Meetings and Mechanisms

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    - (or: how do we hire a new assistant professor?)

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  - How do I sell my house?
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- Mechanism design literature provides answer for monopolistic seller.
  - Organize an auction to extract as much surplus as possible.
- However, competition is a crucial feature of many markets and changes incentives.
  - If I try to extract too much surplus, buyers will go to a competitor.

Search Literature

- Search literature provides a theoretical framework, which has been used to study various aspects of the matching process, e.g.
  - Price determination.
  - Role of information frictions.
  - Dynamic considerations.
- However, competition in a decentralized environment leads to new questions, which remain relatively unexplored:
  - How do buyers and sellers meet in the first place?
  - How does this process affect outcomes?

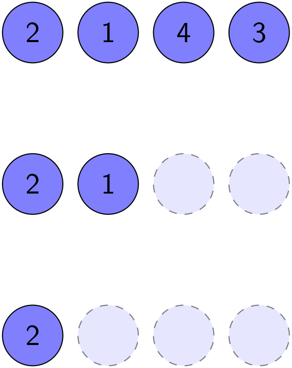
Meeting Technologies

- Markets differ in whether a seller can meet buyers simultaneously.

capacity $\to \infty$ (auction site)

$1 <$ capacity $< \infty$ (labor market)

capacity $= 1$ (bazaar, bar)

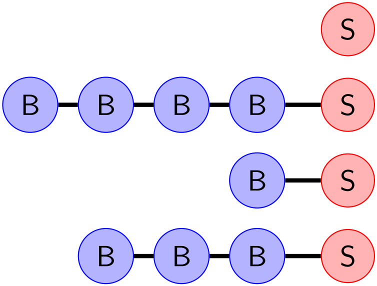


Examples

- Housing market: many-to-one, but viewings are costly.
- Durable consumer goods market: bilateral (e.g. car dealers).
- Online goods/services: many-to-one (eBay) or bilateral (Airbnb).
- Labor market: many-to-one, but firms screen subset of applications.
  - EOPP data: 5 out of 14 applicants.
  - Burks et al. (2014): 10% of 1.4 million applicants.
  - Agrawal et al. (2014): new platforms like Upwork facilitate many-on-one meetings in markets where meetings used to be bilateral, creating scope for different wage mechanisms like auctions.

Standard Approach

- Except for a few exceptions, every paper in the literature simply makes—without too much motivation—one of two assumptions:
  1. urn-ball meetings (Poisson-to-one).
  2. bilateral meetings (one-to-one).



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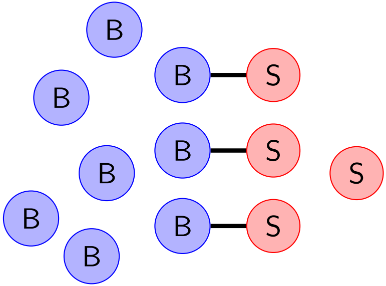

Examples

- Adverse selection and liquidity.
  - **Bilateral**: Guerrieri, Shimer and Wright (2010), Chang (2014).
  - **Urn-ball**: Auster and Gottardi (2016).
- Sorting between heterogeneous agents.
  - **Bilateral**: Shimer and Smith (2000), Eeckhout and Kircher (2010a).
  - **Urn-ball**: Shi (2002), Shimer (2005), Albrecht et al. (2014).
- Macro dynamics
  - **Bilateral**: Menzio and Shi (2011), Lise and Robin (2016).

This Paper: Beyond Urn-Ball and Bilateral

- Standard environment with three ingredients:
  - buyers are (ex ante) heterogeneous in their private valuations;
  - homogeneous sellers compete for these buyers;
  - process by which buyers meet sellers is frictional.
    - directed search: unit supply/demand + symmetric strategies.
- However:
  - arbitrary meeting technologies, as in Eeckhout and Kircher (2010b).

Contribution

- New representation of meeting technologies that simplifies the analysis and allows us to make progress.
- Optimal mechanism for arbitrary meeting technologies.
- Conditions on meeting technology that guarantee unique queue for a given mechanism.
- Efficiency of the equilibrium.
- Two-sided heterogeneity: sorting.
- Spin-off: CGW (2017, JET)
  - Necessary and sufficient conditions for perfect separation / pooling.

