Quantitative Measurement and Management of Liquidity Risk in a Banking Context

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Presentation to PRMIA / CIRANO Luncheon, Montreal, Quebec
November 2010

The views expressed herein are those of the authors and do not necessarily represent the views of the Office of the Comptroller of the Currency or the Department of the Treasury.
Outline

• Introduction to Liquidity Risk Management and Measurement

• Conceptual Considerations in Liquidity Risk

• Quantitative Frameworks for Managing Liquidity Risk

• A Simple Model for Liquidity Risk

• An Option Theoretic Model for Liquidity Risk

• References
“Liquidity” or the related risk is not well-defined, but we try:

- **Definition 1**: Liquidity represents the capacity to fulfill all payment obligations as and when they come due, to the full extent and in the proper currency, on a purely cash basis.

- **Definition 2**: Liquidity risk is the danger & accompanying potential undesirable effects of not being able to accomplish the above.

- This basic definition says that liquidity is neither an amount or ratio, but degree to which a bank can fulfill its obligations, in this sense a qualitative element in the financial strength of a bank.

- Characteristic of liquidity is that it must be available *all the time* (not just on average) – failure to perform, while a low probability event, implies potentially severe or even fatal consequences to the bank.

- While liquidity events occur more frequently and with less severity than extreme stress events, they are sufficiently dangerous to disrupt business and alter the strategic direction of a bank.
Introduction to Liquidity: Review of the Literature

- Knies (1876): stress the necessity for a cash buffer to bridge negative payment gaps between inflows and outflows where timing is uncertain
- Stutzel (1959, 1983): further discussions focusing primarily on basic considerations between liquidity and solvency
- Duttweiler (2008): an holistic view of liquidity risk including quantitative methods (LaR & VaR)
Introduction to Liquidity: Empirics and Stylized Facts*

Table 1.4: Pairwise Correlations for Top 200 and 5 Largest Banks Risk Proxies (Call Report Data 1984-2008)

<table>
<thead>
<tr>
<th>Risk Pair</th>
<th>Aggregate Banks²</th>
<th>JP Morgan Chase</th>
<th>Bank of America</th>
<th>Citigroup</th>
<th>Wells Fargo</th>
<th>PNC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit and Liquidity Risk</td>
<td>53.43%</td>
<td>19.07%</td>
<td>47.87%</td>
<td>31.47%</td>
<td>2.30%</td>
<td>20.85%</td>
</tr>
<tr>
<td>Operational and Liquidity Risk</td>
<td>15.33%</td>
<td>7.37%</td>
<td>-8.55%</td>
<td>11.76%</td>
<td>-4.85%</td>
<td>-10.22%</td>
</tr>
<tr>
<td>Market and Liquidity Risk</td>
<td>11.27%</td>
<td>1.56%</td>
<td>-18.23%</td>
<td>6.29%</td>
<td>-0.94%</td>
<td>-3.21%</td>
</tr>
<tr>
<td>Interest Rate and Liquidity Risk</td>
<td>18.97%</td>
<td>19.96%</td>
<td>9.17%</td>
<td>12.38%</td>
<td>9.14%</td>
<td>12.86%</td>
</tr>
</tbody>
</table>

- Proxies for risk types from quarterly financial data: credit = gross chargeoffs, operational = other non-interest expense, market = deviation in 4-quarter avg. trading revenues, interest rate = deviation in 4-quarter avg. interest rate gap (rate on deposits – loans), liquidity = deviation in 4-quarter avg. liquidity gap (deposits-loans)
- Generally, this measure of liquidity risk exhibits positive and sometimes substantial correlation with other risk types

Introduction to Liquidity: Empirics and Stylized

- Measure as ratio of liquid assets to total value of banking book
- Secular decline in 20 years going into recent crisis (except 1990 spike)
- But now rapid & massive increase from depth to end of crisis?

