Ambiguous Persuasion

by D. Beauchêne, J. Li, and M. Li

Maxim Ivanov

McMaster

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Preliminaries

- Consider communication between a sender and receiver
- Both players hold prior belief p_0 about an unknown state ω
- The sender selects a signal structure $\pi(m|\omega)$ that provides information in message m about ω
- Upon observing m, the receiver takes an action a, which affects players' payoffs
- The sender selects the signal structure, which maximizes her ex-ante payoff

Preliminaries

- Suppose first that both players are *Bayesian* (Kamenica and Gentzkow, 2011)
- Each message m induces a Bayesian posterior belief

$$p = p(m) = \Pr(\omega|m) = \frac{p_0(\omega)\,\pi(m|\omega)}{\tau(m)}$$

- The receiver takes an action that maximizes his posterior payoff

$$a = \hat{a}(p) \in \arg\max_{a \in A} E_p[U(a,\omega)]$$

- Both p and $\hat{a}(p)$ result in the sender's *posterior payoff*

$$V(p) = E_p[v(\hat{a}(p),\omega)]$$

Preliminaries

- Any distribution of posterior beliefs $\{\tau(m), p(m)\}$ must be Bayes plausible

$$E_\tau[p(m)] = p_0.$$

- The *optimal* distribution $\{\tau^*(m), p^*(m)\}$ provides the ex-ante payoff $\bar{V}(p_0)$, where

$$\bar{V}(p) = \sup\{z | (p,z) \in co(V(p))\}$$

  is the concave closure of $V(p)$.

- The persuasion is valuable if $\bar{V}(p_0) > V(p_0)$

Preliminaries: ambiguous signal structures

- Suppose the sender adds another signal structure $\pi'(m|\omega)$ and randomizes between π and π'
  - the receiver is uninformed whether a message m is sent by π or π'
- Randomization does *not* benefit the sender
- A convex combination of signal structures is an (*ambiguous*) signal structure

$$\pi'' = \alpha\pi + (1-\alpha)\pi'$$

Main question

What is the value of ambiguous persuasion if both players have maxmin preferences?

Model: maxmin preferences

- Upon receiving a message m, the receiver builds the *set* of Bayesian posteriors P_m for all signal structures $\{\pi_k\}_{k=1}^K$ in the ambiguous device

$$P_m = \left\{ p_m^k | p_m^k = \frac{p_0(\omega)\,\pi_k(m|\omega)}{\tau_k(m)} \right\}$$

and takes an action

$$\hat{a}(P_m) \in \arg\max_a \min_{p_m^k \in P_m} E_{p_k}[U(a,\omega)]$$

- Similarly, the sender has maxmin preferences. Given a set of signal structures $\{\pi_k\}_{k=1}^N$ in the ambiguous device, his ex-ante payoff is

$$EV = \min_k E_{\tau_k} E_{p_m^k}[v(\hat{a}(P_m),\omega)]$$

Key trade-off

- For maxmin preferences, adding an extra signal structure π' makes a difference
- On one side, the sender can be **hurt** by π':

  - if $\hat{a}\left(P_m\right)$ is unaffected by π', the sender's ex-ante payoff can only decrease

$$EV = \min_k E_{\tau_k} E_{p_m^k}\left[v\left(\hat{a}\left(P_m\right), \omega\right)\right]$$

- On the other side, the sender can **benefit** from π':

  - π' affects the set of Bayesian posteriors P_m

  - a modified P_m can result in the more favorable actions $\hat{a}\left(P_m\right)$ for some message

  - this can potentially increase the sender's ex-ante payoff

The value of ambiguous persuasion

- **Main result 1**: the paper provides the maximum ex-ante payoff EV of the sender across all ambiguous signal structures
- EV has a clear geometric meaning

The value of ambiguous persuasion

- Consider the sender's posterior payoff

$$\upsilon\left(p, P_{-1}\right) = E_p\left[v\left(\hat{a}\left(P\right), \omega\right)\right], \text{ where } P = p \cup P_{-1}$$

for a given posterior belief p and a set of $K-1$ posterior beliefs P_{-1}.

- Denote $V\left(p, P_{-1}\right)$ the concave closure of $\upsilon\left(p, P_{-1}\right)$

$$V\left(p, P_{-1}\right) = \sup\left\{z \in \mathbb{R} \middle| \left(P, z\right) \in co\left(\upsilon\left(p, P_{-1}\right)\right)\right\}$$

- Let $\bar{V}\left(p\right)$ be max projection of $V\left(p, P_{-1}\right)$ on a single dimension of beliefs

$$\bar{V}\left(p\right) = \max_{P_{-1} \in (\Delta\Omega)^{K-1}} V\left(p, P_{-1}\right).$$

