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THE INTERACTIONS BETWEEN PRODUCTION AND ECOLOGICAL NETWORKS



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The Interactions Between Production and Ecological Networks*

Vincent Boucher†

Abstract

I propose a framework to study the intricate interactions between the economy and the environment using an n−sectors endogenous production network and an m−resources ecological network. Markets are contestable, and firms adapt more rapidly than ecosystems. I show that supply chain spillovers strongly affect the nature of the interactions between natural resources, leading to economic and ecological instability. I characterize the optimal tax and show that the (efficient) resources' values can be expressed as an explicit function of the production and ecological networks. I complement the theoretical results with numerical simulations, calibrated using a state-of-the-art ecological network model.

Résumé

Je propose un cadre d'analyse des interactions complexes entre l'économie et l'environnement, fondé sur un réseau de production endogène comprenant n secteurs et un réseau écologique composé de m ressources. Les marchés sont contestables et les entreprises s'adaptent plus rapidement que les écosystèmes. Je montre que les effets de propagation au sein des chaînes d'approvisionnement influencent fortement la nature des interactions entre les ressources naturelles, entraînant une instabilité économique et écologique. Je caractérise la taxe optimale et démontre que les valeurs efficientes des ressources peuvent être exprimées comme une fonction explicite des réseaux de production et des réseaux écologiques. Je complète les résultats théoriques par des simulations numériques, calibrées à partir d'un modèle de réseau écologique de pointe.

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1 Introduction

The economy interacts with the environment in multiple ways. Economic activity produces waste and actively transforms certain ecosystems. But nature also provides inputs for firms, allowing them to produce many goods, from food to cars.[1] The result of these interactions has led to an unprecedented environmental crisis.[2] Escaping this crisis requires elaborating effective public policies that are based on a clear understanding of economic and ecological interactions. Unfortunately, the elaboration of models capturing the diversity and complexity of these interactions between the economy and the environment faces some formidable challenges.

Indeed, the dynamic and complex nature of the problem generally constrains researchers to make painful modeling choices.[3] For example, researchers often consider a few resources (or species), even for models aimed specifically at studying complex interactions, such as biodiversity.[4] Models can also often be solved only using simulations, which implies that the obtained conclusions can also be extremely sensitive to the applied calibration.[5] Crucially, researchers generally consider a single economic sector, keeping technology and market structure constant.[6] I show that this assumption dramatically understates the impact that economic activities have on ecosystems.

The value of examining the interactions between economic sectors when analysing the impact of natural perturbations is well established. Notably, Carvalho et al. (2021) show that the Great East Japan Earthquake had important impact on Japan's GDP through its impact on the supply chains. It is also well documented that the impact of natural disasters spill over globally, affecting trade flows.[7] In this paper, I show that these spillovers, acting through supply chains, also have profound and previously neglected impacts on the ecosystems. In particular, supply chains generate co-variations in the biomass of species belonging to otherwise isolated ecosystems.

In turn, interactions with the ecosystem imply that sectors are not only linked through supply chains: externalities from one sector may, through the ecological network, affect the ecosystem services provided to other sectors, and thus their productivity. This results in a failure of Hulten's theorem. (Hulten, 1978) In the presence of ecological feedbacks, the equilibrium impact of sector-

[1] Dasgupta (2021) offers a comprehensive review of the economics of biodiversity. See in particular Chapter 12 on the value of services provided by nature and Chapter 4 for externalities imposed on nature. See also Ostrom (2009) for a high-level analysis of social–ecological systems.

[2] In particular, four of the seventeen UN Sustainable Development Goals are directly concerned with the human impact on nature and call for immediate action (i.e. goals 12–15). See `https://www.un.org/sustainabledevelopment/`.

[3] See, for example, the excellent discussion in Fenichel et al. (2015).

[4] For example, see Hartwick (2015) for nonrenewable resources. For biodiversity, see the seminal contribution of Brock and Xepapadeas (2003). See also Tschirhart (2009) for a review and Albers et al. (2021) for a recent discussion of this point.

[5] This is in general true for models linking the economy and the environment, such as integrated assessment models, aimed at computing the social cost of carbon. See, for example, the critical reviews by Pindyck (2013) and Stern (2013).

[6] Exceptions include Domominguez-Iino (2025) and Barrozo (2025).

[7] See for example, Osberghaus (2019), Puzzello and Raschky (2014) and Liu et al. (2023).

specific shocks on real GDP is not (only) given by the sector's sale share.

I propose a new framework to analyze the detailed interactions between the economy and the environment, in the presence of supply chain linkages. Leveraging the recent literature on production networks, I study the interactions between the economy, modeled as an $n-$sectors endogenous production network, and the environment, modeled as an $m-$resources ecological network. A powerful advantage of this choice of modelling is that it allows for a wide variety of economic and ecological interactions while remaining tractable, both theoretically, and numerically.

I assume that firms compete in contestable markets, treating the ecological network as a public good. Contestability allows leveraging recent developments in the economics of production networks (Acemoglu and Azar, 2020) and ensures open access to the ecological network, generating inefficiencies. I also assume that firms adapt more rapidly than ecosystems. Whereas the adaptability of market economies has been shown to be extremely valuable, especially in periods of crisis,[8] I show that it may also have dire consequences in the presence of feedbacks between the economy and ecosystems. Specifically, I make the following contributions.

First, I show that, even when the set of suppliers is fixed (exogeneous production network), the dependence between firms creates spillovers that seriously threaten the stability of the ecosystem. Through externalities and ecosystem services, the economy creates an additional source of interactions between resources in the ecological network. For example, consider a farm, whose productivity is increased by the presence of pollinators. Suppose that this farm also sells inputs used by another farm (perhaps located in another country) whose production pollutes a nearby river, reducing its population of fish. Thus, through the supply chain, an increase in the population of pollinators leads to a decrease in the population of fish. I show both theoretically and using simulations that this additional layer of interactions generates large perturbations to the ecosystems and, in particular, threatens their stability.

Second, I show that the endogenous nature of the production network disrupt stability even more. The ability of firms to modify their set of suppliers creates discontinuities in the dynamic process governing the evolution of the stock of resources in the ecological network. Indeed, small modifications in the stock of available resources can lead to changes at the extensive margin. These changes create discontinuities that can prevent the economy from reaching a steady state and lead to economic cycles, even in the absence of shocks. Crucially, the discontinuities created by the firms' choices are structural in the sense that they cannot be eliminated using taxation.

Third, I characterize the optimal tax and describe the efficient allocation. I show that the optimal

[8] For example, the adaptability of the automobile industry was crucial during World War II (Baime, 2014). There is also a consequent literature documenting a robust association between economic freedom and resilience to economic crises (Bjørnskov, 2016) and, in particular, pandemics (Candela and Geloso, 2021).

tax comprises two distinct components: i) a technology tax (acting at the extensive margin), which incentivizes firms to produce using the efficient set of inputs; and ii) a unit production tax (acting at the intensive margin) levied on the production of goods whose production imposes externalities on resources having a high (social) value. These two components of the optimal tax are due to the fact that resources have many potential usages (e.g. trees can be harvested to be used in wood stoves or to produce lumber). Thus, the social planner may prefer to prevent one of these usages (e.g. prohibiting the use of wood stoves).

At the efficient steady state, the resources' values are a function of two elements: the resources' direct values for production and their values for the preservation of the ecological network. In particular, the resources' values can be expressed as an explicit function of the production and ecological networks, which allows bridging the economic (e.g. Acemoglu and Azar (2020)) and ecological (e.g. Allesina and Pascual (2009)) literature.[9] Resources have higher value if they are central in the ecological network or if their provide ecosystem services to central economic sectors.

Fourth, I show that the framework developed in this paper can integrate a large variety of realistic ecological dynamics, while remaining computationally tractable. In particular, I use a state-of-the-art Allometric Trophic Network model (Glaum, Cocco, and Valdovinos, 2020) to study the impact of production networks on marine ecosystem stability. I show that production interdependencies lead to more instability. These simulation results are coherent with, and complement the theoretical results in revealing the importance of considering supply-chain interactions when assessing the impact of economic activities on the environment, and in particular on biodiversity.

Related literature

This paper contributes generally to the vast literature on the economics of the environment and natural resources. Contributions to the literature on the extraction of natural resources are typically distinguished by the type of resource considered: renewable, such as fish (Gordon, 1954) and forests (Hartman, 1976), or nonrenewable (Hotelling, 1931), such as oil and minerals.[10] A particular interest lies in the impact of the market structure (e.g. open access vs monopoly) on the extraction rate and the dynamic evolution of prices in a partial equilibrium framework. Similarly, contributions to the economics of the environment generally consider a single pollutant.[11]

The approach in this paper contrasts sharply with this literature. Instead of focusing on stable economic environments and optimal resource management, I assume that firms adapt more rapidly than the environment. In particular, I allow firms to react to changes in the ecological network by

[9] In Appendix E, I also present an extension in which I allow for a direct consumption (e.g. cultural) value for resources.

[10] Notable exceptions include the study of biodiversity, see below.

[11] See e.g. Hartwick (2015) for nonrenewable resources or Holland (2015) for renewable resources; for pollution, see e.g. the review by Aronsson and Sjögren (2018).

adapting their production technology. I show that the interactions between economic sectors and natural resources has profound impacts on how ecosystems evolve and that the structure of the production network may have a larger impact on the steady-state stock of resource than the amount of externality exerted (see in particular Corollary 1).

By its nature, this paper also contributes specifically to the literature on the economics of biodiversity.[12] Whereas early contributions defined biodiversity in terms of species diversity (e.g. genetic, see Weitzman (1998)), recent contributions have emphasized the role of the ecological functions of resources (Brock and Xepapadeas, 2003). Resource value is generally accepted as comprising its direct consumption value (e.g. cultural or religious), ecosystem service value (value for production), and ecological function value (e.g. maintaining ecosystem stability) (Dasgupta (2021), Chapter 2.4).

In this paper, I focus on ecosystem services and ecological function, although my results hold in the presence of consumption value.[13] I show that the resources' values can be summarized by an explicit function of the production network and of the ecological network. Resources have more value if they are central in the ecological network or if they provide ecosystem services to central sectors of the economy.

This paper leverages the recent literature on the economics of production networks.[14] To my knowledge, the only papers harnessing this literature for the study of environmental issues are King et al. (2019) and Blackburn and Moreno-Cruz (2021). King et al. (2019) find that sector-specific carbon taxes, particularly those targeting key sectors, can lead to a larger abatement than a carbon tax. Blackburn and Moreno-Cruz (2021) show that sectorial energy efficiency improvements, particularly in central sectors, spill over through the production network, resulting in overall efficiency gains. Both papers assume that the production network is exogenous.

Models allowing for endogenous production networks mainly emphasize the impact of changes at the extensive margin on economic cycles and growth (e.g. firms leaving or entering the market or changing their set of suppliers; see for example Acemoglu and Azar (2020), Elliott et al. (2022), and **?**). In particular, small changes in economic conditions in one sector can lead to large changes in aggregate values.

In this paper, dynamics in the evolution of resources lead to changes (through ecosystem services) in firms' productivity, causing discrete endogenous changes to the production network. In particular, whereas the literature usually considers only market-based interactions (e.g. input–output linkages), I show that firms are also linked through ecological networks because firms rely on ecosystem services and production generates externalities.

[12] See in particular the impressive review in Dasgupta (2021).
[13] See Appendix E for a formal treatment.
[14] See in particular Gabaix (2011), Acemoglu et al. (2012), Carvalho and Tahbaz-Salehi (2019), and Elliott and Golub (2022).

1.1 Notation

Unless otherwise stated, all vectors are column vectors and are denoted by bold lowercase Arabic letters (e.g. $\mathbf{a}$). Matrices are denoted by bold uppercase Arabic letters (e.g. $\mathbf{A}$). For any n-dimensional vector $\mathbf{a}$ and m-dimensional vector $\mathbf{b}$, I let $\frac{\partial \mathbf{a}}{\partial \mathbf{b}}$ be the $n \times m$ matrix such that $\left[\frac{\partial \mathbf{a}}{\partial \mathbf{b}}\right]_{ij} = \frac{\partial a_i}{\partial b_j}$. For any two vectors (or matrices) of the same dimension, I define $\mathbf{a} \geq \mathbf{b}$ as the partial order such that $a_i \geq b_i$ for all i and $\mathbf{a} > \mathbf{b}$ as the partial order such that $a_i > b_i$ for all i. Finally, the transformation $\ln(\mathbf{a})$ applies element-wise.

The remainder of this paper is organized as follows. In Section 2, I present an overview of the model. In Section 3, I present the production side of the economy. In Section 4, I present the resource dynamics. In Section 5, I derive the optimal tax. In Section 6, I present a simulation exercise using an Allometric Trophic Network model. I conclude briefly in Section 7.

2 Overview

In this section, I present a high-level overview of the framework. Formal assumptions and results are presented in sections 3–5.

Production follows closely Acemoglu and Azar (2020). There are n production sectors. The technology is assumed to be homogeneous within sectors; therefore, I will use firms, industry and sectors interchangeably. Firms produce using labor and goods produced by other firms. In addition to firms, there are also m natural resources. Resources include a wide range of renewable and nonrenewable resources (such as minerals, microbes, plants, animals, or habitat) in the spirit of Allometric Trophic Networks (Martinez, 2020).

Resources are not owned by any agent and are therefore modeled as public goods. I assume the presence of *externalities* and *ecosystem services*; production may affect the stock of resources, which in turn may affect production. Whereas externalities deplete the stock of resources, ecosystem services do not (see below).

I assume that the economy evolves in discrete time $t = 0, 1, \ldots$. Each time period is divided into two phases (see Figure 1). The first phase is the *production* phase, in which the stock of resources $\mathbf{x}_t$ is taken as given. Once an equilibrium is reached and production is realized, the second phase occurs (the *reproduction* phase). The stock of resources $\mathbf{x}_{t+1}$ for the following period is generated as a function of the current stocks $\mathbf{x}_t$ and current (equilibrium) production levels $\mathbf{y}_t$.

This configuration reflects the idea that the adaptability of firms is far greater than that of resources (e.g. species). It also implies that the rivalry in accessing natural resources can be neglected in the short run (i.e. at time t). This assumption is of course dependent on the time frame

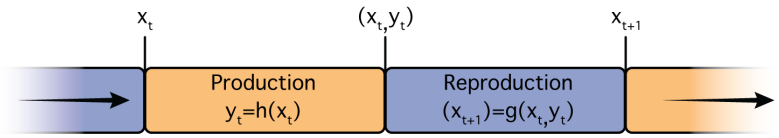
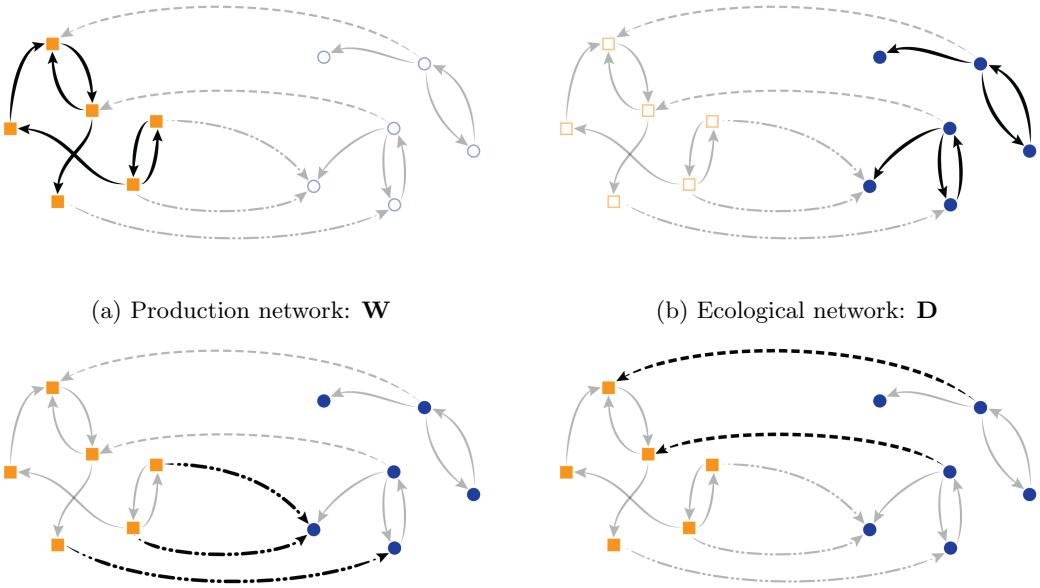


Figure 1: Discrete-time dynamics

considered. While it may be unrealistic if t and $t+1$ are decades apart, I argue that is is reasonable for shorter time frames (say, one, or a few years). Moreover, I will show that this assumption helps to deliver important insights in the dynamics of the interactions between the economy and the ecosystem and on the economic value of natural resources.

2.1 Networks

It will be helpful to describe the interactions between firms and resources using networks: the *production network*, the *ecological network*, the *externality network*, and the *network of ecosystem services* (see Figure 2).



(a) Production network: **W**

(b) Ecological network: **D**

(c) Externality network: **C**

(d) Network of ecosystem services: **B**

Figure 2: Four types of networks

The production network describes the relationships between firms (e.g. buyer–seller) and is

represented by the $n \times n$ matrix $\mathbf{W}$. The ecological network describes the ecological interactions (e.g. predator–prey) between resources and is represented by the $m \times m$ matrix $\mathbf{D}$. The externality network describes the relationships between firms and resources (e.g. pollution) and is represented by the $m \times n$ matrix $\mathbf{C}$. (Araujo, 2024) Finally, the network of ecosystem services describes the interactions between resources and firms (e.g. soil/water quality) and is represented by the $n \times m$ matrix $\mathbf{B}$. Importantly, networks $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{W}$ are, in general, *not* static and can evolve over time as a function of firms' choices. Formal definitions are presented in Section 3 (Assumption 2) and Section 4 (Assumption 4).