Introduction to Liquidity Risk Management and Measurement

Short-term Liquidity
• Capability of a bank to fulfill payment obligations as and when they occur (“situation-specific” liquidity)
• Strong secondary condition related to profitability (“classical view” concerning liquidity)

Long-term liquidity
• Capacity to borrow long-term funds at appropriate spreads to support asset growth (“structural liquidity”)
• At present, very much the focus of most banks

Tradability
• Permanent tradability of capital market products without undue price concessions
• The focus of supervisors and academic researchers in the 1990’s

Market liquidity
• Market capacity to provide the base for borrowing in money and capital markets
• Focus is on danger cause by major events (e.g., 9/11, Lehman Bankruptcy)

• The different types / meanings of liquidity and how they are related to each other (adapted from Bartetzky, 2008, page 9)
Liquidity Risk in Relation to the Risk Management Universe

- Operational Risk
  - Inadequate organizational structures
  - Incorrect data
  - Inadequate models

- Credit Risk
  - Lending risk
  - Counterparty risk
  - Issuer risk

- Customer Risk
  - Call risk
  - Forward risk
  - Behavioral risk

- Event Risk
  - Legal risk
  - Political risk
  - Country risk

- Business Risk
  - Strategic risk
  - Reputational risk

- Market Risk
  - Interest rate risk
  - Price risk
  - Option risk

- Liquidity Risk
  - Call liquidity risk
  - Term liquidity risk
  - Funding liquidity risk
  - Market liquidity risk

- Liquidity in relation to other banking risks (Bartetzky, 2008, page 11)
Liquidity risk is generally thought to be a function of these:

- Volume & tenor of assets as dictated by business policy
- Extent of tenor mismatch between assets & liabilities
- Stability of deposit base (e.g., retail but short term & low volume)
- Optionality of bank assets (e.g., undrawn commitments)
- Persistence of liquidity gap implies bank will have to get funding
- Terms of later financing not known in advance
- Even marketable assets can have different liquidity over time
- Willingness of market to fund depends on future bank state
- Bank financial state & perception depends upon inter-related data (e.g., risk profile, solvency, profitability & trend)
- Bank’s forecast of self state & market perception of this are both risky & uncertain
Conceptual Considerations: How to Look at Liquidity

• *Level of aggregation* includes several dimensions (amount, currency, time): advisable to maintain as detailed information as possible, even if not required by controllers or supervisors

• *Natural* (legal maturities) vs. *artificial* liquidity (shiftability & marketability—"business view") & meeting customers’ needs & keep the franchise intact (not just surviving as a legal entity)

• *Optionalities*: apply not only to traded or OTC options but any bank commitment that puts the bank in a position of an option seller exposed to unpredictable cash requirements

• The impact of *business policy* on liquidity: the need for a dynamic perspective & accounting vs. actual cash flows
Conceptual Considerations: 
Liquidity Risk & Other Risks

- **Solvency** is the condition of sufficient capital to cover loss, a pre-condition but not sufficient for **liquidity** (*perceived* solvency can impact liquidity through investors/customers actions)

- **Liquidity vs. interest rate risk**: it seems that they can be treated similarly with respect to gap analysis, but for a typical bank’s exposure it is unlikely that they form a predictable relationship
  - E.g., prepayment: excess liquidity but reinvestment rate risk; CP line triggered: may have hedged rates with FRA but cash changes hands

- **Liquidity vs. market liquidity risk**: the ability to transform marketable assets into cash depends upon the breadth and depth of markets
  - Depends on quality of issuer, market conditions & asset characteristics

- Ways to close the liquidity gap: international FIs-”asset driven”, savings banks-”liability driven” (but latter depends on behavior of lenders – depends on name of institution & market)
## Segway to Quantification: The Liquidity Balance Sheet