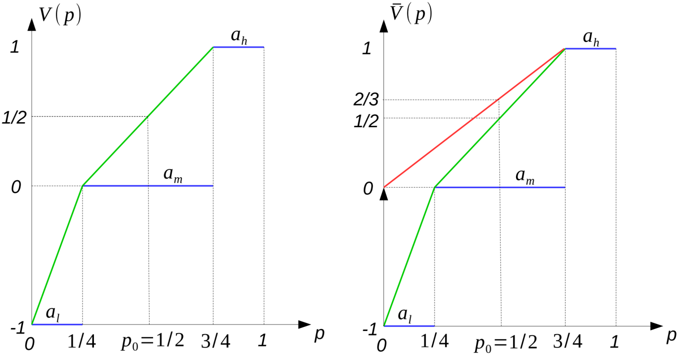
- Then, the sender's maximum ex-ante payoff is $\bar{V}\left(p_0\right)$

The value of ambiguous persuasion: leading example

	ω_l	ω_h
a_l	$-1, 3$	$-1, -1$
a_m	$0, 2$	$0, 2$
a_h	$1, -1$	$1, 3$

- Two states: ω_l, ω_h
- Prior belief: $p_0 = \Pr\{\omega_h\} = \frac{1}{2}$
- Sender's preferences: $v(a_h) > v(a_m) > v(a_l)$
- Receiver's preferences:
  - a_l, a_h are risky
  - a_m is safe

The value of ambiguous persuasion: leading example

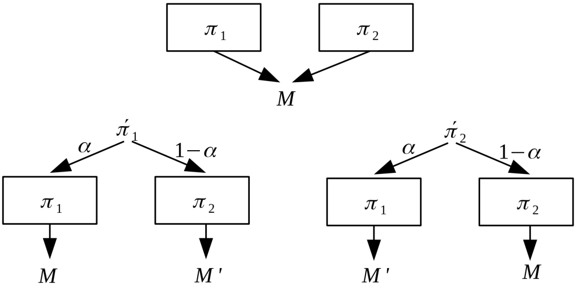


- $\pi_1 \rightarrow p\left(m_l\right) = 0, p\left(m_h\right) = 3/4$; $\pi_2 \rightarrow p\left(m_l\right) = 1/4, p\left(m_h\right) = 3/4$
- Suppose the sender uses the ambiguous device: $\{\pi_1, \pi_2\}$
  - Good news: $\hat{a}\left(m_l\right) = \hat{a}\left(0, 1/4\right) = \hat{a}\left(1/4\right) = a_m$
  - Bad news: $EV = \min\left\{EV\left(\pi_1\right), EV\left(\pi_2\right)\right\} = \min\left\{2/3, 1/2\right\} = 1/2$

Tool: synonyms

- Thus, EV can potentially achieve 2/3
- This requires modifying signal structures. How?
- A solution: using synonyms
  - (Strong synonyms) messages m and m' induce identical sets of posterior beliefs $P_m = P_{m'}$
  - (Weak synonyms) messages m and m' induce identical receiver's actions $\hat{a}(P_m) = \hat{a}(P_{m'})$

Synonyms



- $\pi_1' = \alpha\pi_1 \oplus (1-\alpha)\,\pi_2, \pi_2' = (1-\alpha)\,\pi_2 \oplus \alpha\pi_1$
- Naturally, $EV(\pi_i') = \alpha EV(\pi_1) + (1-\alpha)\,EV(\pi_2)$
- $P_{m_l} = P_{m_l'} = \{0, 1/4\}, P_{m_h} = P_{m_h'} = \{3/4, 3/4\},$
- As $\alpha \to 1$, both $\pi_1' \to \pi_1$ and $\pi_2' \to \pi_1$.
- Hence, $\min\{EV(\pi_1'), EV(\pi_2')\} \to EV(\pi_1) = 2/3$

Synonyms are necessary

- **Main result 2**: If optimal ambiguous persuasion is valuable, then weak synonyms are *necessary*
  - Intuitively, synonyms are needed to hedge against low-payoff signal structures
  - They preserve the desired sets of posteriors (or receiver's actions) across messages
- How many signal structures are needed for the optimal ambiguous persuasion? Only two.

Conclusion

- The paper provides the sharp characterization of optimal persuasion with maxmin preferences of players
- It provides the necessary and sufficient tools for the solution
- It demonstrates how synonyms and ambiguity in messages appear endogenously in communication
- Ideas are clear and intuitive *ex-post*, but (very) non-trivial *ex-ante*

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Comments

- Ambiguous persuasion is more effective than Bayesian persuasion, but it is more complicated
- It requires more complicated signal structures and a bigger message space (as dictated by maxmin preferences of the sender)

  - this problem can be relaxed in the case of the Bayesian sender
- It requires randomizing among signal structures (as dictated by maxmin preferences of the receiver)

  - An ambiguous device is a mixture over signal structures. It is an element in

$$\Delta\pi = \Delta(\Delta p) = \Delta(\Delta(\Delta\Omega))$$

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- How to implement ambiguous devices in practice?
- If the marginal cost of implementation is C, is it lower than the marginal benefit of ambiguous persuasion:

$$\bar{V}(p_0) - V(p_0) \gtrless C$$

- What can be achieved with simple signal structures, say, deterministic ones?

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