Special Cases

Related Literature

- Eeckhout and Kircher (2010b).
  - introduce framework to think about arbitrary meeting technology.
  - sufficient conditions for pooling and separating.
- Lester, Visschers and Wolthoff (2015).
  - ex post heterogeneity.
- Cai (2016).
  - random search + bargaining.

Environment

Agents

- Static model.
- Measure 1 of risk-neutral sellers, indexed by $j \in [0, 1]$.
- Measure Λ of risk-neutral buyers.
- Unit supply / demand of an indivisible good.
- Sellers' valuation: $y = 0$.
  - Extension: $y \sim H(x)$ with $0 \leq y \leq 1$.
- Buyers' valuation: $x \sim G(x)$ with $0 \leq x \leq 1$.
  - Privately observed before making decisions.

Search

- Each seller posts and commits to a direct mechanism.
  - A mechanism specifies for each buyer i ...
    - a probability of trade $\chi(x_i, x_{-i}, n)$
    - an expected transfer $t(x_i, x_{-i}, n)$

  as a function of ...
    - number n of buyers meeting the seller
    - the valuation x_i reported by buyer i
    - the valuations x_{-i} reported by the $n-1$ other buyers.
- Buyers observe all mechanisms and choose one.
- Restriction: symmetric and anonymous strategies.
- All agents choosing a particular mechanism form a *submarket*.

Meeting Technologies

- Consider a submarket with b buyers and s sellers.
- Ratio of buyers to sellers is the *queue length* $\lambda = \frac{b}{s}$.
- Meetings governed by a CRS *meeting technology*, summarized by

$$P_n(\lambda) = \mathbb{P}[\text{seller meets } n \text{ buyers} | \lambda] \text{ for } n \in \{0, 1, 2, \ldots\}.$$

Assumptions

- Assumptions on $P_n(\lambda)$.
  - Exogenous.
  - Twice continuously differentiable.
  - Consistency: $\sum_{n=0}^{\infty} nP_n(\lambda) \leq \lambda$.
  - Type independence:
    - Suppose $\mu \in [0, \lambda]$ buyers in the submarket are blue.
    - Then, $\mathbb{P}$[seller meets i blue buyers and $n - i$ other buyers] =

$$P_n(\lambda) \binom{n}{i} \left(\frac{\mu}{\lambda}\right)^i \left(1 - \frac{\mu}{\lambda}\right)^{n-i}.$$

Better Representation

- Submarket with μ blue buyers and $\lambda - \mu$ other buyers.
- Define $\phi(\mu, \lambda) = \mathbb{P}$[seller meets *at least one* blue buyer].
- Given type independence,

$$\phi(\mu, \lambda) = 1 - \sum_{n=0}^{\infty} P_n(\lambda) \left(1 - \frac{\mu}{\lambda}\right)^n .$$

- Use of ϕ simplifies the derivation and presentation of our results.

Lemma

There exists a one-to-one relationship between $\phi(\mu, \lambda)$ and $\{P_n(\lambda)\}$.

▸ Proof

Properties of ϕ

- Increase in μ makes it easier for seller to meet a high-type buyer.
  - $\phi_\mu > 0$ and $\phi_{\mu\mu} \leq 0$.
- However, increase in λ makes meeting a high-type buyer ...
  - $\phi_\lambda < 0$: harder;
  - $\phi_\lambda = 0$: neutral;
  - $\phi_\lambda > 0$: easier.

Examples of Meeting Technologies

Example (Urn-Ball)

- Number of buyers at each seller is $Poi\left(\lambda\right)$, i.e. $P_n\left(\lambda\right) = e^{-\lambda}\frac{\lambda^n}{n!}$.
- Micro-foundation: each buyer is randomly allocated to a seller.
- $\phi\left(\mu, \lambda\right) = 1 - e^{-\mu}$. Note: $\phi_\lambda = 0$.

Example (Bilateral)

- Number of buyers at each seller is 0 or 1, i.e. $P_0\left(\lambda\right) + P_1\left(\lambda\right) = 1$, where $P_1(\lambda)$ is strictly increasing and concave.
- Micro-foundation: random pairing of agents.
- $\phi\left(\mu, \lambda\right) = P_1\left(\lambda\right)\frac{\mu}{\lambda}$. Note: $\phi_\lambda < 0$.