<table>
<thead>
<tr>
<th><strong>Liquid assets</strong> (up to 5 days)</th>
<th><strong>Short and medium term liabilities</strong> (maturities up to 1 year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intra-day:</td>
<td>Deposits from banks and similar groups</td>
</tr>
<tr>
<td>Cash and collateral at central banks</td>
<td>Short term securities like CD and CP</td>
</tr>
<tr>
<td>Further:</td>
<td>Maturing long-term securities</td>
</tr>
<tr>
<td>Assets at banks and similar groups, available in 5 days</td>
<td>Repo transactions coming due</td>
</tr>
<tr>
<td>Marketable assets within 5 days and unencumbered</td>
<td>Subordinated debts and comparable instruments falling due &lt; 1 year</td>
</tr>
<tr>
<td>Maturing reverse repo transactions</td>
<td>Senior debts falling due &lt; 1 year</td>
</tr>
<tr>
<td>Marketable assets within 1 year and unencumbered:</td>
<td></td>
</tr>
<tr>
<td>for repo transactions</td>
<td></td>
</tr>
<tr>
<td>for sales</td>
<td></td>
</tr>
<tr>
<td>for securitization</td>
<td></td>
</tr>
<tr>
<td>Portion contingent liabilities with maturities up to 1 year</td>
<td>Retail deposits not stable</td>
</tr>
<tr>
<td>Customer assets falling due within 1 year and exceeding customer franchise</td>
<td>Wholesale deposits not stable</td>
</tr>
<tr>
<td>Trading portfolios reducible within 1 year</td>
<td></td>
</tr>
<tr>
<td>Tangible fixed assets close to sale</td>
<td><strong>Stable funding</strong> (to stay &gt; 1 year)</td>
</tr>
<tr>
<td>Stable funding</td>
<td>Shareholder capital and reserves stable for &gt; 1 year</td>
</tr>
<tr>
<td><strong>Less liquid assets</strong> (6 days up to 1 year)</td>
<td></td>
</tr>
<tr>
<td>Assets at banks and similar groups, available in 5 days</td>
<td></td>
</tr>
<tr>
<td>Maturing reverse repo transactions</td>
<td></td>
</tr>
<tr>
<td>Marketable assets within 1 year and unencumbered:</td>
<td></td>
</tr>
<tr>
<td>for repo transactions</td>
<td></td>
</tr>
<tr>
<td>for sales</td>
<td></td>
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<tr>
<td>for securitization</td>
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<tr>
<td>Portion contingent liabilities with maturities up to 1 year</td>
<td></td>
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<tr>
<td>Customer assets falling due within 1 year and exceeding customer franchise</td>
<td></td>
</tr>
<tr>
<td>Trading portfolios reducible within 1 year</td>
<td></td>
</tr>
<tr>
<td>Tangible fixed assets close to sale</td>
<td></td>
</tr>
<tr>
<td><strong>Assets not liquid</strong> (&gt; 1 year)</td>
<td></td>
</tr>
<tr>
<td>Tangible assets to be kept or planned</td>
<td>Subordinated debts and comparable instruments maturing &gt; 1 year</td>
</tr>
<tr>
<td>Due from banks and similar groups with maturities &gt; 1 year</td>
<td>Secured senior debts due longer than 1 year</td>
</tr>
<tr>
<td>Portion of contingent liabilities &gt; 1 year</td>
<td>Unsecured senior debts due longer than 1 year</td>
</tr>
<tr>
<td>Portion conduits or other units externally financed</td>
<td>Other funds due longer than 1 year</td>
</tr>
<tr>
<td>Level of franchise on secured basis</td>
<td></td>
</tr>
<tr>
<td>Level of franchise on unsecured basis</td>
<td></td>
</tr>
<tr>
<td>Customer assets which exceed franchise with maturities &gt; 1 year</td>
<td></td>
</tr>
<tr>
<td>Reserves in hand of liquidity management</td>
<td></td>
</tr>
<tr>
<td>Stable funding</td>
<td></td>
</tr>
</tbody>
</table>
A Quantitative Framework for Managing Liquidity Risk

- Quantification of liquidity risk encompasses 4 key aspects:
  - Behavior of the balance sheet structure under various circumstances
  - Basic conditions required when applying a dynamic business perspective
  - Forecasting the most likely liquidity gaps
  - Linking these findings to liquidity policy

- 2 basic occurrences lead to increased liquidity risk:
  - Asset: marketable assets lose quality or segment for which is congested
  - Liability: funding potential declines

- Key components of a quantitative framework:
  - Dynamic (planned & future developments need to be accounted for)
  - Distinguish between normal & stressed environments
  - Stressed circumstances can vary in quality
  - Heterogeneity of impact upon particular banks
  - Forward looking orientation
A Quantitative Framework for Managing Liquidity Risk (cont.)

