risk - resulting in very few or large numbers of obligor defaults - make Typical measures very volatile and difficult to interpret.
Price and Credit Risk
Price and Credit Risk

- A security’s price depends on a complex interaction of several risk factors:
  - Credit Risk (default, transition, recovery, correlation, exposure size)
  - Interest Rate Risk
  - Liquidity
  - Demand and Supply Conditions
  - Other factors such as reinvestment / prepayment risks, etc. depending on the type of security (MBS for example).

- Rarely is it possible to fully ascertain the contribution of each risk driver to a security’s price – even for simple securities

- Arbitrage pricing models make certain assumptions related to price –

  **Example: CDS spreads reflect default risk**
  - $E [ PV \text{ of spread payments } ] \text{ (fixed leg)} = E [ PV \text{ of losses } ] \text{ (floating leg)}$
  - Expected payment values depend on probability of default
  - Pricing Theory: Cashflow discounting done using risk-free rate while probabilities of default and expected recoveries are computed under the ‘Risk Neutral’ measure
  - Any impact of factors such as liquidity, demand/supply imbalances, systemic risk and market price of risk, etc. that impact CDS spreads get embedded in the implied risk neutral default intensities (measure of credit risk)
• Model interpretations of default risk, perhaps adequate representations of reality in normal times, may break down in stressed environments.
  – CDS spreads for very stable firms impacted in similar ways to more risky firms at the peak of the crisis
  – Across the board re-pricing of all types of structured securities, regardless of the type of assets backing these securities.
    ▪ Senior CLO tranches being quoted at spreads wider than the average spread of the underlying loan portfolio.
Price and Credit Risk

• Bond markets showed similar patterns

Source: Bloomberg.
© Standard & Poor’s 2010.
Case Study: Senior CLO Tranche Prices During Credit Crisis

• Some notes:
  – Structured securities trades typically executed over the counter that make prices opaque (i.e. absence of a marketplace that could enable price discovery).
  – Inherently complex: Not only require predicting the likely evolution of the underlying collateral in the future but also accurately modeling the structure waterfall itself.
  – More prone to ‘model risk’

• Analyze the possible range of default rates, discount premia, market Sharpe ratio, etc. implied by quoted prices of senior CLO (AAA) tranches
  – Several CLO structures were analyzed with senior tranches quoted about 70¢.
  – Horizon of 10 years.
  – Results are presented here for a specific structure with the lowest break-even default cushion
  – Collateral of CLOs is typically a mix of ‘BB’ and ‘B’ rated loans
I. Implied Constant Annual Default Rates (CADR):

- Assume a range of CADR scenarios from 0% to 25% in increments of 1% and 4 levels of Recovery Rates (RR), (50%, 40%, 30% and 20%)
  - Historical average realized recoveries on typical loan portfolios have been in excess of 60% (makes even the most benign 50% recovery case a stressed scenario from a historical perspective)
- For each pair of CADR-RR combination, estimate the cashflows to the senior tranche in the form of interest and principal payments (incorporating all specifics of the transactions such as principal payments due to OC triggers, etc.)
- Value series of tranche cash flows for each CADR-RR pair using a range of Discounting Spreads (DS).
Case Study: Senior CLO Tranche Prices During Credit Crisis

• Combination of CADR-RR-DS that produce a discounted cash flow value of 70¢
II. Defaults occurring early in the life of a transaction have more severe impact – Front Loaded defaults

– During stressed times expectations in the market are usually of a spike in defaults rates followed by a gradual recovery from the crisis.

• Consider speculative grade default rates from the 1930s (historical peak) and sort defaults in descending order to create our base FL default case.

  – Historically (from what is available from public records), the highest default rate recorded over a 10 yr period in the US was decade of the 1930s.

• Generate tranche cash flows conditional on the FL default scenario and value using a range of discounting spreads.

• Result:

  – Even if the worst 10 year period in US economic history repeated itself, senior CLO tranches (investigated) did not suffer any loss of promised principal or interest payments.

  – Stressing recoveries further, senior tranches did not take hits except for 20% recovery case where it suffered loss of a few cents
Case Study: Senior CLO Tranche Prices During Credit Crisis

- Stress default rates from the Depression era by a factor of 25% and 50%
  - Even under these extreme scenarios, only under the 30% and 20% recovery cases did senior tranches suffer small losses
- Similar results when FL scenario created from 10 highest historical annual speculative grade default years (sorted in descending order and corresponding to the years 1933, 1932, 2001, 1991, etc.)

<table>
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<th>Discount Spread (Bps)</th>
<th>'Great Depression'</th>
<th>'Great Depression' x 1.25</th>
<th>'Great Depression' x 1.5</th>
<th>10 Worst Default Years</th>
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Price Implied Market Sharpe Ratio

- Estimate a market Sharpe ratio (market price of risk) in the context of the Merton (1974) structural model.
  - Structural model provides a framework for pricing securities through a transformation between physical probabilities of default (PD) and their risk neutral counterparts

\[ P_t^Q = \Phi \left( \Phi^{-1}(P_t) + \Lambda \sqrt{\rho_t} \right) \]

- Approach: Start with assumption that market expectation is a repeat of the Great Depression era default rates.

- Uncertainty around this expectation modeled in the form of ‘residual’ correlation among the underlying loans in the CLO asset pool (pool is assumed to be 200 homogenous names).