2.2 Equilibrium Networks

Without *feedbacks* (i.e. externalities *and* ecosystem services), the production and ecological networks can be studied independently. With feedbacks, I show that the production and ecological networks are transformed as follows:

$$\text{(Production network)} \qquad \mathbf{W} \underset{\text{feedbacks}}{\longrightarrow} \mathbf{W} + \mathbf{B}(\mathbf{I} - \mathbf{D})^{-1}\mathbf{C}, \tag{1}$$

and

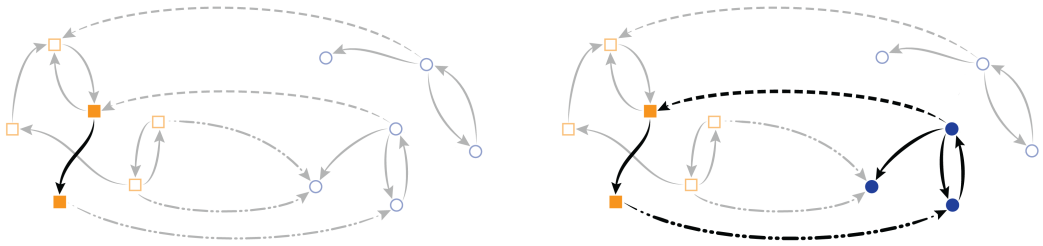
$$\text{(Ecological network)} \qquad \mathbf{D} \underset{\text{feedbacks}}{\longrightarrow} \mathbf{D} + \mathbf{C}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{B}. \tag{2}$$

First consider the transformation of the production network in Equation (1), represented graphically in Figure 3. Without feedbacks, interactions between firms are represented by the production network and reflect (mostly observable) buyer-seller relationships.[15] The presence of feedbacks adds an additional layer of interactions. The relationship between any two firms now depends in a complex way on the ecological network.

For example, consider a fruit farm, relying on ecosystem services ($\mathbf{B}$) from wild pollinators (e.g. bees, butterflies...), and a distributor, buying produce from the farm (through a series of buyer-seller relationships in the production network $\mathbf{W}$), but polluting ($\mathbf{C}$) the neighboring air and water. Then, the impact of a positive productivity shock on the farm has two distinct impacts on the distributor. First, it can provide cheaper inputs (produce) since productivity shocks spread through the production network. Second, the increased distribution operations further pollute the air and water, which through the ecological network $\mathbf{D}$, reduces the population of wild pollinators, and thus

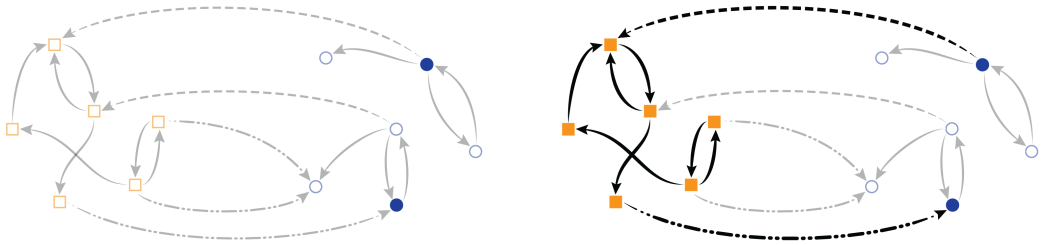
[15] For example, a common practice is to calibrate the production network using input–output matrices (e.g. Acemoglu et al. (2012)) or, when available, buyer–seller firm-level data (e.g. Carvalho et al. (2021)).

reduces the productivity of the farm. This implies that Hulten's theorem fails: due to feedbacks through the ecological networks, firms' sale shares are not sufficient to describe the impact of firm-specific productivity shocks on real GDP.



(a) Production network: $\mathbf{W}$ (b) Production network: $\mathbf{W} + \mathbf{B}(\mathbf{I} - \mathbf{D})^{-1}\mathbf{C}$

Figure 3: Production network (a) without and (b) with feedbacks



(a) Ecological network: $\mathbf{D}$ (b) Ecological network: $\mathbf{D} + \mathbf{C}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{B}$

Figure 4: Ecological network (a) without and (b) with feedbacks

Lets now turn to the transformation of the ecological network in Equation (2), represented graphically in Figure 4. Without feedbacks, the interaction between any two resources (e.g. species) is determined by $\mathbf{D}$. The presence of feedbacks opens up new types of interactions through the production network. In particular, the ecological network in Figure 4 contains two ecosystems that are not ecologically connected (Figure 4a) but that become connected because of feedbacks through the production network (Figure 4b).

Consider again the same example but now assume that there is a positive shock on the stock of wild pollinators. This shock increases ecosystem services ($\mathbf{B}$) to farms, increasing their productivity and their sales (through the production network $\mathbf{W}$). In particular, it increases distribution operations, polluting ($\mathbf{C}$) the air and water. Thus, an increase in the population of wild pollinators leads to decreased water and air quality.

I now present the model in detail.

3 Production

The framework is borrowed from Acemoglu and Azar (2020). Each firm $i \in N = \{1, ..., n\}$ chooses its set of suppliers $S_{i,t} \subseteq N$ and input choices $(l_{i,t}, \{z_{ij,t}\}_{j \in S_{i,t}})$, where $l_{i,t}$ is the labor used by sector, i and $z_{ij,t}$ are goods produced by sector j and used by sector i.[16] The production function for sector i is given by $f_i(S_{i,t}, l_{i,t}, \mathbf{z}_{i,t}, \mathbf{x}_t)$, where $\mathbf{x}_t$ is the stock of natural resources.

It is important to stress that firms do not directly *use* resources. At any time t, the stock of resources $\mathbf{x}_t$ is a public good and affects production (see Assumption 2 below). However, production at time t only affects the stock of resources at $t+1$. This is described in detail in Section 4.

The representative consumer has a unit labor endowment, and their instantaneous preferences at time t are represented by the utility function $u(\gamma_{1,t}, ..., \gamma_{n,t})$, where $\gamma_{i,t}$ is the consumption of goods produced by sector i at time t. The intertemporal utility is therefore $\sum_{t=0}^{\infty} \delta^t u(\gamma_{1,t}, ..., \gamma_{n,t})$, where $\delta \in (0,1)$ is the discount factor.[17]

The production from firm i is subject to the distortion μ_i, which acts as an ad valorem tax. A fraction λ_i of the revenues from the distortions is rebated to the representative consumer so that their budget constraint is

$$\sum_i p_{i,t}\gamma_{i,t} \leq 1 + \sum_i \lambda_i \frac{\mu_i}{1+\mu_i} p_{i,t} y_{i,t} \tag{3}$$

where $y_{i,t}$ is the production of sector i at time t. The consumption is therefore financed by wages (normalized to 1) and tax revenues.

On the production side, the cost minimization problem occurs in two stages. In the first stage, firms choose the optimal quantity of inputs for a fixed set of suppliers. The (conditional) unit cost function is therefore defined as

$$\begin{aligned} \kappa_i(S_{i,t}, \mathbf{p}_t, \mathbf{x}_t) \quad &\equiv \quad \min_{\mathbf{z}_i, l_i} \; l_i + \sum_{j \in S_i} p_{j,t} z_{ij} \\ \text{s/c} \quad &\quad f_i(S_{i,t}, l_i, \mathbf{z}_i, \mathbf{x}_t) = 1. \end{aligned} \tag{4}$$

In the second stage, firms choose the optimal set of inputs as

$$S^*_{i,t} = \arg\min_{S_i} \kappa_i(S_i, \mathbf{p}_t, \mathbf{x}_t). \tag{5}$$

[16] There is representative firm for each sector, so I use the terms "firm" and "sector" interchangeably.

[17] I assume that the preferences of the representative consumer do not depend on the stock of resources $\mathbf{x}$. In Appendix E, I present an extension allowing for a consumption value of the resources and show that the main results still hold.

I assume that markets are contestable, which in this framework translates to

$$p^*_{i,t} = (1+\mu_i)\kappa_i(S^*_{i,t}, \mathbf{p}^*_t, \mathbf{x}_t). \tag{6}$$

I also maintain the same classical assumptions as in Acemoglu and Azar (2020), which are summarized in Assumption 1.

Assumption 1. *The production function $f_i(S_{i,t}, l_{i,t}, \mathbf{z}_{i,t}, \mathbf{x}_t)$ is strictly quasi-concave, exhibits constant return to scale in $(l_{i,t}, \{z_{ij,t}\}_{j\in S_{i,t}})$ and is increasing and continuous in $l_{i,t}$ and $z_{ij,t}$. Labor is essential so that $f_i(S_{i,t}, 0, \mathbf{z}_{i,t}, \mathbf{x}_t) = 0$, and production is always possible using only labor. The production function is also twice continuously differentiable in $(l_{i,t}, \mathbf{z}_{i,t}, \mathbf{x}_t)$. The utility function $u(\gamma_{1,t}, ..., \gamma_{n,t})$ is twice continuously differentiable, increasing, strictly quasi-concave, and such that all goods are normal.*

Note that Assumption 1 does not impose any restriction on how f_i varies with $\mathbf{x}_t$. While it is intuitive to assume that f_i increases with $\mathbf{x}_t$, none of the results are contingent on that assumption. Assumption 1 however implies that the production of all goods is possible using only labor, i.e. $f_i(\emptyset, l_i, \mathbf{x}_t) > 0$ if $l_i > 0$, for any $\mathbf{x}_t$. This is a technical assumption that allows in particular to define an upper-bound on the set of admissible equilibrium prices. While it does implies that (formally) resources are not necessary for production, it is of little practical importance. Indeed, the technology allowing to produce using only labor can be set to arbitrarily inefficient levels (and would thus never chosen by firms).

I follow Acemoglu and Azar (2020) and define an equilibrium as follows:

Definition 1. *For a given period t and a vector of resources' stocks $\mathbf{x}$, an equilibrium tuple $(\mathbf{p}^*, S^*, \boldsymbol{\gamma}^*, \mathbf{l}^*, \mathbf{Z}^*, \mathbf{y}^*)$ is such that:*

1. *For each $i = 1, ...n$, $p^*_i = (1+\mu_i)\kappa_i(S^*_i, \mathbf{p}^*, \mathbf{x})$;*
2. *The consumer maximizes its utility subject to budget constraints in Equation (3);*
3. *For each $i = 1, ..., n$, conditional demands l^*_i and $\mathbf{z}^*_i$ solve Equation (4) conditional on S^*, and the set of suppliers S^*_i solves Equation (5), given $\mathbf{p}^*$;*
4. *For each $i = 1, ..., n$, $\sum_i y^*_i l^*_i = 1$, $1 = f_i(S^*_i, l^*_i, \mathbf{z}^*_i, \mathbf{x})$, and $\gamma^*_i / y^*_i + \sum_j \frac{y^*_j}{y^*_i} z^*_{ji} = \left(1 - (1-\lambda_i)\frac{\mu_i}{1+\mu_i}\right)$.*

Condition 1 is the contestability condition. Under constant returns, profits are linear; thus, firms must earn zero profit. Condition 2 imposes that the consumer maximizes their utility. Because the consumer does not have any saving/borrowing opportunity, the budget constraint must be saturated

at each period. Condition 3 implies that firms maximize profits, and Condition 4 ensures that all markets clear. Acemoglu and Azar (2020) show the following:

Proposition 1 (Acemoglu and Azar (2020), theorems 1 and 2)**.** *Under Assumption 1, there exists a unique equilibrium price vector and a generically unique equilibrium tuple.*

The (non-generic) multiplicity arises when firms are exactly indifferent between two sets of suppliers. Thus, under Proposition 1, the equilibrium production can be written as $\mathbf{y}_t^* = \mathbf{y}^*(\mathbf{x}_t)$, assuming some arbitrary tie-breaking rule.[18]

A powerful feature of this model is that the equilibrium can be found rapidly. Indeed, Acemoglu and Azar (2020) show that Condition 1 in Definition 1 is necessary and sufficient to characterize equilibrium prices (see their Lemma 1). Moreover, they show that $(1+\mu_i)\kappa_i(S_i^*, \mathbf{p}, \mathbf{x})$ is non-decreasing in $\mathbf{p}$ (see their Lemma A1) so the unique fixed point can be found iteratively and quite rapidly.[19]

For concreteness, Assumption 2 considers a well-studied special case.

Assumption 2 (Log-linear preferences and technologies)**.** *Following Acemoglu and Azar (2020), let us assume a Cobb–Douglas production function with Hicks-neutral technology:*

$$y_{i,t} = \frac{1}{\left(1-\sum_{j\in S_{i,t}} w_{ij}\right)^{1-\sum_{j\in S_{i,t}} w_{ij}} \Pi_{j\in S_{i,t}} w_{ij}^{w_{ij}}} l_{i,t}^{1-\sum_{j\in S_{i,t}} w_{ij}} \Pi_{j\in S_{i,t}} z_{ij,t}^{w_{ij}} \left[a_i(S_{i,t}) \Pi_k x_{k,t}^{b_{ik}(S_{i,t})}\right],$$

where the term in brackets represents the technology and is a combination of the total factor productivity (TFP) $a_i(S_{i,t})$ and ecosystem services $\Pi_k x_{k,t}^{b_{ik}(S_{i,t})}$ (e.g. Gordon (1954)). Let us also assume $u(\boldsymbol{\gamma}_t) = \sum_i q_i \ln(\gamma_{i,t})$, with $q_i > 0$ for all i, and $\sum_i q_i = 1$.

One of the advantages of Assumption 2 is that it allows expressing prices, production, and welfare as explicit functions of the equilibrium production network $\mathbf{W}_t^*$ and the equilibrium network of ecosystem services $\mathbf{B}_t^*$.[20]

Assumption 2 makes it clear that, at time t, the stock of resources acts as a public good affecting productivity. By selecting its set of suppliers $S_{i,t}$, the firm also chooses its technology, which is a function of both TFP and ecosystem services. Thus, the network of ecosystem services describes

[18] Note that we can easily allow the functions u and f_i and the quantities μ_i and λ_i to vary over time. In this case, the function $\mathbf{y}^*$ would be indexed by t. However, this would prevent the overall system (i.e. the production and reproduction phases) from reaching a steady state and would therefore unnecessarily obscure the discussions of Section 4. In Section 5, I allow $\boldsymbol{\mu}$ to be a function of $\mathbf{x}$ and S, chosen by the policy maker.

[19] The design of efficient algorithms in this context (i.e. Tarski fixed points) is an active area of study. Recently, Fearnley et al. (2022) showed that the equilibrium can be found in $O(log^2\bar{p}/\epsilon)$ queries, where $\bar{p}$ is the upper-bound on the price and ϵ is the tolerance threshold. Note that this bound is independent on the number of sectors. For many applied contexts however, a simple iteration from $\bar{p}$ reaches the equilibrium rapidly enough.

[20] While for presentation purposes, examples assume that $b_{ik}(S_{i,t})$ is positive, none of the results (unless explicitly stated) are contingent on this assumption.

how the stock of resources $\mathbf{x}_t$ translates into ecosystem services (i.e. productivity gains) given the chosen technology $S_{i,t}$.

Proposition 2 presents the main equilibrium quantities at time t. For clarity, only the case without distortions (i.e. $\boldsymbol{\mu} = \mathbf{0}$) is presented. Proposition 9 in Appendix B presents the general case (along with the proof).

Proposition 2. *Let $\boldsymbol{\mu} = \mathbf{0}$. Under Assumption 2, the unique equilibrium is such that*

$$\ln \mathbf{p}_t^* = -(\mathbf{I} - \boldsymbol{W}_t^*)^{-1}[\ln \boldsymbol{a}_t^* + \boldsymbol{B}_t^* \ln \mathbf{x}_t].$$

Moreover,

$$\ln \mathbf{y}^*(\mathbf{x}) = \ln[(\mathbf{I} - \boldsymbol{W}_t^{*\top})^{-1}\mathbf{q}] - \ln \mathbf{p}_t^*,$$

and

$$u^* = \mathbf{q}^\top \ln(\mathbf{q}) - \mathbf{q}^\top \ln \mathbf{p}_t^*,$$

where $\mathbf{B}_t^* = \mathbf{B}(S_t^*)$, $w_{ij,t}^* = w_{ij}$ if $j \in S_{i,t}^*$ and 0 otherwise.

Proposition 2 describes the short-term impact of firms' productivity on prices, quantities, and real GDP.[21] In particular, ecosystem services act as positive TFP shocks, reducing prices, and increasing production and GDP (holding the production network $\mathbf{W}^*$ constant).

Indeed, the production network $\mathbf{W}^*$, specifically the Leontief inverse $(\mathbf{I} - \mathbf{W}^*)^{-1}$, describes how productivity shocks (through TFP and ecosystem services) affect equilibrium prices, production, and GDP. In particular, the quantity $\mathbf{q}^\top(\mathbf{I} - \boldsymbol{W}_t^*)^{-1}$ is equal to the vector of *Domar weights*, i.e. the ratio of total sales to nominal GDP.[22]

Proposition 2 therefore implies that Hulten (1978)'s theorem holds in the short term: the change in log GDP implied by a change in log TFP for a given sector is equal to that sector's sale share. More importantly, at time t, Hulten (1978)'s theorem also holds for ecosystem services, which implies a short-term value for ecosystem services. Indeed, taking $\mathbf{x}_t$ as given, we can rewrite real GDP at time t as:

$$u^* = \underbrace{\mathbf{q}^\top(\mathbf{I} - \boldsymbol{W}_t^*)^{-1}}_{\text{Domar weights}} \quad \underbrace{\mathbf{B}_t^* \ln \mathbf{x}_t}_{\substack{\text{Ecosystem services} \\ \text{(direct)}}} \quad + u_0^*, \tag{7}$$

where u_0^* does not depend on $\mathbf{x}_t$.