Examples of Meeting Technologies

Example (Truncated Urn-Ball)

- Urn-ball, but seller can meet $1 < N < \infty$ buyers.
- Note: $\phi_\lambda < 0$.

Example (Geometric; Lester, Visschers and Wolthoff, 2015)

- $P_n(\lambda) = \frac{\lambda^n}{(1+\lambda)^{n+1}}$ and .
- Micro-foundation: agents are randomly positioned on a circle and buyers walk clockwise to the nearest seller.
- $\phi(\mu, \lambda) = \frac{\mu}{1+\mu}$. Note: $\phi_\lambda = 0$.

Planner's Problem

Planner's Problem

- Planner aims to maximize surplus, subject to the meeting frictions.
- Planner can observe buyers' types (WLOG, as we will show).
- Two decisions
  1. Allocation of buyers: queues for each seller.
  2. Allocation of the good: trading rule after arrival of buyers.
- Solve in reverse order.

Allocation of the Good

- Trivial solution: allocate good to the buyer with the highest valuation.

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Lemma

Surplus at a seller with a queue λ of buyers with type cdf $F(x)$ equals

$$S(\lambda, F) = \int_0^1 \phi(\lambda(1 - F(x)), \lambda)\, dx.$$

Allocation of Buyers

- For each seller $j \in [0, 1]$, planner chooses a queue length $\lambda(j)$ and a distribution of buyer types $F(j, x)$ to maximize total surplus

$$\mathcal{S} = \int_0^1 S(\lambda(j), F(j, x))\, dj.$$

- Planner cannot allocate more buyers of a certain type than available.
- Terminology:
  - A submarket is *active* if it contains buyers and sellers.
  - A submarket is *idle* if it contains either only buyers or only sellers.

Participation

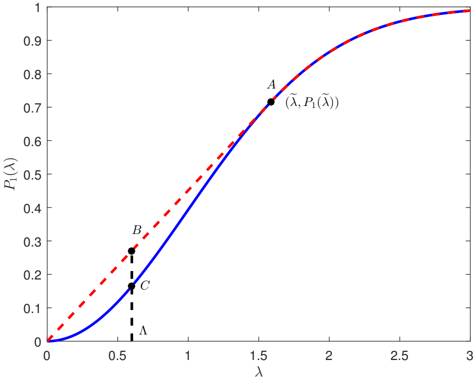
Lemma

If $\phi_\lambda(\mu, \lambda) \geq 0$ *(≤ 0 resp.) for all* $0 < \mu < \lambda$*, then the planner will require all buyers (sellers resp.) to be active in the market.*

Number of Submarkets

Proposition

If there are $n \in \mathbb{N}$ buyer types, the planner's problem can be solved with (at most) $n+1$ submarkets, including one potentially idle submarket.

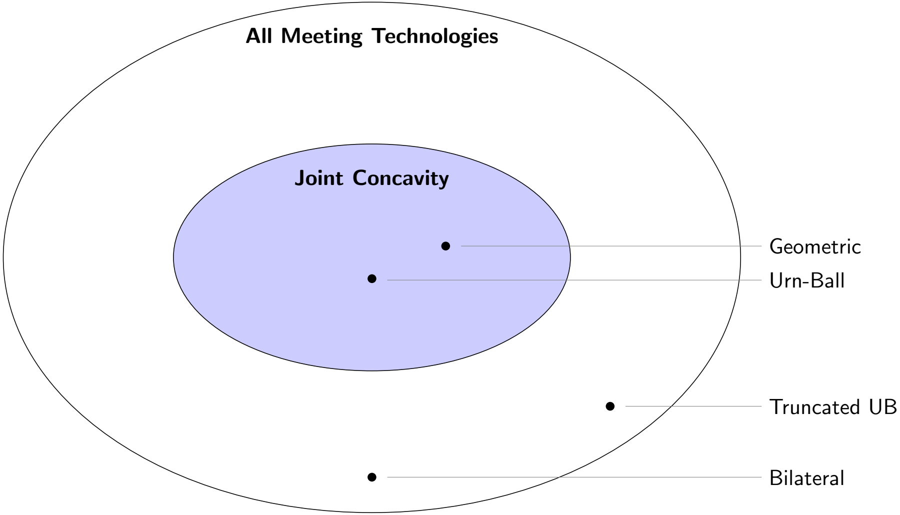


CGW (2017, JET)