• The *Liquidity Condition*: the capability to fulfill all obligations as and when they come due in each currency & period:

\[ ELE_t - LaR_t^\alpha + CBC_t > 0 \]

• ELE\(_t\) is the *Expected Liquidity Exposure* in time \(t\), the difference between expected negative and positive cash flow:

\[ ELE_t = ECF^+_t - ECF^-_t \]

• LaR\(_t^\alpha\) is the *Liquidity at Risk*, the deviations of in- and out-flows due to specific circumstances in period \(t\), which like VaR focuses on the downside (i.e., danger of outflows exceeding inflows at some high confidence level 1-\(\alpha\))

• CBC\(_t\) is the *Counter-Balancing Capacity* containing asset buffers which can be readily converted to liquidity (e.g., sale, repo, collateralization, etc.) or capability to renew existing contracts or new funds from other 3\(^{rd}\) parties
• CBC\(_t\) may be decomposed into the sum of asset (or funding) liquidity A, sale liquidity S and repo liquidity R (the latter 2 comprising balance sheet liquidity):

\[ CBC\(_t\) = A + (S + R) \]

• We may state this equivalently as that CBC (e.g., asset reserves or funding slack) needs to exceed the sum of future exposures:

\[ CBC\(_t\) > -\left(ELE\(_t\) - LaR\(_t\)\right) \]

• We can adjust the formula for nostro balances kept for payment purposes, which at day end if positive (negative) we will invested (borrow):

\[ CBC\(_t\) > -\left(FLE\(_t\) - LaR\(_t\)\right) = -\left([ELE\(_t\) + FLE\(_{t-1}\)] - LaR\(_t\)\right) \]

• Where FLE\(_t\) is *forward liquidity exposure* in period t

• Further adjustments to these are made to make this dynamic
Calculating \( \text{ELE}_t \) requires analysis of instruments with respect to amounts and timings of cash flows, which may be random.

- **Case 1**: time & amount determined (e.g., interest/principle payments on fixed rate funded loan commitments & deposits)
- **Case 2**: time determined & amount random (e.g., dividends, floating legs of swap arrangements)
- **Case 3**: time random & amount determined (e.g., loans with flexible payment agreements - PIKs)
- **Case 4**: time & amount random (e.g., unfunded revolving commitments, savings deposits, own account trading exposure, clearing for 3\(^{rd}\) parties)

**Key Adjustment**: based upon the above analysis, decompose the \( \text{ELE}_t \) into deterministic and random components:

\[
\text{ELE}_t = \text{ECF}^D_t + \text{ECF}^R_t = \text{ECF}^D_t + \left( \text{ECF}^F_t + \text{ECF}^V_t + \text{ECF}^H_t \right)
\]

- \( F \): floating (linked to market prices), \( V \): virtual (not related to market or observable factors), \( H \): hypothetical (new business, credit or operational risk events)
Firm can refinance in any period with any maturity but cannot repurchase bonds early.

If cannot roll-over debt then a restructuring cost of 16 (project’s low market liquidity increases this risk, a type of “funding illiquidity”).

2 strategies, short/long or long/short: while interest rate management gives no guidance, we can show that the former is optimal!

A 3 period project costing 100 & NPV = 103 follows a binomial tree with probability ½ of up/down move.

Owner has no capital and must issue only zero coupon debt of maturity 1 or 2.