- Numerically solve for \( \Lambda \) in equation above for values of \( \rho \) given base assumption that \( P_t \) corresponds to the 10 year ‘Great Depression’ speculative grade default rate witnessed during the 1930’s.
  - \( \Lambda \) estimation done subject to the constraint that risk neutral default distribution of collateral portfolio prices the representative senior CLO tranche to about 70¢.
Price Implied Market Sharpe Ratio

- Market Sharpe ratio corresponding to various correlation assumptions (5%-20%) estimated in the range of 1.3 to 2.6.
  - Historically, $\Lambda$ from corporate bond prices estimated in a range of 0.25 to 0.5 [Bohn(2000)]
  - Estimates of ex-post Sharpe ratios from equity markets computed around 0.3 [French et al (1987), Fama and French (1989)]

- Market prices of risk/Sharpe ratios could diverge across different asset classes with capital markets known to be less than perfectly efficient - however, extent of deviation for CLO tranche price based measure points to a level of risk aversion not supported by ‘fundamentals’.

- Currently senior CLO tranches trade in the low 90s.

Prices reveal useful information related to credit risk; market inefficiencies/sentiment however cloud the picture of actual credit risk.
CDS Spread-based Models for Predicting Credit Transitions
CDS Market Derived Signals

• **Modeling Objective**
  – Use CDS spreads to determine a market view of credit quality and evaluate relative to a fundamental credit analysis benchmark (S&P Credit Ratings)

• **Model**: Use CDS data to determine a statistical relationship between CDS spreads and S&P rating levels
  – Adjust for CW/OL, Industry (GICS classification), Region, and Document Type

• **Model Output**:
  – Representative CDS spreads for each rating level (credit curve)
  – The market-implied credit ranking for a firm given its CDS spread and the value of the other covariates (CW/OL etc)
CDS Market Derived Signals: Model

• Ordinary Least Squares regression to determine the coefficients that best explain the relationship between observed log-spreads and rating, firm and contract details.
  – \( \text{Log(Spread)} = \text{Piecewise linear function of S&P rating (on a numeric scale)} \)
    + Adjustment for CW/OL status
    + Adjustment for GICS classification
    + Adjustment for Document Type
    + Error term

• Model made more robust through use of Bayesian prior on the model parameters
  – Ensure there is always a solution, even when data is missing for some ratings, GICS, CW/OLs or DOCs
  – Create an appropriate balance between continuity to past data and fidelity to current data
    ▪ If today’s data are few, then rely to some degree on previous day’s results
    ▪ Let historical relationship between parameters have some influence on how today’s parameters relate to each other

• Compute CDS implied rank for a firm by solving for numerical rank that gives actual spread in above model with zero error.
CDS Market Derived Signals: Discrepancy

- Discrepancy is difference between the computed rank and the actual S&P rating (Computed Rank – S&P Rating).

- Some of the questions we try to investigate:
  - Does a large positive discrepancy suggest that the market is anticipating an S&P downgrade or rating action?
    - Similarly, does a large negative discrepancy suggest that the market is anticipating an upgrade?
  - Are day-to-day discrepancies as informative as persistent discrepancies?
  - How may large discrepancies be interpreted?
    - Does the CDS market have a substantially different perspective on the firm’s credit risk?
    - Is the CDS market responding to non-credit aspects regarding the firm (liquidity, price volatility, and non-credit arbitrage/relative value opportunities)?
CDS Market Derived Signals: Results

- Discrepancy on a given day vs. subsequent change in rating within 6 months

- Discrepancies appear to convey more information on adverse credit changes than credit improvements

- Larger the discrepancy, greater the proportion of downgraded names
CDS Market Derived Signals : Results

• Issues with the approach
  – Daily discrepancies are volatile
    ▪ Large discrepancies on a given day very often decline in magnitude before the actual rating action.
    ▪ Number of instances where discrepancies are large is small i.e. fraction of downgraded/upgraded names based on small number of events

• Look at persistent discrepancies to address volatility issues
  – Compute a moving average of discrepancy to smooth the deviations
  – Define certain criteria for persistence
    ▪ Average/Min/Max of (MA of) discrepancy exceeds some threshold for a chosen period length (Signal Persistence Period) before the credit change
  – If persistent discrepancy preceded a credit change, consider MDS information as relevant
CDS Market Derived Signals : Results

- We see instances in the data where CDS MDS seems to provide informative signals of credit changes
CDS Market Derived Signals: Results

- But we see (more) instances of false positives i.e. discrepancies reducing over time and CDS MDS converging to S&P rating.

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For Fortune Brands Inc. and CenturyTel Inc., the graphs show the trend of ratings over time. The blue line represents the MDS (Market-Driven Signals) and the green line represents the rating. The graphs illustrate how the MDS converges to the rating over time.
CDS Market Derived Signals: Results

• Also, S&P rating changes are very frequently accompanied by announcements related to Outlook/Credit Watch
  – CDS market expects subsequent rating actions in the future and prices this expectation
  – Future changes in credit driven primarily by CW/OL status

• Controlling for CW/OL, less than 10% of persistent discrepancies (of 4 notches or more) precede credit changes for sub-investment grade names.
  – Corresponding number for investment grade names is about 6%

• Overall, instances of signals unrelated to credit quality changes are very high
CDS Market Derived Signals: Results

• **Accuracy Ratio for MDS as predictor of credit changes**
  
  – Rank order names on the basis of discrepancy and compute AR from the proportion of names that actually had a credit change
    
    
    ▪ For downgrades, the optimal signal had an AR = 45%.
    
    ▪ For upgrades, AR = 30%
    
    ▪ ARs for high yield downgrades (50%) and investment grade upgrades (40%) are higher
    
    ▪ ARs may be biased upwards as signals that lead to opposite credit changes are not penalized

• **The caveats discussed earlier about Accuracy Ratio as a measure of credit model performance apply here also.**
References and Credits


Many thanks to:

S&P’s Quantitative Analytics Research Group