- Conditions on the meeting technology that are necessary and sufficient to obtain ...
  - perfect separation (i.e. n submarkets)
  - perfect pooling (i.e. 1 submarket)

  for any Λ and G.
- These conditions are
  - separation $\Longleftrightarrow$ meetings are bilateral.
  - pooling $\Longleftrightarrow$ meetings satisfy joint concavity of ϕ in (μ, λ).

Classification of Meeting Technologies



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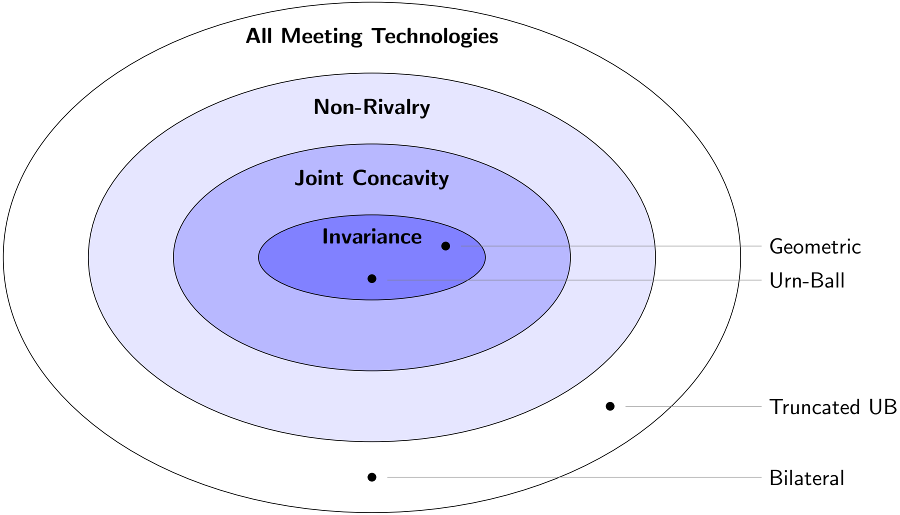


Market Equilibrium

Notation

- In a submarket with mechanism m and a queue of buyers (λ, F):
  - $R(m, \lambda, F)$ = expected payoff of a seller
  - $U(x, m, \lambda, F)$ = expected payoff of a buyer with valuation x.
  - $\overline{U}(x)$ = the market utility function, i.e.

$$\overline{U}(x) = \max_{j \in [0,1]} U\left(x; m(j), \lambda(j), F(j, \cdot)\right).$$

Equilibrium Definition

Definition

A directed search equilibrium is a mechanism $m(j)$ and a queue $(\lambda(j), F(j, \cdot))$ for each seller $j \in [0, 1]$, and a market utility $\overline{U}(x)$ for each type of buyer x, such that ...

1. each $(m(j), \lambda(j), F(j, \cdot))$ maximizes $R(m, \lambda, F)$ subject to

$$U(x, m, \lambda, F) \leq \overline{U}(x), \text{ with equality for } x \text{ in the support of } F.$$

2. aggregating queues across sellers does not exceed the total measure of buyers of each type;
3. incentive compatibility is satisfied, so buyers report their valuations truthfully.

Market Utility Condition

- Market utility: seller posting m expects a queue (λ, F) satisfying

$$U(x, m, \lambda, F) \leq \overline{U}(x), \text{ with equality for } x \text{ in the support of } F.$$

- Complication: not obvious that this condition has a unique solution.

Optimism

- Standard solution: assume that sellers are optimistic and expect the solution that maximizes their revenue (see e.g. McAfee, 1993; Eeckhout and Kircher, 2010b; Auster and Gottardi, 2016; CGW, 2017).
- This makes deviations maximally profitable and may therefore help to limit the set of equilibria.
- Our contribution: derive (weak) conditions which jointly imply a unique solution.