The value $\mathbf{B}_t^* \ln \mathbf{x}_t$ is the direct impact of $\mathbf{x}_t$ on productivity. Thus, the network of ecosystem

[21] In the context of this model, the utility of the representative consumer is equivalent to real GDP.

[22] For completeness, I provide a short proof of this statement in Appendix C, Corollary 3. Similar results can be found for example in **?**, Corollary 1, or Acemoglu et al. (2012), Appendix A.

services $\mathbf{B}$ converts the stock of resources $\mathbf{x}_t$ (e.g. expressed in units of biomass) into TFP units. The Domar weights $\mathbf{q}^\top(\mathbf{I} - \boldsymbol{W}_t^*)^{-1}$ then translate productivity growth into GDP growth. Thus, a resource is more valuable when it provides ecological services to sectors with higher sales shares.

In particular, Equation (7) allows to measure the impact of a shock on the stock of some resource (say k) on short-term GDP, i.e. $\mathbf{q}^\top(\mathbf{I} - \boldsymbol{W}_t^*)^{-1}\mathbf{b}_k$, where $\mathbf{b}_k$ is the k'th column of the matrix $\mathbf{B}$. At the sectoral level, $\mathbf{q}$ (final consumption shares) and $\mathbf{W}$ (intermediate input shares) are readily available from input-output tables (as used in Acemoglu et al. (2012) or Acemoglu and Azar (2020)). While more scattered and recent, many countries also provide comprehensive information on ecosystem services at the sectoral level.[23]

Equation (7) is, however, a naive (or short sighted) definition of the "value" of ecosystem services for two reasons. First, it fails to capture the fact that firms can adapt to changes in $\mathbf{x}_t$ by altering their production technology (i.e. the production network is endogenous). For example, a resource may have no value under existing technology but possess a high value under an alternative (future) technology.[24]

Second, it disregards the value of resources in maintaining the ecosystem. That is, even if a resource does not directly affect firms' productivity, it may nonetheless be essential for the long-term sustainability of other productive resources (such as plankton for fish). This value is not considered here because the value is conditioned on the current stock of resource $\mathbf{x}_t$. I describe the evolution of the stock of resources in the next section.

4 Reproduction

I assume that the stock of resources evolves according to

$$\mathbf{x}_{t+1} = \mathbf{g}(\mathbf{x}_t; \mathbf{y}_t, S_t),$$

where $\mathbf{g}$ is twice-continuously differentiable in $\mathbf{x}$ and in $\mathbf{y}$, for any given S. Resources can potentially include both renewable and nonrenewable resources. A nonrenewable resource k is such that $g_k(\mathbf{x}; \mathbf{y}, S) \leq x_k$ for all $\mathbf{x}$, $\mathbf{y}$, and S. Nonrenewable resources can include resources such as oil but also resources that are typically not used in the production process, such as species' habitat.

Importantly, note that the impact of production on the evolution of the stock of resources is not only a function of the production levels $\mathbf{y}$ but also of the set of suppliers S, as in King et al. (2019).

[23] For example, the Ecosystem Services Valuation Database (https://www.esvd.net/) includes valuation for many ecosystem services that could be linked to input-output matrices. The GLORIA dataset (Lenzen et al., 2017) also includes Satelites accounts for 120 economic sectors which includes information on ecosystem perturbations (e.g. water stress).

[24] This is closely linked to the "Biological prospecting" value of species discussed e.g. in Polasky et al. (2005).

This allows capturing situations in which different production technologies have distinct impacts on the ecosystem.

Before presenting the analysis, I introduce two main assumptions and some definitions. The first assumption is a compacity requirement.

Assumption 3 (Finite Earth)**.** *Let $K \subseteq M$ be the set of renewable resources. There exists $\underline{x}$ such that for all k, $g_k([x_k, \mathbf{x}_{K-k}, \mathbf{x}_{-K}]; \mathbf{y}^*(x_k, \mathbf{x}_{K-k}, \mathbf{x}_{-K}), S) \leq x_k$, for all $x_k \geq \underline{x}$ and $\mathbf{x}_{K-k}$, and for all S.*

Assumption 3 implies that resources cannot grow indefinitely. Fixing the stock of nonrenewable resources results in a maximal sustainable stock of renewable resources. Note that Assumption 3 is in part an assumption on the production technology. It implies that the economy cannot grow resources indefinitely.

The second assumption focuses on a specific type of dynamics and is made both for simplicity and exposition purposes. Many of the results hold more generally and a formal treatment is presented in Appendix D.

Assumption 4 (Generalized Lotka–Volterra)**.** *There exists a set $X^* \subseteq \{\mathbf{x} | \mathbf{x} > \mathbf{1}\}$ and $\Delta > 0$ such that for all $\mathbf{x} \in X^*$, we have*

$$\ln g_k(\mathbf{x}; \mathbf{y}, S) = \ln x_k + \Delta \ln x_k (r_k + [\boldsymbol{D} \ln \mathbf{x}]_k - \ln x_k + [\boldsymbol{C} \ln \mathbf{y}]_k) \tag{8}$$

for all k, where $(\mathbf{I} - \mathbf{D})$ is invertible, and $\mathbf{y} > \mathbf{1}$.

Assumption 4 postulates that there is a set for which the evolution of the stocks is governed by Equation (8).[25] This specification corresponds to the m-species Lotka–Volterra model (Lotka, 1925; Volterra, 1926), in which species abundance is measured on a logarithmic scale. Given its simplicity and parsimony, the m-species Lotka–Volterra model is well studied in ecology.[26] For any k, r_k describes the growth rate of the resource. The ecological network is represented by the matrix $\mathbf{D}$ and describes the strength and nature of the interaction between species. For example, if $d_{kl} < 0$ and $d_{lk} > 0$, it reflects a predator–prey relationship. The externality network $\mathbf{C}$ describes how production impacts resources' growth rates.

Here, the parameter $\Delta > 0$ controls the steps of the discrete-time dynamics. As Δ goes to zero, the GLV in Assumption 4 converges to its continuous-time counterpart. Recalling Figure 1, the

[25] Assuming that $\mathbf{y}^* > \mathbf{1}$ is without the loss of generality conditional on $\mathbf{y}^* > \mathbf{0}$ because no units are specified. Therefore, Assumption 4 imposes that, around the steady state, all goods have a strictly positive demand. A simple way to ensure this is to assume positive final demand as in Assumption 2. This assumption is made for exposition purposes and could be relaxed at the expense of some additional notation.

[26] See e.g. Hofbauer and Sigmund (1998) Chapter 5 for a textbook treatment as well as Fort (2018) and Serván and Allesina (2021) for recent contributions.

economic interpretation of Δ is that it captures the adaptability of the economy. Indeed, if the economy reaches an equilibrium rapidly, then we can model the dynamics of the system using a small value of Δ. Thus, I will use the terminology: *in fast enough economies* as equivalent to: for $\Delta > 0$ small enough.

The m-species Lotka–Volterra model is both flexible and parsimonious: it can replicate all dynamics (convergence, limit cycles, chaos...) but can only feature two types of steady state.[27] The first type of steady state, in which some resources are extinct (in particular when all resources are extinct: $\ln \mathbf{x}^* = \mathbf{0}$), is explicitly ruled out by Assumption 4. The second and more interesting type of steady state is described (implicitly) by

$$\ln \mathbf{x} = [\mathbf{I} - \mathbf{D}]^{-1}[\mathbf{r} + \mathbf{C}^* \ln \mathbf{y}^*], \tag{9}$$

where $\mathbf{C}^* = \mathbf{C}^*(S(\mathbf{x}))$ and $\ln \mathbf{y}^* = \ln \mathbf{y}^*(\mathbf{x})$. Because $\mathbf{y}^*$ and $\mathbf{C}^*$ can be a function of $\mathbf{x}$, the existence of a steady state therefore depends on whether Equation (9) has one or many solutions in X^*. This is discussed in detail in Section 4.1 and Section 4.2 below.

Stability

A large part of the analysis focuses on stability. To do so, I will use the following standard stability notions (see e.g. Kelley and Peterson (2001)).

Definition 2 (Stability)**.** *A steady state* $\mathbf{x}^*$ *is* stable *if for any* $\epsilon > 0$ *there exists* $\beta = \beta(\epsilon) > 0$ *such that whenever* $\|\mathbf{x}_0 - \mathbf{x}^*\| < \beta$*, then* $\|\mathbf{x}_t - \mathbf{x}^*\| < \epsilon$ *for all* t*.*

A steady state $\mathbf{x}^*$ is stable if the stock of resources remains in a neighborhood of $\mathbf{x}^*$ following a small perturbation.

Definition 3 (Asymptotic stability)**.** *A steady state* $\mathbf{x}^*$ *is* asymptotically stable *if it is stable and if there exists* $\beta > 0$ *such that* $\lim_{t\to\infty} \mathbf{x}_t = \mathbf{x}^*$ *whenever* $\|\mathbf{x}_0 - \mathbf{x}^*\| < \beta$*.*

A steady state $\mathbf{x}^*$ is asymptotically stable if the stock of resources converges to $\mathbf{x}^*$ following a small perturbation (i.e. when $\mathbf{x}_0$ is in a neighborhood of $\mathbf{x}^*$).

Definition 4 (Domain of attraction)**.** *The* domain of attraction *of a steady state* $\mathbf{x}^*$ *is the set* X *such that* $\lim_{t\to\infty} \mathbf{x}_t = \mathbf{x}^*$ *whenever* $\mathbf{x}_0 \in X$*.*

It follows directly that if a steady state is asymptotically stable, then its domain of attraction is non-empty. A common tool to find conditions for stability is the following.[28].

[27] Some dynamics require at least five species; see Barabás et al. (2016) for specific examples.
[28] For example, Lyapunov diagonal stability is presented in Barker et al. (1978). Note that this version of Lyapunov

Definition 5 (Strict Lyapunov diagonal stability)**.** *A square matrix* $\boldsymbol{A}$ *is* Lyapunov-diagonally stable *if there exists a strictly positive diagonal matrix* $\boldsymbol{\Lambda}$ *such that* $\boldsymbol{\Lambda A} + \boldsymbol{A}'\boldsymbol{\Lambda}$ *is positive definite.*

Before I present the formal analysis, I briefly discuss the importance of stability for welfare. Recall that from Equation (7), real GDP at time t can be expressed as:

$$u_t = u_{0,t} + \mathbf{q}^\top (\mathbf{I} - \boldsymbol{W}_t)^{-1} \mathbf{B}_t \ln \mathbf{x}_t.$$

Evaluated at some steady state $\mathbf{x}^*$, this expression is simply:

$$u^* = u_0^* + \mathbf{q}^\top (\mathbf{I} - \boldsymbol{W}^*)^{-1} \mathbf{B}^* \ln \mathbf{x}^*,$$

which does not vary with time.

Suppose that steady state $\mathbf{x}^*$ is asymptotically stable. Then, after a small perturbation of the steady state, real GDP (i.e. u_t) will converge back to u^*. In particular, this means that the volatility of real GDP implied by this small perturbation will eventually vanish.

Now, suppose that the steady state is *not* stable. Then, the impact of a vanishingly small perturbation can have two effects. First, the dynamics can converge to some other steady state. In that case, volatility of real GDP will eventually vanish, but real GDP at the (new) steady state can be quite different. Second, the economy may never converge. In that case, volatility of real GDP will not vanish and may evolve in some chaotic way. Thus, when the steady state is unstable, *vanishingly small shocks can have large, and permanent impacts on welfare.*

Below, I first present an analysis of the dynamics of the economy assuming an exogenous production network. Then, I present an extension to endogenous networks in Section 4.2.

4.1 Exogenous production network

In this section, I assume that the production is fixed. This implies that the matrices $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{W}$ are also fixed and do not depend on $\mathbf{x}$. This setting is sufficient to ensure the existence of a steady state.

Proposition 3. *Fix* S *and suppose that assumptions 1 and 3 hold. Then, there always exists at least one steady state.*

The proof, presented in Appendix B, relies on the application of the Poincaré–Miranda theorem.[29]

diagonal stability is usually reserved for continuous-time settings. Given Assumption 4, its application therefore requires that Δ be small enough. See in particular Lemma 3

[29] For convenience, Appendix A reproduces known results from the literature that are used in this paper.

While Proposition 3 guarantees the existence of a steady state, I show that feedbacks between the economy and the ecosystem seriously threaten their stability.[30] Proposition 4 formalizes.[31]

Proposition 4. *Let* $\boldsymbol{\mu} = \mathbf{0}$ *and suppose that assumptions 2 and 4 hold. Fix* S*, consider* X^* *around:*

$$\ln \mathbf{x}^* \quad = \quad [\mathbf{I} - \mathbf{D} - \mathbf{C}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{B}]^{-1}[\mathbf{r} + \mathbf{C} \ln \tilde{\mathbf{y}}], \tag{10}$$

where $\ln \tilde{\mathbf{y}} = \ln[(\mathbf{I} - \boldsymbol{W}^\top)^{-1}\mathbf{q}] + (\mathbf{I} - \mathbf{W})^{-1} \ln \mathbf{a}$ *does not depend on* $\mathbf{x}$*. Then* $\mathbf{x}^*$ *is the unique steady state in* X^**. Moreover, in fast enough economies,* $\mathbf{x}^*$ *is asymptotically stable whenever* $[\mathbf{I} - \mathbf{D} - \mathbf{C}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{B}]$ *is Lyapunov-diagonally stable.*

Note that $\tilde{\mathbf{y}}$ in Proposition 4 is not equal to the equilibrium production level but rather represents the part of production that is independent from the ecosystem (see below). Note also that in the absence of feedbacks (that is $\mathbf{B} = \mathbf{0}$ or $\mathbf{C} = \mathbf{0}$), the, stability condition requires $(\mathbf{I} - \mathbf{D})$ to be Lyapunov-diagonally stable. In that sense, feedbacks have profound impacts on the stability of the steady-state.[32]

Proposition 4 provides important insights regarding the impact of the economy on the dynamics of resources. In particular, it shows that the impact that the economy has on reshaping the nature of the interactions between resources might be more important than the scale of the externality imposed. In order to expose this point more clearly, Corollary 1 and Corrolary 2 consider two of implications of Proposition 4.

A first implication of Proposition 4 is that even in economies with arbitrarily low levels of TFP and final demand, the steady-state stock of resources is bounded away from the level that would be observed in the absence of interactions between the economy and the ecosystem. Corollary 1 formalizes.

Corollary 1 ($\epsilon-$economies)**.** *Fix* S *and assume that* $\mathbf{C}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{B} \neq \mathbf{0}$*. Consider* $\ln \mathbf{x}^*$ *as defined in Equation (10) and denote* $\ln \mathbf{x}_0 = (\mathbf{I} - \mathbf{D})^{-1}\mathbf{r}$ *the steady-state in the absence of externalities (i.e.* $\mathbf{C} = \mathbf{0}$*). Then, there exists* $\Delta > 0$ *such that* $\| \ln \mathbf{x}^* - \ln \mathbf{x}_0 \| \geq \Delta$ *for any* $\mathbf{a}$ *and* $\mathbf{q}$ *such that* $\ln \tilde{\mathbf{y}}$ *is arbitrarily close to* $\mathbf{0}$*.*

[30] The general argument made in this section is similar to the one in Beaudry et al. (2020): non-market elements (in their case financial frictions, here public goods) lead to locally unstable steady states and (in Beaudry et al. (2020)) limit-cycle dynamics.

[31] Again, I present the general case in Proposition 10 in Appendix B. Also, in order to lighten the text, I assume throughout that the matrix

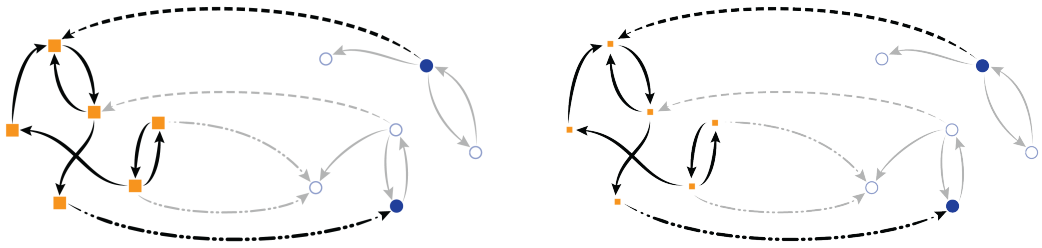
$$\boldsymbol{\Gamma} = \begin{bmatrix} \mathbf{I} - \boldsymbol{W} & \boldsymbol{B} \\ \boldsymbol{C} & \mathbf{I} - \boldsymbol{D} \end{bmatrix}$$

is invertible, which guarantees the the expressions in propositions 4–6 are well defined. See Theorem 2 in Appendix A.

[32] The focus on Lyapunov stability could be seen as restrictive. I discuss local stability conditions in Section 6 and Appendix D.

Recall that $\ln \tilde{\mathbf{y}} = \ln[(\mathbf{I} - \boldsymbol{W}^\top)^{-1}\mathbf{q}] + (\mathbf{I} - \mathbf{W})^{-1} \ln \mathbf{a}$ does not depend on $\mathbf{x}$. Thus, $\mathbf{C} \ln \tilde{\mathbf{y}}$ captures the impact of the size (through the impact of final demand $\ln[(\mathbf{I} - \boldsymbol{W}^\top)^{-1}\mathbf{q}]$) and productivity spillovers (i.e. $(\mathbf{I} - \mathbf{W})^{-1} \ln \mathbf{a}$) on the steady state. Corrolary 1 considers economies for which this quantity gets arbitrarily close to zero: the case of small, unproductive economies. To provide a simple example, consider the consumption of berries, harvested from wild berries. Berries will be produced even with very low final demand, and very poor technology. (Picture a child playing and stumbling uppon a patch of wild berries.)