Continuum of risk neutral bondholders who can invest in the risk-free asset at zero interest rate.
• Bonds sold at 1b will be at a discount to value 101 to compensate for default at 3d

• Therefore, in period 0 firm pursues short-long strategy, firm value is 103, bonds are worth 100 and there is equity of 3

• This illustrates the common case where for new projects, firms may opt for initial short financing until states of nature are revealed

• Under long-short financing, if wind up in 2c can’t refinance ("debt overhang") -> value = 99 - 16 = 83 -> initial value = .25*107+.5*103+.25*83 = 99 -> can’t finance project

• Short-long: if land on 1b -> period 0 NPV = 103 -> can sell 2 period discount bonds worth 100 to cover this
A Simple Model of Liquidity Risk (continued)

• Assume discrete time indexed by \( t \) and an infinitely lived project with value \( X_t \) that is a Martingale (the increments \( X_{t+1}/X_t \) are i.i.d.) & there is continuous compounding at constant rate \( r \):
  \[
  X_t = e^{-r} E_t[ X_{t+1} ]
  \]

• Assume that the project matures at random time \( T \) that is distributed geometric, and independent of \( X_t \), having survival function:
  \[
  \Pr(m) = \Pr(T > t + m | T > t)
  \]

• The project is financed with zero-coupon bonds through maturity \( m \), bonds of only one kind are issued in any given period, there are no repurchases prior to maturity and the firm holds no cash

• Suppose outstanding bonds of face value \( F_t \) in period \( t \) when the project has not matured – if the firm cannot roll over at \( t < T \) the pays a restructuring cost of \( \lambda X_t \) for \( 0 < \lambda < 1 \). If we denote the borrowing capacity by \( V_t \), then then rollover constraint is defined by:
  \[
  V_t \geq F_t
  \]
A Simple Model of Liquidity Risk (continued)

• It follows that the borrowing capacity of a firm issuing \(m\) period bonds with face value \(F_{t+m}\) at time \(t\) is the sum of the value of the project minus the present value of future restructuring costs:

\[
V_t(m) = (1 - \Pr(m))E_t\left[e^{-r_m}X_{t+m}\right] + \Pr(m)E_t\left[e^{-r_m}(V_{t+m} - 1_{V_{t+m} < F_{t+m}})\lambda X_{t+m}\right]
\]

\[
= X_t - \Pr(m)E_t\left[e^{-r_m}(1_{V_{t+m} < F_{t+m}})\lambda X_{t+m} - V_{t+m} + X_{t+m}\right]
\]

• The bond price is then the expected payoff at maturity minus the current cost of restructuring:

\[
D_t(m) = (1 - \Pr(m))E_t\left[e^{-r_m}\min(F_{t+m}, X_{t+m})\right] + \\
\Pr(m)E_t\left[e^{-r_m}\min(F_{t+m}, V_{t+m} - 1_{V_{t+m} < F_{t+m}})\lambda X_{t+m}\right] - 1_{V(m) < F_t}\lambda X_t
\]

• If the rollover constraint is satisfied in period \(t\), then there exists a maturity \(m^*\) and face value \(F_{t+m^*}\) such that \(D_t(m^*) = F_t\)

• This implies that face value is endogenous variable dependent on financing policy that is increasing in in rollover risk

• It can be shown that the optimal policy is for the firm to maximize borrowing capacity \(V_t(m)\) subject to the rollover constraint
A Simple Model of Liquidity Risk (continued)

- Proposition: An optimal financing strategy is to issue an m period bond if borrowing capacity minus restructuring costs, $V_t(m) - 1_{(V_t(m) - F_t)} \lambda X_t$, is maximized. Additionally, net worth subtracts the bond value, $V_t(m) - D_t(m) - 1_{(V_t(m) - F_t)} \lambda X_t$.

- If the rollover constraint is satisfied, net worth is maximized by maximizing $V_t(m)$ subject to $D_t(m) = F_t$.

- If not, the firm restructures, it is still a matter of maximizing the value of debt $D_t(m) = V_t(m) - \lambda X_t$ in $V_t(m)$, therefore we get:

$$V^*_t = \max_{m \in \{1, \ldots, M\}} V_t(m)$$

- Proposition: An optimal financing strategy is to issue
  - (i) bonds of the shortest maturity as long as subsequent rollover is guaranteed, and
  - (ii) bonds of maximum maturity when net worth is zero and restructuring is inevitable.