Decentralization

Proposition

For any meeting technology, the planner's solution $\{\lambda(j), F(j,x)\}$ *can be decentralized as a directed search equilibrium in which seller* j *posts a second-price auction and a meeting fee equal to*

$$\tau(j) = -\frac{\int_0^1 \phi_\lambda\left(\lambda(j)\left(1 - F(j,x)\right), \lambda(j)\right) dx}{\phi_\mu\left(0, \lambda(j)\right)}.$$

Intuition

- Market utility implies that sellers are residual claimants on surplus.
- Hence, incentive to implement planner's solution; this requires ...
  1. Efficient allocation of buyers to sellers.
  2. Efficient allocation of the good.
- Auction fulfills second condition.
- First condition requires that each buyer receives a payoff equal to marginal contribution to surplus.
- Meeting fee ensures this by pricing the meeting externality.
  - Denominator: probability of meeting a seller.
  - Numerator: externality on meetings between seller and other buyers.

Implication

- Ranking of surplus (decreasing order):
  1. Planner who knows buyers' valuations.
  2. Planner who does not know buyers' valuations.
  3. Market equilibrium.
- Equivalence of 1 and 3 therefore implies equivalence of all three.

Uniqueness

- Second-price auction can be replaced by first-price auction, etc.
  - Allocation or payoffs remain the same.
- For some meeting technologies, multiple allocations generate the same surplus.
  - Allocation may vary, but surplus and payoffs remain the same.
- For some meeting technologies, multiple queues can be compatible with market utility.
  - Allocation, surplus and payoffs may vary.

Beliefs

- When are queues uniquely determined by market utility?
- Consider the case in which the support of $G(x)$ is $[0, 1]$.
  - (weaker condition in the paper).
- Define ...
  - $Q_0(\lambda) = \mathbb{P}[\text{buyer fails to meet a seller}]$.
  - $Q_1(\lambda) = \mathbb{P}[\text{buyer meets a seller without other buyers}]$.
- Both probabilities can readily be calculated from $P_n(\lambda)$ or $\phi(\mu, \lambda)$.

Assumptions

Assumption

A1. $Q_1(\lambda)$ *is strictly decreasing in* λ.

A2. $1 - Q_0(\lambda)$ *is (weakly) decreasing in* λ.

A3. $\frac{Q_1(\lambda)}{1-Q_0(\lambda)}$ *is (weakly) decreasing in* λ.

- Not restrictive: satisfied for each of our examples.

Uniqueness of the Queue

Proposition

Under A1, A2 and A3, for a seller posting an auction with entry fee t, there is a unique queue (λ, F) compatible with market utility.

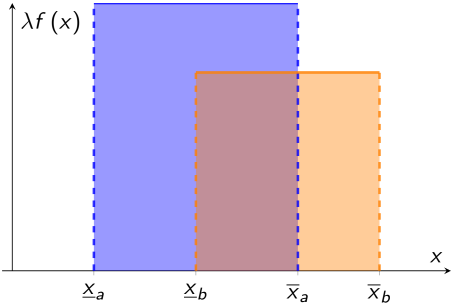
- Main idea
  - Market utility $U(x)$ is strictly convex.
  - Slopes in $\underline{x}$ and $\overline{x}$ are $Q_1(\lambda)$ and $1 - Q_0(\lambda)$, respectively.
  - Hence, one-to-one relation between λ, $\underline{x}$ and $\overline{x}$.
  - A3 is required to establish one-to-one relation with t.

Characterization of the Queue

Proposition

Under A1, A2 and A3, for a seller posting an auction with entry fee t, ...

- *the support of F is an interval $[\underline{x}, \overline{x}]$.*
- *if $t_a < t_b$, then $\lambda^a > \lambda^b$, $\underline{x}_a \leq \underline{x}_b$, and $\overline{x}_a \leq \overline{x}_b$.*



Strengthening the Assumption ...

Assumption

A4. $\phi_{\mu\lambda}(\mu, \lambda) \leq 0$ *for* $0 \leq \mu \leq \lambda$.