Corollary 1 says that if the structure of the integrated economic and ecological network is fixed, then the steady-state stock of resources will be different than the one with the ecological network alone, even as the size of the economy becomes arbitrarily small. Figure 5 illustrates. Note the size of the nodes (the economic sectors), becomes smaller (as $\ln \tilde{\mathbf{y}}$ becomes arbitrarily close to $\mathbf{0}$) but the size of the edges are constant (since $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{W}$ are held constant).



(a) Initial values for $\ln \tilde{\mathbf{y}}$ (b) Smaller values for $\ln \tilde{\mathbf{y}}$

Figure 5: Illustration of Corollary 1: $\ln \tilde{\mathbf{y}}$ goes to $\mathbf{0}$

A second implication of Proposition 4 is that, as discussed above, feedbacks affect the conditions under which the steady-state is stable. Indeed, in general, even if $(\mathbf{I} - \mathbf{D})$ is Lyapunov-diagonally stable (the stability condition in the absence of feedbacks), the matrix $[\mathbf{I} - \mathbf{D} - \mathbf{C}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{B}]$ might not be. Thus, the condition for stability is altered by feedbacks from the economy, i.e. $\mathbf{C}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{B}$. This impact of feedbacks has been neglected by large parts of the literature on biodiversity and natural resource extraction, which have implicitly focused on special cases for which stability is unaffected.

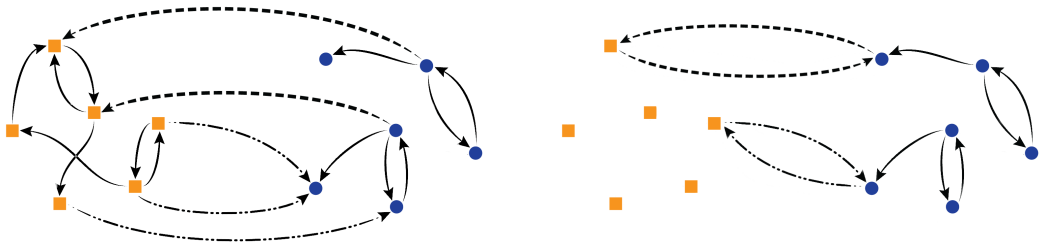
Indeed, a special case for which Lyapunov-diagonal stability is unaffected by feedbacks is when $\mathbf{C}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{B}$ is diagonal with non-positive entries.[33] While, this case is obviously non generic for networked economies, it is naturally implied when focusing on disconnected resource economies. That is, economies in which each sector is isolated, and such that (1) each resource k provides

[33] To see this, note that it implies that $-\boldsymbol{\Lambda}\mathbf{C}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{B}$ is diagonal with positive entries for any $\boldsymbol{\Lambda}$ as in Definition 5, and thus Lyapunov-diagonal stability follows. Of course, this is only a sufficient condition.

ecosystem service to a single economic sector i_k, and (2) only suffers externalities from this sector. In particular, this is the case for most of the literature on resource extraction. Figure 6 illustrates and Corollary 2 formalizes.

Corollary 2 (Disconnected resource economy)**.** *Fix S, and assume that $\mathbf{W} = \mathbf{0}$ and that $c_{ki}b_{jl} = 0$ for all $k \neq l$ and that $c_{ki}b_{ik} \leq 0$ for all k and i. Then, in fast enough economies, if $[\mathbf{I} - \mathbf{D}]$ is Lyapunov-diagonally stable, $[\mathbf{I} - \mathbf{D} - \mathbf{C}(\mathbf{I} - \mathbf{W})^{-1}\mathbf{B}]$ is also Lyapunov-diagonally stable.*

It is important to note that Corollary 2 holds *irrespective of the structure of the ecological network*. By focusing on isolated extracting sectors, the economic literature has implicitly ignored the impacts that the feedbacks between the economy and the ecosystem have on stability.



(a) A generic economy (b) A disconnected resource economy

Figure 6: Illustration of Corollary 2

Corollary 2 therefore highlights the benefits of interacting the four types of networks introduced in this paper. In doing so, we are able to capture a previously neglected effect of the interactions between the economy and the environment: the joint effect of externalities and ecosystem services on ecosystem stability in networked economies. In Section 6, I show that this insight holds for more general ecological dynamics.[34]

4.1.1 Implications for the economy

Of course, the perturbed dynamics of the ecosystem also affects the economy. Proposition 5 formalizes.

Proposition 5. *Fix S, and let $\boldsymbol{\mu} = \mathbf{0}$. Suppose that assumptions 2 and 4 hold, and consider X^* as*

[34] See also Appendix D.

defined in Proposition 4. Then, the unique steady-state equilibrium price vector is given by:

$$\begin{aligned}\ln \mathbf{p}^* &= -\left[\mathbf{I}-\mathbf{W}-\mathbf{B}(\mathbf{I}-\mathbf{D})^{-1}\mathbf{C}\right]^{-1}\times \\ &\quad \Big(\underbrace{\ln \mathbf{a}}_{TFP}+\underbrace{\mathbf{B}(\mathbf{I}-\mathbf{D})^{-1}\mathbf{C}\ln[(\mathbf{I}-\boldsymbol{W}^{\top})^{-1}\mathbf{q}]}_{Demand}+\underbrace{\mathbf{B}(\mathbf{I}-\mathbf{D})^{-1}\mathbf{r}}_{Ressources\ growth}\Big). \qquad (11)\end{aligned}$$

The general case is presented in Proposition 11 in Appendix B. The expressions for $\ln \mathbf{y}^*$ and u^* are directly implied by Proposition 2 and are therefore omitted.

Proposition 5 implies that feedbacks have important implications for comparative statics, even locally. For example, consider the impact of small, persistent productivity (TFP) changes on prices:

$$\frac{\partial \ln \mathbf{p}^*}{\partial \ln \mathbf{a}} = -\left[\mathbf{I}-\mathbf{W}-\mathbf{B}(\mathbf{I}-\mathbf{D})^{-1}\mathbf{C}\right]^{-1}.$$

Without feedbacks, positive productivity shocks necessarily translate into lower prices because $[\mathbf{I}-\mathbf{W}]^{-1}$ is a positive matrix.[35] With feedbacks, however, a small positive productivity shock may well result in price increases, even in sectors affected by productivity gains.[36] This implies, in particular, that Hulten (1978)'s theorem no longer holds.

At the steady state, economic sectors are not only linked through the production network, but also through the ecological feedbacks. As such, TFP shocks spill over through both types of links (economic, and ecological). This implies that the economic size of a sector (e.g. given by its sale share) is no longer an indicator of its importance (in terms of propagating sector-specific TFP shocks).

Consider for example a positive TFP shock on some mining sector. This positive shock spreads through the entire economy due to buyer-seller relationships. But, it may also spread through ecological feedbacks. Indeed, suppose that the mining sector is releasing harmful pesticides in the air, which in turns reduces the population of wild pollinators (e.g. bees, butterflies...). Since pollinators provide ecosystem services to fruit farms, the positive TFP shock on the mine leads to a negative productivity shock on the farm. As a result, Hulten's theorem fails in the long run: Domar weights are not sufficient to describe the impact of firm-specific TFP shocks.[37]

Proposition 5 also provides a more complete (although still unsatisfactory) picture of the value of ecosystem services (the "Ressources growth" part of Equation (11)). Proposition 5 informs us

[35] This result also holds for large shocks, affecting the optimal input choices S, see Acemoglu and Azar (2020), Theorem 4.

[36] See Proposition 13 in Appendix D for a similar result that does not rely on assumptions 2 and 4.

[37] Steady-state equilibrium prices are also affected by final demand shifts ("Demand" part in Equation (11)). Indeed, if the consumer demands more of some good ($\ln[(\mathbf{I}-\boldsymbol{W}^{\top})^{-1}\mathbf{q}]$), this exerts externality ($\mathbf{C}$) on the ecosystem ($(\mathbf{I}-\mathbf{D})^{-1}$), which impacts firms' productivity (through ecosytem services, $\mathbf{B}$), thus acting as a TFP shock at the steady state.

about the value of ecosystem services generated by the growth rate of resources. Using Proposition 2, we find:[38]

$$u^* = \underbrace{\mathbf{q}^\top(\mathbf{I}-\mathbf{W})^{-1}}_{\text{Domar weights}}\mathbf{B}\underbrace{\left[\mathbf{I}-\mathbf{D}-\mathbf{C}(\mathbf{I}-\mathbf{W})^{-1}\mathbf{B}\right]^{-1}\mathbf{r}}_{\text{Total growth}} \quad +u_0, \tag{12}$$

where u_0 does not depend on $\mathbf{r}$. The "total growth" part of Equation (12) gives the steady-state impact on the stock of resources of the growth rates, accounting for resource interactions because of feedbacks throughout the production network. When pre-multiplied by $\mathbf{B}$, this translates into the effects on productivity. When pre-multiplied again by the Domar weights, it impacts real GDP.

Note that the presence of the externality network $\mathbf{C}$ implies that externalities affect the value of the growth rate of resources. For example, a larger growth rate may be worth less, because some part of that growth is lost because of production externalities.

Although the expression in Equation (12) implies a value of ecosystem services that constitutes an improvement over the one implied by Equation (7), it remains unsatisfactory. First, Equation (12) does not give the value of resources themselves but rather that of their growth rates. Second, and more importantly, it describes the *equilibrium* value of ecosystem services derived from the growth rate of resources (at the steady state). However, as markets are inefficient because of feedbacks between the production and the ecological network, this does not inform us on the socially optimal value of ecosystem services. I study corrective taxation and the social planner's problem in Section 5.

In the next section, I study the impact of allowing firms to change their set of suppliers endogenously.

4.2 Endogenous Production Network

The previous section assumed that the production network was fixed. In general, however, firms can adapt to changes in the stock of resources. That is, $S^*(\mathbf{x})$ can potentially vary with $\mathbf{x}$, and as a consequence, so do the matrices $\mathbf{W}(S^*(\mathbf{x}))$, $\mathbf{B}(S^*(\mathbf{x}))$, $\mathbf{C}(S^*(\mathbf{x}))$, and the vector $\mathbf{a}(S^*(\mathbf{x}))$. This has important consequence for the existence, uniqueness and stability of a steady-state.

4.2.1 Existence

Proposition 3 relies on the continuity of the dynamic process governing the evolution of the stock of resources, i.e. $\mathbf{g}(\mathbf{x};\mathbf{y}(S,\mathbf{x}),S)$ is continuous in $\mathbf{x}$ when S is fixed. This is no longer true if $S(\mathbf{x})$ varies with $\mathbf{x}$. Indeed, the dependence of $\mathbf{y}$ and S on $\mathbf{x}$ implies that the mapping $\mathbf{g}(\mathbf{x};\mathbf{y}^*(S^*(\mathbf{x}),\mathbf{x}),S^*(\mathbf{x}))$

[38] This expression uses the equality: $(\mathbf{I}-\boldsymbol{W})^{-1}\mathbf{B}(\mathbf{I}-\boldsymbol{D}-\mathbf{C}(\mathbf{I}-\boldsymbol{W})^{-1}\mathbf{B})^{-1} = (\mathbf{I}-\boldsymbol{W}-\mathbf{B}(\mathbf{I}-\boldsymbol{D})^{-1}\mathbf{C})^{-1}\mathbf{B}(\mathbf{I}-\boldsymbol{D})^{-1}$, see Theorem 2 in Appendix A.

is typically not continuous in $\mathbf{x}$ because small changes in $\mathbf{x}$ might cause firms to change their input choices. A minimal example showing the failure of existence of a steady state is presented in Appendix C.2.

If no steady state exists, the evolution of resources may lead to (potentially chaotic) economic cycles, even in the absence of shocks. To illustrate, consider the following (very stylized) economy. There are two sectors: fish production and manufacturing. There is a single renewable resource: wild fish. Manufacturing goods are produced using only labor, and production does not interact with the population of wild fish. Fish can either be produced using only labor (i.e. fishing or capture) or be produced using the manufacturing sector (i.e. aquaculture/fish farms). Fishing depletes the stock of wild fish, but fish farms do not.

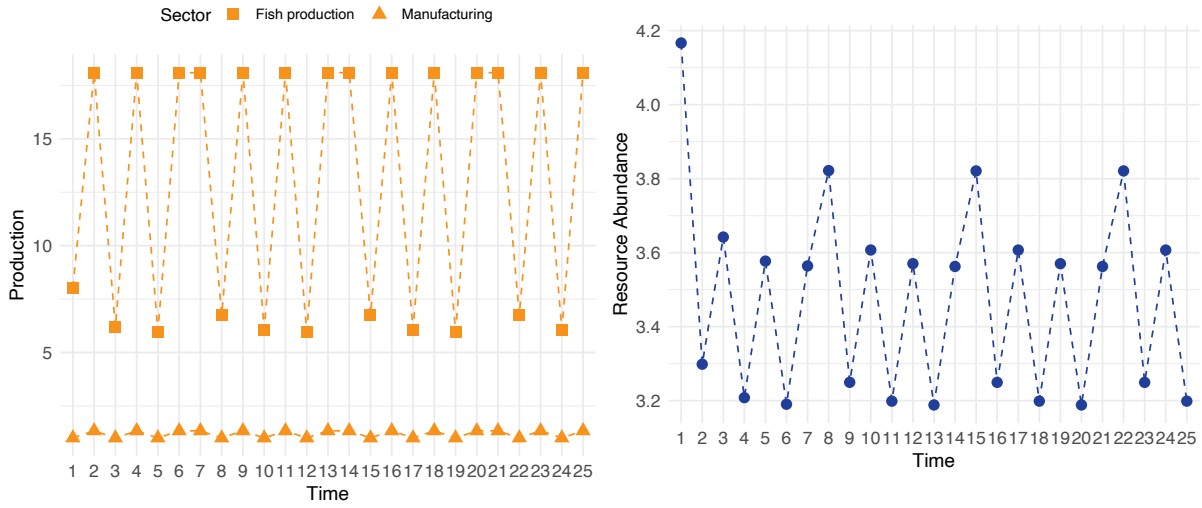


Figure 7: Simple one-species, two-sector economy

Note: See Appendix C.1 for calibration details.

This simple setting is sufficient for the emergence of cycles because of alternating technologies in the fish production sector. Figure 7 illustrates. When the stock of wild fish is high, production occurs through fishing, productivity is high, and there is weak demand for the manufacturing sector. This high productivity comes, however, at the cost of severe depletion of the stock of wild fish. When there is insufficient wild fish, fish production shifts to fish farms, which increases demand for the manufacturing sector and allows time for the stock of wild fish to recover. When the stock of

wild fish is sufficient, fishing becomes profitable again.

Although this scenario is a simple example, the general argument extends to more complex economies. These effects will be important, in particular, if the sectors that are more likely to depend on ecosystem services are also central in the sense that they are more likely to propagate sectorial shocks (Acemoglu et al., 2012), leading to aggregate fluctuations.[39] Note also that these cycles are not (solely) the result of market failures. As long as small changes in the stock of resources lead to changes in the set of suppliers S, a steady state may not exist. In that sense, this is a structural implication of the interactions between firms and the environment and is *not* a result of market failures.[40]

4.2.2 Stability

The endogeneity of the production network also affects the stability and uniqueness of a steady-state, provided one exists. Proposition 6 formalizes.[41]

Proposition 6. *Let $\boldsymbol{\mu} = \mathbf{0}$ and suppose that assumptions 2 and 4 hold. Consider X^* as defined in Assumption 4 and consider the following fixed-point equation for $\mathbf{W}^* = \mathbf{W}^*(S(\mathbf{x}))$, $\mathbf{B}^* = \mathbf{B}^*(S(\mathbf{x}))$, $\mathbf{C}^* = \mathbf{C}^*(S(\mathbf{x}))$, and $\mathbf{a}^* = \mathbf{a}^*(S(\mathbf{x}))$:*

$$\ln \mathbf{x} \quad = \quad [\mathbf{I} - \mathbf{D} - \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^*]^{-1}[\mathbf{r} + \mathbf{C}^* \ln \tilde{\mathbf{y}}(\mathbf{x})], \tag{13}$$

where $\ln \tilde{\mathbf{y}}(\mathbf{x}) = \ln[(\mathbf{I} - \boldsymbol{W}^{*\top})^{-1}\mathbf{q}] + (\mathbf{I} - \mathbf{W}^*)^{-1} \ln \mathbf{a}^*$.
Then, any steady state in X^* must solve Equation (13). Moreover, consider any solution to Equation (13) such that $\mathbf{x}^* \in X^*$. Then (i) $\mathbf{x}^*$ is a steady state in X^*; and (ii) generically and in fast enough economies, $\mathbf{x}^*$ is asymptotically stable whenever $[\mathbf{I} - \mathbf{D} - \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^*]$ is Lyapunov-diagonally stable.

Proposition 6 differs from Proposition 4 in some important ways. First, existence is no longer guaranteed. This is because the right-hand side in (13) is a function of $\mathbf{x}$ and a fixed point is not guaranteed to exist. Second, and relatedly, uniqueness is no longer guaranteed. However, two different steady states cannot have the same set of suppliers S. There is thus at most one steady-state for every possible production network structure. Third, the stability condition can only hold locally. Indeed, the matrix $[\mathbf{I} - \mathbf{D} - \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^*]$ is only constant in a neighborhood of the

[39] Here, following Acemoglu et al. (2012), centrality is defined as $\mathbf{1}'(\mathbf{I} - \mathbf{W})^{-1}$.

[40] That is, except in the limit cases in which efficiency requires the total absence of externality or ecosystem services or if the optimal set of suppliers $S(\mathbf{x})$ is constant for all $\mathbf{x}$.