- Firms should rollover short-term debt if subsequent refinancing is guaranteed in subsequent periods, but as soon as rollover risk becomes significant hedge this risk by seeking long term financing.
An Option Theoretic Model for Liquidity Risk

- We postulate a reference process (e.g., an equity index) that follows G.B.M.:
  \[
  \frac{dS_t}{S_t} = (\mu - r)dt + \sigma dW_t, \quad \ln \left( \frac{S_t}{S_0} \right) \sim N \left( \left( \mu - \frac{1}{2} \sigma^2 - r \right)t, \sigma^2 t \right)
  \]

- This shadows perceived value of the illiquid asset, and we are concerned that long-term investors may withdraw funds if it falls below a threshold.

- Let \( B \leq S_0 \): the barrier, \( c = B/S_0 \): the complement of the % decline that triggers the option and \( \tau \) be the maturity, so that 1st passage option payoff is:
  \[
  \min_{t \in [0, \tau]} S_t = \begin{cases} 
  > cS_0 = B & \Rightarrow 0 \\
  \leq cS_0 & \Rightarrow e^{r\tau}B = K
  \end{cases}
  \]

- The risk-neutral (\( \mu = r \)) price of this option is given by:
  \[
  \Pi_{FP} \left( S_0, B, \sigma, r, \tau \right) = e^{-r\tau} \left( KN \left( -d_2 \right) + S_0 e^{r\tau} N \left( -d_1 \right) \right)
  \]

  \[
  d_1 = \frac{1}{\sigma \sqrt{\tau}} \ln \left( \frac{S_0}{K} \right) + \tau \left( r + \frac{\sigma^2}{2} \right) = d_2 + \sigma \sqrt{\tau}
  \]

- The price per $1 notional is given by:
  \[
  \Pi_{FP} \left( 1, B, \sigma, r, \tau \right) = \frac{\Pi_{FP} \left( S_0, B, \sigma, r, \tau \right)}{S_0} \leq 1
  \]
An Option Theoretic Model for Liquidity Risk (continued)

• Assume a T period investment horizon with m equally spaced liquidity intervals of length \( \tau = T_i - T_{i-1} \):

\[
0 = T_0 \leq T_1 \leq \cdots \leq T_m = T = m\tau
\]

• Model an illiquid instrument as a long position in the liquid reference process with price \( S_{\text{disc}} < S_0 \), and a short position in m-period liquidity option of value \( S_0 - S_{\text{disc}} \), a collection of m-1 1st passage options (a “cliquet option”)

• If we define a cash call event as a log return of -100% over the entire period T, then we can solve for the barrier as a proportion of initial value as:

\[
c = \frac{B}{S_0} = \exp\left( -\sqrt{\frac{\tau}{T - \tau}} \right)
\]

• This shows the barrier to be higher (lower) for investors with shorter (longer) liquidity intervals \( \tau \), and to be increasing in maturity \( T \)

• Denote \( C(t) \) as the cash in the replicating portfolio at period \( t \): then for \( T_i < T \) (at \( T \) the asset matures and no payoff) the payoff on the \( i \)th option is:

\[
\min_{t \in [T_{i-1}, T_i]} S_t = \begin{cases} 
> c C(T_{i-1}) \Rightarrow 0 \\
\leq c C(T_{i-1}) \Rightarrow e^{r(T_i - T_{i-1})} C(T_{i-1})
\end{cases}
\]
An Option Theoretic Model for Liquidity Risk (continued)

• The price of the option in the 1st interval is just as before:

\[ \Pi_{FP,1}(S_0, B, \sigma, r, \tau) = \Pi_{FP}(1, B, \sigma, r, \tau)S_0 - \Pi S_0 \]

• The expected cash position at the end of the initial period is:

\[ E_0(C_1) = S_0e^{rT} - Be^{rT} \text{Pr}\left(\min_{t\in[0, T_i]} S_t \leq B\right) = S_0e^{rT_i}(1 - \Pi) \]