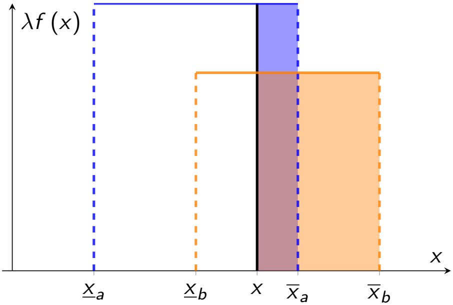
- Interpretation: low-type buyers exert a (weakly) negative externality on high-type buyers.
- A4 $\implies$ A2.

… Strengthens the Characterization

Proposition

Under A1, A3 and A4, if $\lambda^a > \lambda^b$ and $\underline{x}_b < \overline{x}_a$, then for any $x \in [\underline{x}_b, \overline{x}_a]$,

$$\lambda^b \left(1 - F^b(x)\right) \geq \lambda^a \left(1 - F^a(x)\right).$$



Further Strengthening the Assumption ...

Assumption

Invariance. $\phi_\lambda(\mu, \lambda) = 0$ *for* $0 \leq \mu \leq \lambda$.

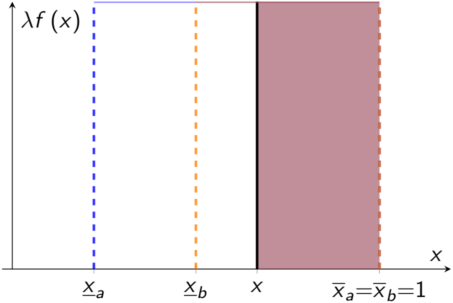
- Interpretation: meetings with high-type buyers are unaffected by the presence of low-type buyers.
- Invariance $\implies$ (A1,A2,A3,A4).

... Further Strengthens the Characterization

Proposition

If meetings are invariant, then for $x \in [\underline{x}_b, 1]$,

$$\lambda^a \left(1 - F^a(x)\right) = \lambda^b \left(1 - F^b(x)\right).$$



Two-Sided Heterogeneity and Sorting

Two-Sided Heterogeneity and Sorting

- Suppose sellers differ in their valuation $y \sim H(x)$ with $0 \leq y \leq 1$.
- Earlier results regarding uniqueness and efficiency carry over.
- Characterizing sorting patterns requires additional (weak) assumption.

Assumption

A6. *$P_0(\lambda)$ is strictly decreasing in λ.*

Proposition (Positive Assortative Matching)

Under A1, A3, A4 and A6, $y_a < y_b$ implies $\lambda^a \geq \lambda^b$, $\underline{x}_a \leq \underline{x}_b$, $\overline{x}_a \leq \overline{x}_b$, and the earlier results regarding characterization.

Conclusion

- We analyze an environment in which ...
  - sellers compete for heterogeneous buyers by posting mechanisms;
  - buyers direct their search;
  - meetings are governed by a frictional meeting technology.
- We introduce a transformation (ϕ) of the meeting technology which allows us to extend and clarify many existing results in competing auctions literature.

Appendix Slides

Special Cases

- Urn-ball (e.g. Peters and Severinov, 1997)
  - all sellers post auctions.
  - buyers randomize between all sellers (in equilbrium).
  - perfect pooling: single market.
  - equilibrium is constrained efficient.
- Bilateral (e.g. Eeckhout and Kircher, 2010b)
  - sellers post different prices.
  - buyers select market that is optimal for their type.
  - perfect separation: # markets = # types.
  - equilibrium is constrained efficient.

Back

Proof of One-to-One Relation between ϕ and P_n

Proof.

- Define probability-generating function (pgf) of $P_n(\lambda)$, i.e.

$$m(z,\lambda) \equiv \sum_{n=0}^{\infty} P_n(\lambda) z^n = 1 - \phi(\lambda(1-z), \lambda).$$

- Then, by the properties of pgfs,

$$P_n(\lambda) = \frac{1}{n!}\frac{\partial^n}{\partial z^n} m(z,\lambda)\Big|_{z=0} = \frac{(-\lambda)^n}{n!}\frac{\partial^n}{\partial \mu^n}(1-\phi(\mu,\lambda))\Big|_{\mu=\lambda}.$$

□

Back