[41] Proposition 6 uses the term "generically" in the same sense as in Proposition 1: unless for cases in which a firm is indifferent between two different sets of suppliers. Acemoglu and Azar (2020) show that this happens for a Lebesgue measure zero of the TFP vector $(a_1, ..., a_n)$. In the context of Proposition 6, the generic condition is that $S^*(\mathbf{x}^*)$ is constant in a neighborhood of $\mathbf{x}^*$.

steady state. In that sense, the endogeneity of the production network weakens the stability of the steady-state, provided one exists.

This section shows that feedbacks between the economy and the ecosystem have first order effects on the nature and stability of the long-term dynamics. In the next section, I analyse the social planer problem and the optimal taxation scheme.

5 Corrective Taxation

In this section, I show that efficiency can be restored by imposing taxes. To describe the optimal corrective tax, I expand the model presented in Section 3 slightly by allowing the vector of distortions $\boldsymbol{\mu}$ to be a function of $\mathbf{x}$ and S, i.e. $\boldsymbol{\mu}(\mathbf{x}, S)$. This implies that distortions—interpreted here as taxes—can potentially change at every period. It also implies that these distortions affect firms' choices of inputs, i.e. the cost-minimization for firm i at time t becomes: $S^*_{i,t} = \arg\min_{S_i} (1 + \mu_i(\mathbf{x}_t, S_i))\kappa_i(S_i, \mathbf{p}_t, \mathbf{x}_t)$. I will also let $\tau_i(\mathbf{x}, S) \equiv \mu_i(\mathbf{x}, S)\kappa_i(S_i, \mathbf{p}^*(\mathbf{x}, S), \mathbf{x})$ be the amount of tax levied on the production of one unit of good i.

The problem of the social planner is to maximize the discounted sum of per-period utility over time.[42] This problem can be written recursively as follows:

$$v(\mathbf{x}) = \max_{S,\boldsymbol{\gamma},\mathbf{Z},\mathbf{l}} \{u(\boldsymbol{\gamma}) + \delta v[\mathbf{g}(\mathbf{x}; \mathbf{y}, S)]\}$$

subject to (1) $y_i = f_i(S_i, \mathbf{a}_i(S_i), l_i, \mathbf{z}_i, \mathbf{x}) \quad \forall i$, (2) $\sum_i l_i \leq 1$, and (3) $y_j \geq \gamma_j + \sum_i z_{ij} \quad \forall j$, and where $v(\mathbf{x}$ is the Bellman value function.

At any period, the state is completely determined by the stock of resources $\mathbf{x}$. The objective is thus to choose the set of suppliers $S(\mathbf{x})$, consumption $\boldsymbol{\gamma}(\mathbf{x})$, and conditional factor demands $\mathbf{Z}(\mathbf{x})$ and $\mathbf{l}(\mathbf{x})$, subject to the feasibility constraints.

In Proposition 7 below, I characterize the optimal tax $\boldsymbol{\mu}(\mathbf{x}, S(\mathbf{x}))$ so that the market equilibrium coincides with the solution of the social planner.[43]

Proposition 7. *Let $\boldsymbol{\lambda} = \mathbf{1}$. Under Assumption 1, there exists $\boldsymbol{\mu}^E(\mathbf{x}, S(\mathbf{x}))$ such that the market equilibrium is efficient. Moreover, generically:*

(1) Let $S^E(\mathbf{x})$ be the efficient set of inputs when the current stock of resources is $\mathbf{x}$. Then, for all

[42] Equivalently, the discounted sum of real GDP over time.

[43] Proposition 7 and Proposition 8 below use the term "generically" which here means that the results hold whenever the stock of resources $\mathbf{x}$ is such that no firm is exactly indifferent between two set of suppliers. This implies that the derivatives exists at $\mathbf{x}$ and holds generically since the set of potential suppliers (and thus its power set) is finite.

i,

$$\tau_i(\mathbf{x}, S^E) = -\frac{\delta}{\phi} \sum_k \frac{\partial v}{\partial x_k}(\mathbf{x}, S^E) \frac{\partial g_k(\mathbf{x}, S^E)}{\partial y_i}, \tag{14}$$

where ϕ is the Lagrange multiplier for the budget constraint of the representative consumer.

(2) Assume that there exists a steady state for the social planner's problem $\mathbf{x}^E$ and let $\boldsymbol{\nu}^E = \frac{\partial v^E}{\partial \mathbf{x}^\top}$, $\boldsymbol{\mathcal{D}}^E = \frac{\partial \mathbf{g}^E}{\partial \mathbf{x}}$, and $\boldsymbol{\zeta}^E = \sum_i \frac{\partial f_i}{\partial \mathbf{x}^\top} / \frac{\partial f_i}{\partial l_i}$. Then,

$$\boldsymbol{\nu}^{E^\top} = \boldsymbol{\zeta}^{E^\top} [\mathbf{I} - \delta \boldsymbol{\mathcal{D}}^E]^{-1} \tag{15}$$

whenever the inverse exists.

The optimal tax $\boldsymbol{\mu}^E(\mathbf{x}, S(\mathbf{x}))$ is a function of the stock of resources and the set of suppliers. This implies that the optimal tax must act both at the intensive and extensive margins. At the intensive margin, efficiency is achieved by setting higher taxes on goods whose production reduces the stock of resources having high marginal social values (Equation (14)). However, this is insufficient because production must use the optimal set of inputs S^E (the optimal technology). The optimal tax achieves this by setting arbitrarily large taxes on inefficient sets of suppliers.

The choice of the efficient production network (obtained by taxing at the extensive margin) is affected by two main forces. First, the production network affects TFP, and thus the social planer would prefer production networks leading to higher TFP. Second, the production network also affects the interactions with the ecological network (ecological services and externalities). Thus, the social planer would also prefer cleaner technologies, or those that rely on ecosystem services from abundant resources at the steady state.

At the intensive margin, the optimal tax is closer to a standard Pigouvian tax, typically expressed for a fixed technology. Looking at Equation (14), we see that the tax on sector i is given by the sum, for all resources, of the externalities generated by production of good i. Here, all resources may be affected heterogeneously by all sectors so there is potentially a total of $m \times n$ different "pollutant" emitted. In Appendix C.4, I derive the Pigouvian taxes assuming that these pollutant can be monitored and taxed and show that Equation (14) can be naturally rewritten as a function of these Pigouvian taxes: the amount paid by Sector i corresponds to the sum of the Pigouvian taxes for the externalities generated by its production.

Proposition 7 also shows that if a steady state exists, then there exists an explicit expression for the marginal social value of resources (i.e. Equation (15)). This value is composed of two elements, one direct (i.e. $\boldsymbol{\zeta}^E$) and one indirect (i.e. $[\mathbf{I} - \delta \boldsymbol{\mathcal{D}}^E]^{-1}$). The vector $\boldsymbol{\zeta}^E$ is simply the sum over all sectors of the marginal rate of technical substitution between resources and labor. If a resource can

easily be substituted with labor in all sectors, then it is less valuable.

The matrix $[\mathbf{I} - \delta \boldsymbol{\mathcal{D}}^E]^{-1}$ summarizes the interactions in the ecological network. We can write, for any integer $K > 0$: $(\mathbf{I} - \delta^K \boldsymbol{\mathcal{D}}^K)[\mathbf{I} - \delta \boldsymbol{\mathcal{D}}^E]^{-1} = \sum_{k=0}^{K-1} \delta^k \boldsymbol{\mathcal{D}}^k$.[44] Since $\boldsymbol{\mathcal{D}}$ is the Jacobian for the evolution of the biomass of resources, this gives the disounted impact of a change in the biomass of some resource within the ecological network. In particular, if there was no heterogeneity in the marginal rates of technical substitution, resources' values would be proportional to their Katz–Bonacich centrality (Katz, 1953; Bonacich, 1972, 1987) in the ecological network, i.e. $\mathbf{1}^\top [\mathbf{I} - \delta \boldsymbol{\mathcal{D}}^E]^{-1}$.

In Appendix C.3, I show, using an argument similar to Galeotti et al. (2006), that Equation (15) implies that the contribution of the indirect effect on a resource's value is relatively more important for complex, highly nested ecological networks.[45]

Although Proposition 7 is general, more insight can be obtained under the conditions of assumptions 2 and 4. Proposition 8 follows.

Proposition 8. *Under assumptions 2 and 4, we generically have*

$$\frac{\partial v}{\partial \ln \mathbf{x}} = \underbrace{\mathbf{q}^\top [\mathbf{I} - \mathbf{A}^{-1}\mathbf{W}^E]^{-1}\mathbf{A}^{-1}}_{Demand} \underbrace{\mathbf{B}^E}_{Ecosystem\ services} \underbrace{[(1-\delta)\mathbf{I} + \delta diag(\ln \mathbf{x}^e)(\mathbf{I} - \mathbf{D})]^{-1}}_{Ecosystem\ Function} \tag{16}$$

at any steady state, where $\mathbf{B}^E = \mathbf{B}(S^E)$, $\mathbf{W}^E = \mathbf{W}(S^E)$, *and* $\mathbf{A} = \mathbf{I} + diag(\boldsymbol{\mu}^E)$.

Equation (16) captures the impact on welfare of a small variation of the stock of resources (in log). It is helpful to compare this expression to the corresponding effect on real GDP, without taxation, at any period. From Equation (7), we obtain

$$\frac{\partial u^*}{\partial \ln \mathbf{x}} = \mathbf{q}'(\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^*. \tag{17}$$

There are two main differences between Equation (16) and Equation (17). First, the production technology chosen by the social planner (leading to $\mathbf{W}^E$ and $\mathbf{B}^E$) is typically different from the one chosen by firms (leading to $\mathbf{W}^*$ and $\mathbf{B}^*$). Also, optimal taxes affect the final and intermediate demands (through $\mathbf{A}$) relative to the no-tax equilibrium.[46] Thus, at any point in time, the impact of changes in the stock of resources is different under the optimal tax (Equation 16) policy than without taxation (Equation 17).[47] Second, the social planner anticipates changes that occur through

[44] The expression converges to $[\mathbf{I} - \delta \boldsymbol{\mathcal{D}}^E]^{-1} = \sum_{k=0}^{\infty} \delta^k \boldsymbol{\mathcal{D}}^k$ for $K \to \infty$ whenever the norm of $\boldsymbol{\mathcal{D}}$ is strictly less than one.

[45] The *cosine similarity* between $\boldsymbol{\nu}$ and $\boldsymbol{\zeta}$ is bounded by the eigenvalues of $\boldsymbol{\mathcal{D}}$ (in absolute values, for well conditioned eigenbasis), whereas a large spectral radius is indicative of complexity and nestedness (Staniczenko et al., 2013; Strydom et al., 2021).

[46] Although the optimal tax is not part of the social planner's problem, it is more intuitive (and less cumbersome). See the proof of Proposition 8 for an equivalent statement that only depends on the social planner's program.

[47] In particular, with taxation, Hulten (1978)'s theorem fails, so the "demand" part of Equation (16) is no longer

the ecosystem, leading to the additional term $[(1-\delta)\mathbf{I}+\delta diag(\ln \mathbf{x}^e)(\mathbf{I}-\mathbf{D})]^{-1}$ that implies an additional source of value for resources (ecosystem function).

Implications for existence and stability: Note that Proposition 7 (or Proposition 8) does not guarantee the existence of a steady-state and is silent on the impact of the optimal tax on the long-term stability of the economy. A few remarks are in order. First, as highlighted in the previous section, stability and the existence of a steady-state both depend on the production network (recall that $\mathbf{B}$ and $\mathbf{C}$ are functions of S), and its dynamics.

This implies that taxation *at the extensive margin* strongly affects stability and existence of a steady state. However, nothing guarantees that the optimal choice leads to the existence of a steady-state, or of a stable one. Consider stability. Even if there exists a production network $\tilde{\mathbf{W}}$ such that $[\mathbf{I}-\mathbf{D}-\tilde{\mathbf{C}}(\mathbf{I}-\tilde{\mathbf{W}})^{-1}\tilde{\mathbf{B}}]$ is Lyapunov-diagonally stable, it does not mean that this production network is optimal. For instance, it may lead to very low TFP levels, and thus the social planer may prefer more a productive production network, even if it leads to an unstable steady state. A similar argument holds for the existence of a steady-state: A simple way to ensure existence is to fix the production network, so that Proposition 3 holds. However, this may not be optimal since in general, the optimal solution $S^E(\mathbf{x})$ is a function of $\mathbf{x}$.

This shows that the implications of the interactions between the production and ecological networks have structural implications for the stability and existence of a steady-state. These implications are not (only) due to market failures, but are at the heart of the interactions between the economy and the ecosystem.

6 Application

In this section, I explore further the properties of the model studied in this paper in the context of fishing. This context has been widely studied both in economics (e.g. Grafton et al. (2017)) and ecology (e.g. Jennings et al. (2009)) and is well adapted to the model developed in this paper.

In Section 4, I showed (Corollary 2) that Lyapunov-diagonal stability is unaffected when $\mathbf{C}(\mathbf{I}-\mathbf{W})^{-1}\mathbf{B}$ is diagonal with non-positive diagonal elements. This global stability analysis is possible because of Assumption 4. For more general ecological dynamics, however, there exists no global stability condition, and we need to focus on local stability conditions (Allesina and Tang, 2012).[48]

equal to the Domar weights.

[48] In particular, a steady state $\mathbf{x}^*$ is stable if the *spectral radius* σ of the Jacobian of the dynamic process (evaluated at $\mathbf{x}^*$) is strictly less than one, i.e. $\sigma(\boldsymbol{\mathcal{D}}^*)<1$, where $\boldsymbol{\mathcal{D}}^*=\frac{\partial \mathbf{g}(\mathbf{x}^*,\mathbf{y}^*(\mathbf{x}^*,S),S)}{\partial \mathbf{x}}$ (e.g. Kelley and Peterson (2001), Corollary 4.3). The interested reader can consult Appendix D in which I provide an analysis of the local stability of the steady state, similar to the analysis presented in Section 4. In particular, I discuss the intuition as to why/when Corollary 2 could be expected to hold more generally.

In this section, I study how volatility is affected by an exogenous production networks in realistic ecological networks through numerical simulations, and for a production structure as defined in Assumption 2. As discussed in Section 4, unstable steady states are expected to be more volatile. Calibration details can be found in Table F.1 in Appendix F. I assume that the evolution of resources (here, marine species) is governed by an Allometric Trophic Network, described in the next section. There are 30 marine species and 30 economic sectors, some of which are fishing sectors, each harvesting their own specie.

6.1 Allometric Trophic Networks

Allometric Trophic Network (ATN) theory postulates that interactions between species are primarily driven by consumer–resource relations (trophic networks). The evolution of species biomass is assumed to be a function of these consumer–resource relations and the species' body masses (allometry). The simplicity and flexibility of ATN models, as well as their ability to integrate a wide variety of taxa have contributed to their popularity in recent years (Martinez, 2020).

An ATN model typically consists of two main parts. First, the trophic network, which describes predator–prey relationships. This network is usually unweighted and is assumed to be fixed over time. In this paper, I use the unweighted niche model first presented in Williams and Martinez (2000) and used in particular in Milo et al. (2002), Allesina and Tang (2012), and Glaum et al. (2020). The niche model is a latent position (random network) model in which species, located along a line, are assigned a feeding range. Species can only consume species within their feeding range. If specie k is in the feeding range of specie l, then l may eat k. Species with no other species in their feeding range (such as plants) are called *basal* species. A more detailed description of the niche model can be found in Appendix F.

The second part of an ATN model describes the dynamics of the evolution of species biomass over time. The parameters guiding the evolution of species biomass (e.g. reproduction rates, feeding rates, metabolic loss) are grounded in theory, calibrated, and vary depending on the ecosystem (e.g. marine). In this paper, I use the ATN model proposed by Glaum, Cocco, and Valdovinos (2020) (hereafter referred to as GCV) and based on Yodzis and Innes (1992).[49] To tie my hands as much as possible, I assume the exact specification used in GCV, except for the harvesting (production) function, which follows from the model presented in Section 3, under Assumption 2. I briefly present their model below, but more details are available in Appendix F.

To limit the introduction of too much additional notation, I denote all ecological *parameters* by

[49] GCV are interested in the impact of fishing on the properties of the ecological network, which makes their model a natural benchmark.

ω. Superscripts refer to the notation typically used in the ecological literature and in particular in GCV, whereas subscripts index species. For example, ω_k^y refers to the maximal consumption rate of species k and is denoted by y_k in GCV. Parameter values can be found in Table F.1 of Appendix F. The evolution of biomass for basal and non-basal species is modeled separately.

For basal species, we have

$$g_k(\mathbf{x};\mathbf{y},S) - x_k = \underbrace{\omega_k^r\left(1 - \frac{\sum_{l\in Basal}[x_l]}{\omega^K}\right)}_{\text{Autonomous growth}} - \underbrace{\sum_l d_{lk}\frac{\omega_l^y\omega_l^x}{\omega_{lk}^e}\frac{\omega_{lk}^w\ [x_k^h x_l]}{\omega_0^x + \sum_s d_{ls}\omega_{ls}^w\ [x_s^h]}}_{\text{Loss to predation}}, \tag{18}$$

where $d_{kl} = 1$ if k preys on l, and $d_{kl} = 0$ otherwise. Basal species k has autonomous growth and faces a global carrying capacity ω^K: when the total biomass of basal species is larger than ω^K, the autonomous growth rate is negative. The rate at which species k is eaten by predators depends on its biomass x_k and the biomass of predators x_l, as well as the biomass of other basal species x_s on which predators can also feed.