• So the time 0 price of the second period option with this notional is:

\[ \Pi_{FP,2}(S_0, B, \sigma, r, \tau) = \Pi(1 - \Pi)S_0 \]

• Continuing on this reasoning, and imposing that under risk-neutral measure \( E(S(T_i))=e^{(\mu-r)T_i}S_0=S0 \) as \( \mu=r \), we get for small \( \Pi \) the approximation:

\[ \Pi_{FP,m-1}(S_0, B, \sigma, r, \tau) = S_0\Pi \sum_{k=0}^{m-1} (1 - \Pi)^k = S_0\left(1 - (1 - \Pi)^{m-1}\right) \approx (m - 1)\Pi \]

• If the number of liquidity intervals is \( 1 = T/\tau \), then the barrier is \( c = B/S_0 = 0 \), the value of the liquidity option is zero & there is no price difference between the liquid and illiquid assets

• Conversely, if the investor needs immediate liquidity \( \tau = 0 \) (an infinite number of liquidity intervals), then the barrier is \( c = B/S_0 = 1 \), and the price of the liquidity option is 100% the notional \( \Pi(.)=S_0 \)
An Option Theoretic Model for Liquidity Risk (continued)

- The value of the liquidity option decreases (increases) in duration of the liquidity interval (lockup period), but is more sensitive to decreases (increase) in liquidity interval (lockup period) at longer (shorter) lockup periods (liquidity intervals).

- The value of the liquidity option increases in both the duration of the lock-up period and the volatility, and more sensitive to either volatility or lock-up period at elevated values of lock-up period or volatility.

- The value of the liquidity option decreases in the duration of the liquidity interval and increases the volatility, and more sensitive to volatility at lower levels of the liquidity option, and more sensitive to the liquidity interval at elevated values of volatility.

### Table 1: Liquidity Option Price vs. Lockup Period and Liquidity Interval (Volatility = 20%)

<table>
<thead>
<tr>
<th>Liquidity Interval</th>
<th>Lockup Period (Years)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Week</td>
<td>0.00%</td>
<td>3.67%</td>
<td>43.09%</td>
<td>91.42%</td>
<td></td>
</tr>
<tr>
<td>1 Month</td>
<td>0.00%</td>
<td>0.63%</td>
<td>10.41%</td>
<td>39.70%</td>
<td></td>
</tr>
<tr>
<td>1 Quarter</td>
<td>0.00%</td>
<td>0.09%</td>
<td>2.40%</td>
<td>12.18%</td>
<td></td>
</tr>
<tr>
<td>6 Months</td>
<td>0.00%</td>
<td>0.01%</td>
<td>0.62%</td>
<td>4.27%</td>
<td></td>
</tr>
</tbody>
</table>

### Table 2: Liquidity Option Price vs. Lockup Period and Volatility (Liquidity Interval = 1 Month)

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Lockup Period (Years)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td>0.00%</td>
<td>0.63%</td>
<td>10.41%</td>
<td>39.71%</td>
<td></td>
</tr>
<tr>
<td>30%</td>
<td>0.47%</td>
<td>28.45%</td>
<td>81.32%</td>
<td>98.51%</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: Liquidity Option Price vs. Liquidity Interval and Volatility (Lockup Period = 2 years)

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Liquidity Interval</th>
<th>1 Week</th>
<th>1 Month</th>
<th>1 Quarter</th>
<th>6 Months</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>20%</td>
<td>3.67%</td>
<td>0.63%</td>
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<td>0.01%</td>
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</tr>
<tr>
<td>30%</td>
<td>82.90%</td>
<td>28.45%</td>
<td>6.60%</td>
<td>1.42%</td>
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</tbody>
</table>
References

- Knies, Karl (1876) *Geld und Credit II*, Abteilung Der Credit, Leipzig.
- Reitz, Stefan (2008), Moderne Konzepte zur Liquiditätsrisikos,
  in Baretsky, Peter, Gruber, Walter and When, Carsten (eds), *Handbuch Liquiditätsrisiko: Identifikation, Messung und Steuerung*, Schaffer-Poeschel Verlag, Stuttgart, pages 121-140.