For non-basal species, we have

$$\begin{aligned} g_k(\mathbf{x};\mathbf{y},S) - x_k = & \underbrace{-\omega_k^x\ [x_k]}_{\text{Maintenance loss}} + \underbrace{\sum_l d_{kl}\omega_k^x\omega_k^y\frac{\omega_{kl}^w\ [x_l^h x_k]}{\omega_0^x + \sum_s d_{ks}\omega_{ks}^w\ [x_s^h]}}_{\text{Gain from predation}} \\ & \underbrace{-\sum_l d_{lk}\frac{\omega_l^y\omega_l^x}{\omega_{lk}^e}\frac{\omega_{lk}^w\ [x_k^h x_l]}{\omega_0^x + \sum_s d_{ls}\omega_{ls}^w\ [x_s^h]}}_{\text{Loss to predation}} - \underbrace{\sum_i c_{ki}^* y_i^*}_{\text{Loss to catches}}. \end{aligned} \tag{19}$$

Non-basal species k suffer a maintenance loss so that without predation, they would go extinct. The gains from predation depend on the biomass of species k, the biomass of its prey x_l, and the biomass of other preys x_s that predator k also consumes. The loss to predation is similar to that for basal species, although the parameters' values are typically different.

Finally, loss to catches is simply the externality from production. Here, the externality network determines which species k is used in the production of sector i and allows scaling the units of biomass of resource k into units of production in sector i. I assume that the production side of the economy is as described in Assumption 2, and calibrated parameters can be found in Table F.1 of Appendix F.

6.2 Simulation

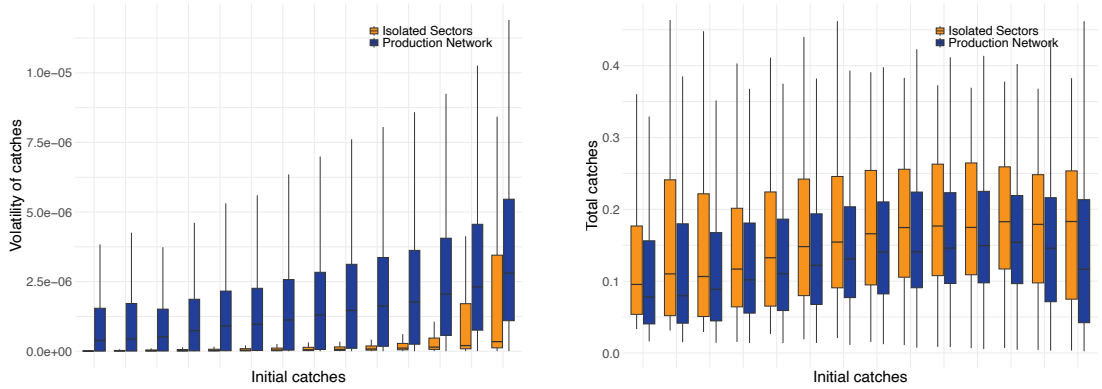
The simulation procedure goes as follows. First, and following GCV, the ecosystem is simulated without any production, and until it reaches a steady state. Second, production is introduced. I

simulate the dynamics under two alternative specifications. The first specification is a disconnected resource economy in which each sector produces independently using only labor. In the second specification, sectors are linked through a random (power law distributed) production network. Details of the simulation exercise are provided in Appendix F. There are 30 sectors and 30 species.

For each specification, and ecosystem, multiple simulations are made, varying the firm's TFP levels. TFP levels are calibrated so that production (and therefore catches) are comparable for the economies with and without a production network. Specifically, TFP levels are calibrated so that the first production period (starting from the production-free steady state) is comparable for the two specifications.

The simulation therefore results in a dataset for which an observation corresponds to a specific specie, in a specific ecosystem, under a specific production technology (TFP and network structure). I first present results for harvested species.

Figure 8 displays the volatility of catches as well as the average quantity harvested over the first 500 periods of production. For similar levels of initial catches, connected economies (i.e. the ones with a production network) display much higher volatility, but extract less resource. Higher volatility of catches (i.e. production) is anticipated since shocks resulting from changes in the biomass of resources are propagated through the network (see Proposition 2).



(a) Volatility of Catches

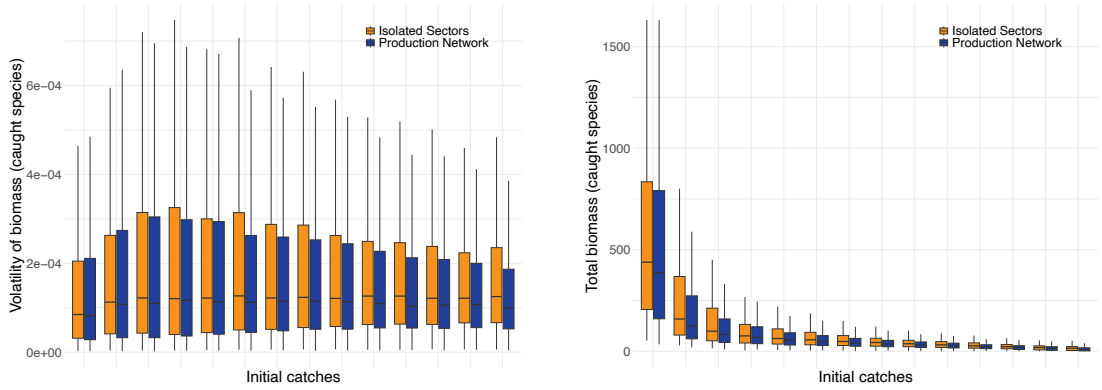
(b) Average Catches

Figure 8: The impact of the production network on catches

Note: Each observation corresponds to a specie (harvested species only), in some ecosystem, and under some production technology (TFP and network structure). The level of initial catches is discretized in 15 categories (by specie) for clarity. Volatility is defined as the standard deviation (in log scale) of catches over the first 500 periods of production. The average is also computed over the first 500 periods of production.

The fact that connected economies extract less resources on the other hand is an indication that production economies may help preserve the resource. This is however not true. Figure 9 displays the average volatility of biomass as well as the average biomass over the first 500 periods

of production. If anything, connected economies lead to lower, and slightly less volatile stocks.



(a) Volatility of Biomass

(b) Average Biomass

Figure 9: The impact of the production network on the biomass of harvested species

Note: Each observation corresponds to a specie (harvested species only), in some ecosystem, and under some production technology (TFP and network structure). The level of initial catches is discretized in 15 categories (by specie) for clarity. Volatility is defined as the standard deviation (in log scale) of biomass over the first 500 periods of production. The average quantity harvested is also computed over the first 500 periods of production.

To understand further the impact of the production network of the ecosystem, it is necessary to aggregate results at the ecosystem level. Table 1 presents the results from fixed effect regressions on the average volatility of catches and biomass, at ecosystem level. Consider columns (1) and (3): Connected economies lead to an increase of roughly 0.485 standard deviation of the aggregate volatility of catches, and 0.015 standard deviation of the aggregate volatility of the ecosystem biomass. Specifications (2) shows that the impact of the production network on catches increases with the quantity harvested. An increase of one standard deviation of the average level of initial catches leads to an additional 0.257 standard deviation of the average volatility of catches in connected economies. To put these number in perspective, note that a maximum of 5 species (out of 30 species total) can be harvested.

These effects are exemplified in Figure 10 which shows the evolution of catches and biomass in Ecosystem #752 under different production technologies.[50] We see that the introduction of catches when sectors are isolated and when sectors are linked through the production network leads to qualitativeley different extraction patterns. With the production network, catches are more volatile, leading to a more volatile ecosystem. These effects are strengthen as production intensifies.

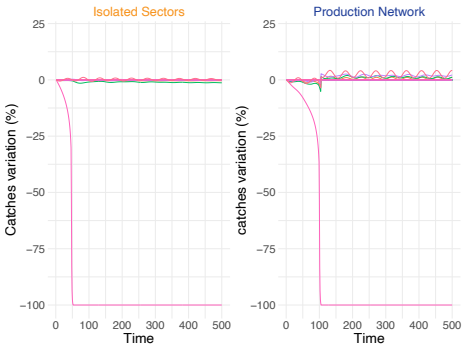
Overall, these simulations confirm the insights from the model that production networks (even exogenous ones) have non-trivial, profound, and previously neglected impacts on the dynamics of ecosystems and economies. The simulations also show that the framework presented in this paper

[50]The ecosytem is selected so that the difference in initial catches (between the economy with the production network and the one with isolated sectors) is the smallest.

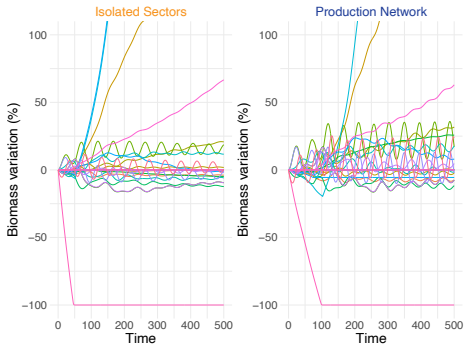
Variable	**Volatility (Catches)**		**Volatility (Biomass)**	
	(1)	(2)	(3)	(4)
Initial catches	0.707	0.592	0.211	0.212
	(0.006)	(0.007)	(0.004)	(0.006)
Production Network	0.484	-0.003	0.015	0.018
	(0.009)	(0.019)	(0.007)	(0.016)
Initial catches × Production Network	-	0.257	-	-0.002
	(-)	(0.011)	(-)	(0.009)
Ecosystem fixed effects	X	X	X	X

Notes: Volatility is defined a the standard deviation (in log scale) over the first 500 periods of production. All variables are defined at the ecosystem level. Continuous variables are averaged over all species, for a given ecosystem, and production technology (TFP and network structure) and are normalized to have a unit variance. All specifications include ecosystem fixed effects so that parameters are identified from variations across production technologies (TFP and production network) only. Heteroskedasticity robust standard errors are in parenthesis.

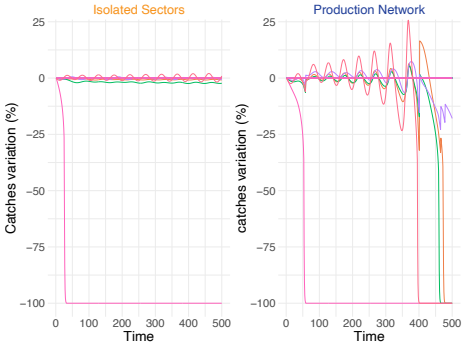
Table 1: The impact of the production network on the volatility of the ecosystems



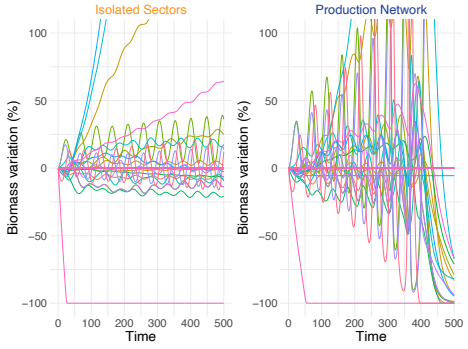
(a) Catches, Ecosystem #752 (low TFP)



(b) Biomass, Ecosystem #752 (low TFP)



(c) Catches, Ecosystem #752 (high TFP)



(d) Biomass, Ecosystem #752 (high TFP)

Figure 10: Example for ecosystem #752 under different production technologies.

Note: Each observation corresponds to a specie. Variations are expressed as percentage deviations from the initial value. Time 0 corresponds to the first period of production from the production-free steady state.

is flexible enough to be combined with state-of-the-art ecological models, leading to integrated calibrated bio-economic models.

7 Conclusion

In this paper, I present a model of the complex interactions between the economy, modeled as an $n-$sectors endogenous production network, and the environment, modeled as an $m-$resources ecological network. Building on Acemoglu and Azar (2020), I assume a contestable market and treat the ecological network as a public good. The interactions between firms create an additional layer of interactions between species, which strongly affects stability.

I show that the optimal tax acts both at the extensive margin, leading firms to choose the efficient set of suppliers, and at the intensive margin, increasing the marginal cost of production. The optimal tax is higher on the production of goods whose production imposes externalities on valuable resources. Resources' values are a function of their marginal rate of technical substitution with labor and of their Katz–Bonacich centrality in the community matrix of the ecological network.

I show that the model presented in this paper is flexible enough to integrate state-of-the-art models of ecological dynamics and present numerical simulations complementing the theoretical results. I show that production interdependencies threaten stability and increase volatility.

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A Known useful results

In this section, I present known results (with references) that are used to prove the propositions from the paper.

Theorem 1 (Poincaré–Miranda)**.** *Consider the rectangle* $\mathcal{X} = [z_1^-, z_1^+] \times \cdots \times [z_m^-, z_m^+]$ *in* $\mathbb{R}^m$*. Let* $\boldsymbol{\psi}(\mathbf{x}) = (\psi_1(\mathbf{x}), ..., \psi_m(\mathbf{x}))$ *be a continuous function from* $\mathcal{X}$ *into* $\mathbb{R}^m$*. Define* $\mathcal{F}_k^- = \{\mathbf{x} \in \mathcal{X} : x_k = z_k^-\}$ *and* $\mathcal{F}_k^+ = \{\mathbf{x} \in \mathcal{X} : x_k = z_k^+\}$*. Assume that for* $k = 1, ..., m$*, either*

$$\psi_k(\mathbf{x}) \begin{cases} \leq 0 & \text{for every } x \in \mathcal{F}_k^- \\ \geq 0 & \text{for every } x \in \mathcal{F}_k^+ \end{cases} \quad \text{or } \psi_k(\mathbf{x}) \begin{cases} \geq 0 & \text{for every } x \in \mathcal{F}_k^- \\ \leq 0 & \text{for every } x \in \mathcal{F}_k^+ \end{cases} . \tag{20}$$

Then, there exists $\mathbf{x}^* \in \mathcal{X}$ *such that* $\boldsymbol{\psi}(\mathbf{x}) = \mathbf{0}$*.*

Proof. The proof has been given by Miranda (1940) using Brouwer's fixed-point theorem. The statement above, for functions on the rectangle, is taken from Fonda and Gidoni (2016). □

The next theorem uses the following notation:

$$\boldsymbol{\Gamma} = \begin{bmatrix} \mathbf{I} - \boldsymbol{W} & \boldsymbol{B} \\ \boldsymbol{C} & \mathbf{I} - \boldsymbol{D} \end{bmatrix},$$

and

$$\boldsymbol{\Gamma}^{-1} = \begin{bmatrix} \boldsymbol{\Gamma}_{11}^{-1} & \boldsymbol{\Gamma}_{12}^{-1} \\ \boldsymbol{\Gamma}_{21}^{-1} & \boldsymbol{\Gamma}_{22}^{-1} \end{bmatrix}.$$

Theorem 2. *Assume that* $\boldsymbol{\Gamma}$ *and* $\mathbf{I} - \boldsymbol{W}$ *are non-singular. Then,*

$$\boldsymbol{\Gamma}_{11}^{-1} = (\mathbf{I} - \boldsymbol{W})^{-1} + (\mathbf{I} - \boldsymbol{W})^{-1}\boldsymbol{B}(\mathbf{I} - \boldsymbol{D} - \mathbf{C}(\mathbf{I} - \boldsymbol{W})^{-1}\mathbf{B})^{-1}\mathbf{C}(\mathbf{I} - \boldsymbol{W})^{-1},$$

$$\boldsymbol{\Gamma}_{12}^{-1} = -(\mathbf{I} - \boldsymbol{W})^{-1}\mathbf{B}(\mathbf{I} - \boldsymbol{D} - \mathbf{C}(\mathbf{I} - \boldsymbol{W})^{-1}\mathbf{B})^{-1},$$

$$\boldsymbol{\Gamma}_{21}^{-1} = -(\mathbf{I} - \boldsymbol{D} - \mathbf{C}(\mathbf{I} - \boldsymbol{W})^{-1}\mathbf{B})^{-1}\mathbf{C}(\mathbf{I} - \boldsymbol{W})^{-1}, \text{and}$$

$$\boldsymbol{\Gamma}_{22}^{-1} = (\mathbf{I} - \boldsymbol{D} - \mathbf{C}(\mathbf{I} - \boldsymbol{W})^{-1}\mathbf{B})^{-1}.$$

Assume that $\boldsymbol{\Gamma}$ *and* $\mathbf{I} - \boldsymbol{D}$ *are non-singular. Then,*

$$\boldsymbol{\Gamma}_{11}^{-1} = (\mathbf{I} - \boldsymbol{W} - \mathbf{B}(\mathbf{I} - \boldsymbol{D})^{-1}\mathbf{C})^{-1},$$

$$\boldsymbol{\Gamma}_{12}^{-1} = -(\mathbf{I} - \boldsymbol{W} - \mathbf{B}(\mathbf{I} - \boldsymbol{D})^{-1}\mathbf{C})^{-1}\mathbf{B}(\mathbf{I} - \boldsymbol{D})^{-1},$$

$$\boldsymbol{\Gamma}_{21}^{-1} = -(\mathbf{I} - \boldsymbol{D})^{-1}\mathbf{C}(\mathbf{I} - \boldsymbol{W} - \mathbf{B}(\mathbf{I} - \boldsymbol{D})^{-1}\mathbf{C})^{-1}, \textit{and}$$

$$\boldsymbol{\Gamma}_{22}^{-1} = (\mathbf{I} - \boldsymbol{D})^{-1} + (\mathbf{I} - \boldsymbol{D})^{-1}\boldsymbol{C}(\mathbf{I} - \boldsymbol{W} - \mathbf{B}(\mathbf{I} - \boldsymbol{D})^{-1}\mathbf{C})^{-1}\mathbf{B}(\mathbf{I} - \boldsymbol{D})^{-1}.$$

Proof. The result for $\boldsymbol{\Gamma}_{11}^{-1}$ and $\boldsymbol{\Gamma}_{22}^{-1}$ is usually referred as the "Woodbury matrix identity" (Woodbury, 1950). The statement above is a direct implication of Theorem 2.1 in Lu and Shiou (2002). □

B Proofs

B.1 Proof of Proposition 2

Proposition 9. *Under Assumption 2, the unique equilibrium is such that*

$$\ln \mathbf{p}_t^* = -(\mathbf{I} - \boldsymbol{W}_t^*)^{-1}[\ln \boldsymbol{a}_t^* + \boldsymbol{B}_t^* \ln \mathbf{x}_t - \ln(\mathbf{1} + \boldsymbol{\mu})].$$

Moreover,

$$\ln \mathbf{y}^*(\mathbf{x}) = \ln[(\mathbf{I} - \ddot{\boldsymbol{W}}_t^{*\top})^{-1}\mathbf{q}] - \ln \mathbf{p}_t^* - \ln \boldsymbol{\alpha},$$

and

$$u^* = \mathbf{q}^\top \ln \left[(\mathbf{I} - \dot{\mathbf{W}}_t^{*\top})(\mathbf{I} - \ddot{\mathbf{W}}_t^{*\top})^{-1}\mathbf{q}\right] - \mathbf{q}^\top \ln \mathbf{p}_t^*,$$

where $\mathbf{B}_t^* = \mathbf{B}(S_t^*)$, $w_{ij,t}^* = w_{ij}$ *if* $j \in S_{i,t}^*$ *and* 0 *otherwise, and where* $\dot{w}_{ij}^* = w_{ij}^*/(1 + \lambda_i\mu_i)$ *and* $\ddot{w}_{ij}^* = (w_{ij}^* + q_j\lambda_i\mu_i)/(1 + \lambda_i\mu_i)$, *while* $\alpha_i = (1 + \lambda_i\mu_i)/(1 + \mu_i)$.

Proof. The expression for the equilibrium prices comes directly from Acemoglu and Azar (2020), Page 42, Equation (9). The other quantities are computed as follows.

The representative consumer maximizes $\sum_i q_i \ln(\gamma_{i,t})$ under the constraint $\sum_i p_{i,t}\gamma_{i,t} = 1 + \sum_i \lambda_i \frac{\mu_i}{1+\mu_i} p_{i,t}y_{i,t}$. We obtain $p_{i,t}\gamma_{i,t} = q_i/\nu_t$, where ν_t is the Lagrange multiplier. Substituting into the budget constraint, we obtain $1/\nu_t = 1 + \sum_i \lambda_i \frac{\mu_i}{1+\mu_i} p_{i,t}y_{i,t}$, and therefore

$$\gamma_{i,t} = \frac{q_i}{p_{i,t}}(1 + \sum_j \lambda_j \frac{\mu_j}{1+\mu_j} p_{j,t}y_{j,t}). \tag{21}$$

The market clearing condition imposes that for all i and t:

$$\gamma_{i,t} + \sum_j z_{ji,t} = \alpha_i y_{i,t}, \tag{22}$$

where $\alpha_i = 1 - (1 - \lambda_i)\frac{\mu_i}{1+\mu_i} = (1 + \lambda_i\mu_i)/(1 + \mu_i)$.

From Acemoglu and Azar (2020) (Proof of Lemma B1), we have $l_{i,t} = y_{i,t}(1 - \sum_j w_{ij,t})p_{i,t}/(1+\mu_i)$ and $z_{ij,t} = y_{i,t}w_{ij,t}l_{i,t}/(1 - \sum_j w_{ij,t})p_j$. We therefore obtain

$$z_{ji,t} = y_{j,t}w_{ji,t}p_j/(1 + \mu_j)p_i. \tag{23}$$

Substituting the values in equations (21) and (23) into the market clearing condition (22), we

can rewrite (after some algebra)

$$q_i + \sum_j y_{j,t} p_{j,t} \alpha_j \left(\frac{\lambda_j q_i \mu_j + w_{ji,t}}{1 + \lambda_j \mu_j} \right) = \alpha_i p_{i,t} y_{i,t}.$$

Let $\tilde{y}_{i,t} = \alpha_i p_{i,t} y_{i,t}$ and $\ddot{w}_{ji,t} = (\lambda_j q_i \mu_j + w_{ji,t})/(1 + \lambda_j \mu_j)$, we can solve

$$\tilde{\mathbf{y}}_t = (\mathbf{I} - \ddot{\mathbf{W}}_t^\top)^{-1} \mathbf{q}.$$

Taking the logarithm element-wise, we have

$$\ln \mathbf{y}_t = \ln[(\mathbf{I} - \ddot{\mathbf{W}}_t^\top)^{-1} \mathbf{q}] - \ln \mathbf{p}_t - \ln \boldsymbol{\alpha}.$$

Finally, using the market clearing condition (22) and substituting the expression for $z_{ji,t}$ in (23), we can rewrite

$$p_{i,t} \gamma_{i,t} = \alpha_i p_i y_i - \sum_j \frac{w_{ji,t}}{1 + \lambda_j \mu_j} \alpha_j p_{j,t} y_{j,t},$$

and therefore

$$\ln \boldsymbol{\gamma}_t = \ln \left[(\mathbf{I} - \dot{\mathbf{W}}^\top)(\mathbf{I} - \ddot{\mathbf{W}}_t^\top)^{-1} \mathbf{q} \right] - \ln \mathbf{p}_t,$$

where $\dot{w}_{ij,t} = w_{ij,t}/(1 + \lambda_i \mu_i)$, and from which we get the consumer utility:

$$u = \mathbf{q}' \ln \left[(\mathbf{I} - \dot{\mathbf{W}}^\top)(\mathbf{I} - \ddot{\mathbf{W}}_t^\top)^{-1} \mathbf{q} \right] - \mathbf{q}' \ln \mathbf{p}_t.$$

□

B.2 Proof of Proposition 3

Proof. Let $\underline{x}_k = \underline{x}$ for all renewable resources $k \in K$, where $\underline{x}$ is defined in Assumption 3. Let $\underline{x}_k = x_k^0$ for all nonrenewable resources $k \notin K$. Consider the function $\boldsymbol{\psi}$ from $[0, \underline{x}_1] \times \cdots \times [0, \underline{x}_m]$ into $\mathbb{R}^m$ such that $\boldsymbol{\psi}(\mathbf{x}) = \mathbf{g}(\mathbf{x}; \mathbf{y}(\mathbf{x})) - \mathbf{x}$. Note that since S is fixed, $\mathbf{y}(\mathbf{x})$ is continuous and so $\boldsymbol{\psi}(\mathbf{x})$ is also continuous.

By the feasibility constraint, we necessarily have that $\psi_k([0, \mathbf{x}_{-k}]) \geq 0$ for all k. Under Assumption 3, we have that $\psi_k[\underline{x}, \mathbf{x}_{-k}]) \leq 0$ for all renewable resources $k \in K$, whereas $\psi_k[\underline{x}, \mathbf{x}_{-k}]) \leq 0$ holds by definition for all nonrenewable resources $k \notin K$. Then, by application of the Poincaré–Miranda theorem (see Theorem 1 in Appendix A), there exists $\mathbf{x}^*$ such that $\boldsymbol{\psi}(\mathbf{x}^*) = \mathbf{0}$. □

B.3 Proof of Proposition 4 and Proposition 6

Lemma 3. *Consider the dynamical process* $\dot{\mathbf{x}}_{t+1} - \dot{\mathbf{x}}_t = \Delta D(\dot{\mathbf{x}}_t)[\mathbf{b} - \mathbf{A}\dot{\mathbf{x}}_t]$*, where* $\dot{\mathbf{x}}_t, \dot{\mathbf{x}}_{t+1} \in \tilde{X} \subset \mathbb{R}^m_{++}$*, and* $\tilde{X}$ *includes the element* $\mathbf{A}^{-1}\mathbf{b}$*, and where* $D(\dot{\mathbf{x}})$ *is a diagonal matrix with* $\dot{\mathbf{x}}$ *on the diagonal and* $\Delta > 0$*. Then, in fast enough economies, if* $\mathbf{A}$ *is Lyapunov Diagonally Stable, then* $\mathbf{x}^* = \mathbf{A}^{-1}\mathbf{b}$ *is asymptotically stable.*

Proof. The generic condition for asymptotic stability in discrete time settings is that $\mathbf{J}^\top \boldsymbol{P}\mathbf{J} - \boldsymbol{P}$ is negative definite for some positive diagonal matrix $\mathbf{P}$, where $\mathbf{J}$ is the Jacobian of the dynamic system, evaluated at the steady state. We immediately find: $\mathbf{J} = \mathbf{I} - \Delta Diag(\mathbf{x}^*)\mathbf{A}$.

Substituting, we find:

$$-\Delta \mathbf{A}^\top Diag(\mathbf{x}^*)\boldsymbol{P} - \Delta \boldsymbol{P} Diag(\mathbf{x}^*)\mathbf{A} + \Delta^2 \mathbf{A}^\top Diag(\mathbf{x}^*)^2 \boldsymbol{P}\mathbf{A},$$

Now, let $\mathbf{P} = \boldsymbol{\Lambda} Diag(\mathbf{x}^*)^{-1}$, we have:

$$-\Delta \mathbf{A}^\top \boldsymbol{\Lambda} - \Delta \boldsymbol{\Lambda}\mathbf{A} + \Delta^2 \mathbf{A}^\top Diag(\mathbf{x}^*)\boldsymbol{\Lambda}\mathbf{A}.$$

Finally, note that as $\Delta \to 0$, the linear term dominates and the result follows. □

Proposition 10. *Suppose that assumptions 2 and 4 hold. Consider* X^* *as defined in Assumption 4 and consider the following fixed point equation for* $\mathbf{W}^* = \mathbf{W}^*(S(\mathbf{x}))$*,* $\mathbf{B}^* = \mathbf{B}^*(S(\mathbf{x}))$*,* $\mathbf{C}^* = \mathbf{C}^*(S(\mathbf{x}))$ *and* $\mathbf{a}^* = \mathbf{a}^*(\mathbf{W}^*)$*:*

$$\begin{aligned} \ln \mathbf{x}^* \quad = \quad & [\mathbf{I} - \mathbf{D} - \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^*]^{-1} \Big[\mathbf{r} + \mathbf{C}^* \ln[(\mathbf{I} - \ddot{\boldsymbol{W}}^{*\top})^{-1}\mathbf{q}] \\ & + \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1}(\ln \mathbf{a}^* - \ln(\mathbf{1} + \boldsymbol{\mu})) - \mathbf{C}^* \ln \boldsymbol{\alpha}\Big]. \end{aligned} \tag{24}$$

Then, any steady state in X^* *must solve (24). Moreover, consider any solution to (24)* $\mathbf{x}^* \in X^*$*, then (i)* $\mathbf{x}^*$ *is a steady state in* X^**, and (ii) if* $S^*(\mathbf{x}^*)$ *is constant in a neighborhood of* $\mathbf{x}^*$*, then* $\mathbf{x}^*$ *is asymptotically stable in fast enough economies whenever* $[\mathbf{I} - \mathbf{D} - \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^*]$ *is Lyapunov-diagonally stable.*

Proof. Equation (24) is obtained by substituting the expression for $\mathbf{y}_t^*$ in Proposition 9 into the fixed point Equation (9) and solving.

Now, let $\tilde{X}$ be a neighborhood around $\ln \mathbf{x}^*$ as defined in Equation (24) and define

$$\begin{aligned}
\mathbf{A} &= \mathbf{I} - \mathbf{D} - \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^* \\
\mathbf{b} &= [\mathbf{r} + \mathbf{C}^* \ln[(\mathbf{I} - \ddot{\boldsymbol{W}}^{*\top})^{-1}\mathbf{q}], + \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1}(\ln \mathbf{a}^* - \ln(\mathbf{1} + \boldsymbol{\mu})) - \mathbf{C}^* \ln \boldsymbol{\alpha} \\
\dot{\mathbf{x}} &= \ln \mathbf{x}.
\end{aligned}$$

If $S(\mathbf{x})$ is constant in a neighborhood around $\mathbf{x}^*$, then $\mathbf{A}$ and $\mathbf{b}$ are constant in $\mathbf{x}$ (on X^*). Thus, Lemma 3 shows the asymptotic stability. □

Proposition 4 and Proposition 6:

Proposition 10 reduces to Proposition 4 when S is fixed and $\boldsymbol{\mu} = \mathbf{0}$. Proposition 10 reduces to Proposition 6 when $\boldsymbol{\mu} = \mathbf{0}$.

B.4 Proof of Proposition 5

Proposition 11. *Suppose that assumptions 2 and 4 hold. Then, the equilibrium associated to any given steady state in X^* (when one exists, see Proposition 10) has the form*

$$\begin{aligned}
\ln \mathbf{p}^* = & -\left[\mathbf{I} - \mathbf{W}^* - \mathbf{B}^*(\mathbf{I} - \mathbf{D})^{-1}\mathbf{C}^*\right]^{-1} \times \\
& \left(\ln \mathbf{a}^* - \ln(\mathbf{1} + \boldsymbol{\mu}) + \mathbf{B}^*(\mathbf{I} - \mathbf{D})^{-1}\mathbf{r} + \mathbf{B}^*(\mathbf{I} - \mathbf{D})^{-1}\mathbf{C}^* \ln[(\mathbf{I} - \ddot{\boldsymbol{W}}^{*\top})^{-1}\mathbf{q}]\right. \\
& \left. -\mathbf{B}^*(\mathbf{I} - \mathbf{D})^{-1}\mathbf{C}^* \ln \boldsymbol{\alpha}\right).
\end{aligned}$$

Proof. The equation for the equilibrium price is obtained by substituting the equilibrium stock of resources in Equation (24) into the price equation given in Proposition 9 and by applying the two following matrix identities (see Theorem 2 in Section A):

$$\begin{aligned}
(\mathbf{I} - \mathbf{W}^*)^{-1} + (\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^* \left[\mathbf{I} - \mathbf{D} - \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^*\right]^{-1} \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1} = \\
\left[\mathbf{I} - \mathbf{W}^* - \mathbf{B}^*(\mathbf{I} - \mathbf{D})^{-1}\mathbf{C}^*\right]^{-1},
\end{aligned}$$

and

$$\left[\mathbf{I} - \mathbf{W}^* - \mathbf{B}^*(\mathbf{I} - \mathbf{D})^{-1}\mathbf{C}^*\right]^{-1} \mathbf{B}^*(\mathbf{I} - \mathbf{D})^{-1} = (\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^*(\mathbf{I} - \mathbf{D}^* - \mathbf{C}^*(\mathbf{I} - \mathbf{W}^*)^{-1}\mathbf{B}^*)^{-1}$$

□

B.5 Proof of Proposition 7

Proof.

Existence and Equation (14)

Start by fixing the sets of input choices, the recursive SP problem is given by

$$v(\mathbf{x}) = \max_{\boldsymbol{\gamma},\mathbf{Z},\mathbf{l}} \{u(\boldsymbol{\gamma}) + \delta v[\mathbf{g}(\mathbf{x};\mathbf{y},S)]\}$$

under the following constraints:

$$y_i = f_i(S_i, \mathbf{a}_i(S_i), l_i, \mathbf{z}_i, \mathbf{x}) \quad \forall i,$$

$$1 - \sum_i l_i \geq 0, and$$

$$y_j - \sum_i z_{ij} - \gamma_j \geq 0 \quad \forall j.$$

The Kuhn–Tucker objective function is

$$\begin{aligned} L \quad = \quad & u(\boldsymbol{\gamma}) + \sum_j \psi_j \left[f_j(S_j, \mathbf{a}_j(S_j), l_j, \mathbf{z}_j, \mathbf{x}) - \sum_i z_{ij,t} - \gamma_j \right] + \psi_0 (1 - \sum_i l_i) \\ & + \delta v \left[\{ g_k(\mathbf{x}, \{ f_j(S_{j,t}, \mathbf{a}_j(S_j), l_j, \mathbf{z}_j, \mathbf{x}) \}_j, S) \right], \end{aligned}$$

where $\{\psi_j\}_{j=0}^n$ are positive multipliers. The first-order conditions are

$$\frac{\partial L}{\partial \gamma_i} = \frac{\partial u}{\partial \gamma_i} - \psi_i = 0 \quad \forall i,$$

$$\frac{\partial L}{\partial z_{ij}} = -\psi_j + \left[\psi_i + \delta \sum_k \frac{\partial v}{\partial x_k} \frac{\partial g_k}{\partial y_i} \right] \frac{\partial f_i}{\partial z_{ij}} = 0$$

whenever $j \in S_i$, and

$$\frac{\partial L}{\partial l_i} = -\psi_0 + \left[\psi_i + \delta \sum_k \frac{\partial v}{\partial x_k} \frac{\partial g_k}{\partial y_i} \right] \frac{\partial f_i}{\partial l_i} = 0.$$

In particular, we have

$$\frac{\partial f_i}{\partial z_{ij}} = \frac{\psi_j}{\psi_i + \delta \sum_k \frac{\partial v}{\partial x_k} \frac{\partial g_k}{\partial y_i}}$$

whenever $j \in S_i$, and

$$\frac{\partial f_i}{\partial l_i} = \frac{\psi_0}{\psi_i + \delta \sum_k \frac{\partial v}{\partial x_k} \frac{\partial g_k}{\partial y_i}}.$$

The market equilibrium solves (for firms)

$$\frac{\partial f_i}{\partial z_{ij}} = \frac{p_j^*(1+\mu_i)}{p_i^*}$$

whenever $j \in S_i$, and

$$\frac{\partial f_i}{\partial l_i} = \frac{(1+\mu_i)}{p_{i,t}^*},$$

as well as (from the consumer's problem)

$$\frac{\partial u}{\partial \gamma_i} = \phi^* p_i^*,$$

where ϕ^* is the multiplier from the budget constraint.

We therefore have $\psi_j = \phi^* p_j^*$; the optimal tax (for a fixed network) solves

$$\tau_i(\mathbf{x}, S) = -\frac{\delta}{\phi^*} \sum_k \frac{\partial v}{\partial x_k} \frac{\partial g_k}{\partial y_i},$$

where the last expression uses the fact that $p_{i,t}^* = (1+\mu_i)\kappa_i^*$. This analysis assumes S and $\mathbf{x}$ are fixed. That is, $\boldsymbol{\mu}^* = \boldsymbol{\mu}^*(\mathbf{x}, S)$. Consider the optimal set of inputs $S^E(\mathbf{x})$. Letting $\boldsymbol{\mu}^*(\mathbf{x}, S) \to \infty$ whenever $S \neq S^E(\mathbf{x})$ guarantees that the set of inputs $S^E(\mathbf{x})$ will be chosen by firms.

Steady state, Equation (15)

For all $\mathbf{x}$, we have

$$v(\mathbf{x}) = u(\{f_i(S_i^E, l_i^E, \mathbf{z}_i^E, \mathbf{x}) - \sum_j z_{ji}^E\}_i) + \delta v(g(\mathbf{x}, \{f_i(S_i^E, l_i^E, \mathbf{z}_i^E, \mathbf{x})\}_i, S^E).$$

By the envelope theorem, we have (generically):

$$\frac{\partial v}{\partial x_k} = \sum_i \frac{\partial u}{\partial \gamma_i} \frac{\partial f_i}{\partial x_k} + \delta \sum_l \frac{\partial v}{\partial x_l} \frac{\partial g_l}{\partial x_k} + \sum_i \left[\delta \sum_l \frac{\partial v}{\partial x_l} \frac{\partial g_l}{\partial y_i} \right] \frac{\partial f_i}{\partial x_k},$$

or substituting using the first-order conditions:

$$\frac{\partial v}{\partial x_k} = \sum_i \psi_i \frac{\partial f_i}{\partial x_k} + \delta \sum_l \frac{\partial v}{\partial x_l} \frac{\partial g_l}{\partial x_k} + \sum_i \left[\left(\frac{\partial f_i}{\partial l_i} \right)^{-1} - \psi_i \right] \frac{\partial f_i}{\partial x_k},$$

and thus,

$$\frac{\partial v}{\partial x_k} = \sum_i \frac{\partial f_i}{\partial x_k} \Big/ \frac{\partial f_i}{\partial l_i} + \delta \sum_l \frac{\partial v}{\partial x_l} \frac{\partial g_l}{\partial x_k},$$

or

$$\frac{\partial v}{\partial x_k} = \zeta_k + \delta \sum_l \frac{\partial v}{\partial x_l} \frac{\partial g_l}{\partial x_k}.$$

Using matrix notation: $\boldsymbol{\nu}^\top = {\boldsymbol{\zeta}^E}^\top + \delta \boldsymbol{\nu}^\top \boldsymbol{\mathcal{D}}^E$, and thus $\boldsymbol{\nu}^\top = {\boldsymbol{\zeta}^E}^\top [\mathbf{I} - \delta \boldsymbol{\mathcal{D}}^E]^{-1}$.

□

B.6 Proof of Proposition 8

Proof. Letting $\tilde{\tau}_i = \delta \sum_k \frac{\partial v}{\partial x_k} \frac{\partial g_k}{\partial y_i}$, the maximization of the social planner's problem leads to (see the proof of Proposition 7)

$$\frac{\partial f_i}{\partial z_{ij}} = \frac{\psi_j}{\psi_i + \tilde{\tau}_i} = \frac{w_{ij} y_i}{z_{ij}}$$

whenever $j \in S_i$, and

$$\frac{\partial f_i}{\partial l_i} = \frac{\psi_0}{\psi_i + \tilde{\tau}_i} = \frac{y_i (1 - \sum_j w_{ij})}{l_i},$$

while

$$\psi_i = q_i / \gamma_i.$$

Substituting z_{ij} and l_i into the market clearing condition $y_j - \sum_i z_{ij} - \gamma_j = 0$, we obtain

$$\psi_j y_j - \sum_i w_{ij} y_i (\psi_i + \tilde{\tau}_i) = q_j.$$

Letting $\tilde{y}_i = y_i(\psi_i + \tilde{\tau}_i)$, we obtain

$$(\mathbf{A} - \mathbf{W}^\top) \tilde{\mathbf{y}} = \mathbf{q},$$

where $\mathbf{A}$ is a diagonal matrix with diagonal elements $a_{ii} = \psi_i / (\psi_i + \tilde{\tau}_i) = (1 + \mu_i^E)$.

Now, the MRTS for the log-stock of resource k in sector i is given by

$$\zeta_{ik,\ln} = b_{ik} y_i \Big/ \frac{\partial f_i}{\partial l_i} = b_{ik} y_i (\psi_i + \tilde{\tau}_i) = b_{ik} \tilde{y}_i.$$

Thus, the MRTS for the log-stock of resources is

$$\boldsymbol{\zeta}^E_{\ln} = \mathbf{B}^\top [\mathbf{A} - \mathbf{W}^\top]^{-1} \mathbf{q}.$$

Now, for the Jacobian for the evolution of resources, under Assumption 4, we have

$$\frac{\partial \ln g_k}{\ln x_l} = d_{kl} \ln x_k$$

for all $l \neq k$ and k, and

$$\frac{\partial \ln g_k}{\ln x_k} = (1 + r_k + [\boldsymbol{D} \ln \mathbf{x}]_k - \ln x_k + [\boldsymbol{C} \ln \mathbf{y}]_k) + \ln x_k (d_{kk} - 1)$$

for all k. Evaluated at the steady state and writing in matrix form, we obtain

$$\boldsymbol{\mathcal{D}}^E_{\ln} = \mathbf{I} + diag(\ln \mathbf{x}^E)(\mathbf{D} - \mathbf{I}).$$

Thus,

$$\boldsymbol{\zeta}^{E^\top}_{\ln}[\mathbf{I} - \delta \boldsymbol{\mathcal{D}}^E_{\ln}]^{-1} = \mathbf{q}^\top [\mathbf{A} - \mathbf{W}^E]^{-1} \mathbf{B}[(1-\delta)\mathbf{I} + \delta diag(\ln \mathbf{x}^e)(\mathbf{I} - \mathbf{D})]^{-1}.$$

Finally, note that we can write $\mathbf{A} - \mathbf{W} = \mathbf{I} - \tilde{\mathbf{W}}$, where $\tilde{w_{ij}} = w_{ij}$, and $\tilde{w_{ii}} = w_{ii} - \mu_i$. □

C Additional results

Corollary 3. *We have:* $[\mathbf{q}'(\mathbf{I}-\mathbf{W}_t)^{-1}]_i = p_{i,t}y_{i,t}/\sum_j \gamma_{j,t}p_{j,t}$

Proof. Without distortions, the budget constraint of the representative consumer in Equation (3) becomes:

$$\sum_i p_{i,t}\gamma_{i,t} = 1.$$

It therefore remains to show that $[\mathbf{q}'(\mathbf{I}-\mathbf{W}_t)^{-1}]_i = p_{i,t}y_{i,t}$. Equivalently,

$$\ln[\mathbf{q}'(\mathbf{I}-\mathbf{W}_t)^{-1}] = \ln \mathbf{p}_t + \ln \mathbf{y}_t$$

Which is implied by expressions in Proposition 2. □

C.1 Stylized economy in Section 4.2.1

I use the specifications in assumptions 2 and 4, and I set the following parameters: $q_1 = q_2 = 1/2$, $a_1(\emptyset) = a_2(\emptyset) = 2$, $a_1(\{2\}) = 10$, $W_{12}(\{2\}) = 0.25$, $B_{11}(\emptyset) = 0.5$, $C_{11}(\emptyset) = -1$,$r_1 = 5$, $D_{11} = -0.2$ and $x_{1,0} = 64.5$. The other parameters are set to 0.

C.2 Failure of existence

Consider an economy with only one sector and one renewable resource. I assume that a variant of Assumption 2 holds for which the technology (including TFP and ecosystem services) is $a_1(\{1\}) = 1$ and $a_1(\emptyset) = bx_t$. Straightforward computations show that the unit cost of production for $S_1 = \emptyset$ is $1/bx_t$, whereas it is 1 for $S_1 = \{1\}$. Production therefore proceeds with technology $S_1 = \emptyset$ iff $x_t \geq 1/b$. In such case, production is given by $1/(bx_t)$

I assume that the evolution of the resource follows a simple logistic growth:

$$x_{t+1} = g(x_t) = \max\{\epsilon, x_t + rx_t(1-x_t/K) - \mathbb{1}\{S_1 = \emptyset\}y_t(x_t)x_t\},$$

where $\epsilon > 0$ but is arbitrarily close to 0.[51] Fixing, $1/b = 0.5$, $K = 2/3$, $r = 1$ for example, there is no x^* such that $g(x^*) = x^*$. Figure C.1 illustrates.

[51] If $\epsilon = 0$, then there exists a trivial steady state at $x = 0$.

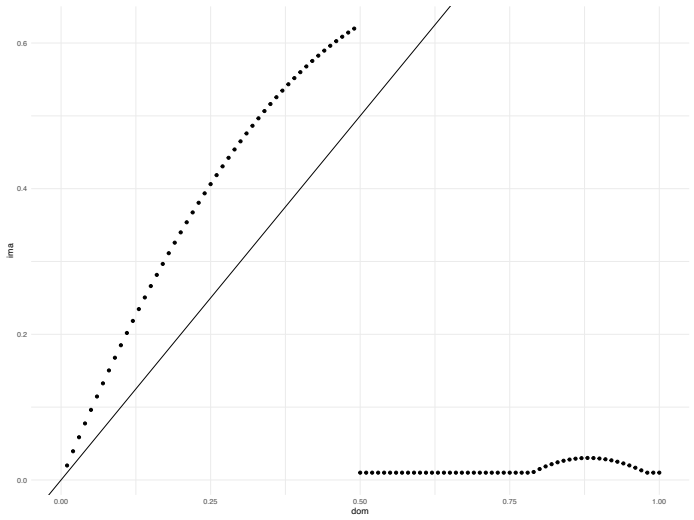


Figure C.1: Simple one-species, one-sector economy

C.3 Cosine similarity

The *cosine similarity* between two (potentially complex) vectors $\mathbf{a}$ and $\mathbf{b}$ is defined as

$$\rho(\mathbf{a}, \mathbf{b}) = \frac{|\langle \mathbf{a}, \mathbf{b} \rangle|}{\|\mathbf{a}\| \|\mathbf{b}\|} \in [0, 1],$$

where $\langle \mathbf{a}, \mathbf{b} \rangle$ is the inner product between $\mathbf{a}$ and $\mathbf{b}$. The cosine similarity is equal to the cosine of the *Hermitian angle* between $\mathbf{a}$ and $\mathbf{b}$, which can be interpreted as a measure of dependence between two vectors.[52] Thus, $\rho(\mathbf{a}, \mathbf{b}) = 0$ means that the vectors are orthogonal, whereas $\rho(\mathbf{a}, \mathbf{b}) = 1$ means that the vectors have the same orientation, given a same-sign restriction. In real vector spaces, cosine similarity has been similarly applied in economics in Galeotti et al. (2020) and Pellegrino (2023).

Proposition 12. *Under the conditions of Proposition 7, let* $\boldsymbol{\nu}^{E\top} = \boldsymbol{\zeta}^{E\top}[\mathbf{I} - \delta \boldsymbol{\mathcal{D}}^E]^{-1}$. *If additionally* $\boldsymbol{\mathcal{D}}^E$ *is diagonalizable, then for* $k = 1, ..., m$,

$$\frac{\rho(\boldsymbol{\zeta}^E, \mathbf{e}_k)}{\rho(\boldsymbol{\nu}^E, \mathbf{e}_k)} \propto |1 - \delta \lambda_k|, \tag{25}$$

where $\mathbf{e}_k$ *and* λ_k *are the* k*'th right eigenvector and* k*'th eigenvalue of* $\boldsymbol{\mathcal{D}}^E$.

Proof. Using the eigen decomposition, we have (dropping the subscript E for simplicity)

$$\boldsymbol{\nu}^\top[\mathbf{I} - \delta \mathbf{E}\boldsymbol{\Lambda}\mathbf{E}^{-1}] = \boldsymbol{\zeta}^\top = \boldsymbol{\nu}^\top \mathbf{E}[\mathbf{I} - \delta \boldsymbol{\Lambda}]\mathbf{E}^{-1}.$$

[52] See e.g. Scharnhorst (2001) for a recent discussion of the difference between Hermitian and Euclidean angles in complex vector spaces. For a textbook treatment on Hermitian geometry, see Gorodentsev (2016).

Equivalently,

$$\boldsymbol{\zeta}^\top \mathbf{E} = \boldsymbol{\nu}^\top \mathbf{E}[\mathbf{I} - \delta\boldsymbol{\Lambda}].$$

For any given eigenvector k, we have

$$\boldsymbol{\zeta}^\top \mathbf{e}_k = \boldsymbol{\nu}^\top \mathbf{e}_k[1 - \delta\lambda_k].$$

$$\frac{\|\boldsymbol{\zeta}\|}{\|\boldsymbol{\nu}\|}\frac{\rho(\boldsymbol{\zeta}, \mathbf{e}_k)}{\rho(\boldsymbol{\nu}, \mathbf{e}_k)} = |1 - \delta\lambda_k| \leq 1 + \delta|\lambda_k|.$$

□

Equation (25) provides better insight into the role of the ecological network.[53] Recall that $\boldsymbol{\mathcal{D}}^E$ is the Jacobian of the dynamic process governing the evolution of resources. Thus, its eigenvectors give the orientations such that a small change in the stock of resources at time t leads to a proportional change in the stock of resources at time $t+1$. Equation (25) means that ratio between $\rho(\boldsymbol{\nu}^E, \mathbf{e}_k)$ and $\rho(\boldsymbol{\zeta}^E, \mathbf{e}_k)$ is proportional to $|1 - \delta\lambda_k|$. Thus, the impact of the ecological network on resources value is mostly impacted by the absolute values of its eigenvalues associated with eigenvectors that are cosine similar to resources' marginal rates of technical substitution.

For concreteness, consider a naive policy aimed at increasing the stock of resources with direct productive value, i.e. an increase in the stock that is proportional to $\boldsymbol{\zeta}$. Thus, the difference between this naive policy and a policy aimed at increasing the value of resources $\boldsymbol{\nu}$ is given by $|1 - \delta\lambda_k|$. Consequently, provided the eigenvectors are not highly skewed, less dispersed eigenvalues tend to keep the naive policy closer to the optimal policy.

Beyond this interpretation, Equation (25) has an additional benefit. Indeed, while we do not know the details of interaction matrix $\boldsymbol{\mathcal{D}}^E$ itself, we do know a fair amount about the distribution of its spectrum, which is of a lower dimension.[54] In particular, Equation (25) provides a natural heuristic: in networks with a reasonably well-conditioned eigenbasis, overall similarity between $\boldsymbol{\nu}$ and $\boldsymbol{\zeta}$ is greater when the spectral radius of $\boldsymbol{\mathcal{D}}^E$ is small. The spectral radius has been found to be a good measure of the complexity and nestedness of the ecological network (e.g. Staniczenko et al. (2013); Strydom et al. (2021)). This implies that for complex, highly nested ecological networks, the social value of the resources is less influenced by their substitutibility in the production process. I expore this further in Section 6.

[53] Contrary to Galeotti et al. (2020), $\boldsymbol{\mathcal{D}}^E$ is in general non-symmetric, and the eigenvectors cannot be interpreted as principal components because the eigenvalue decomposition and singular value decomposition of $\boldsymbol{\mathcal{D}}^E$ do not coincide.

[54] For example, the empirical findings and theoretical simulations of Barabás and Allesina (2015), Staniczenko et al. (2013) and Strydom et al. (2021) imply that the real parts of the eigenvalues of $(\boldsymbol{\mathcal{D}}^E - \mathbf{I})$ are negative. Eigenvalues with small (in absolute value) real parts are more likely to be complex. Note also that because $\boldsymbol{\mathcal{D}}^E$ is a real matrix, complex eigenvalues (eigenvectors) come in conjugate pairs, implying that the distribution of eigenvalues is symmetric relative to the real line in the complex plane. This also reduces the number of "sufficient statistics" for Equation (25).

C.4 Link with Pigouvian taxation

In this section, I consider the more structured environment in which externalities are mediated by explicit pollutants. Define the function $\xi_{ik}(y_i, S_i)$ as the pollutant, impacting good k and emitted by the production of good i.

Note that by definition we can rewrite:

$$\frac{\partial g_k(\mathbf{x}, S)}{\partial y_i} = \frac{\partial g_k(\mathbf{x})}{\partial \xi_{ik}} \frac{\partial \xi_{ik}(y_i, S_i)}{\partial y_i}.$$

Then Equation (14) can be written as:

$$\tau_i(\mathbf{x}, S^E) = -\frac{\delta}{\phi} \sum_k \frac{\partial v}{\partial x_k}(\mathbf{x}, S^E) \frac{\partial g_k(\mathbf{x})}{\partial \xi_{ik}} \frac{\partial \xi_{ik}(y_i^E, S_i^E)}{\partial y_i}.$$

Suppose now that ξ_{ik} is observed/measured so that a tax can be levied on ξ_{ik}. The "Pigouvian" tax on the externality would therefore be:

$$\tau_{ik}^P(\mathbf{x}, S^E) = -\frac{\delta}{\phi} \frac{\partial v}{\partial x_k}(\mathbf{x}, S^E) \frac{\partial g_k(\mathbf{x})}{\partial \xi_{ik}},$$

from which we obtain:

$$\tau_i(\mathbf{x}, S^E) = \sum_k \tau_{ik}^P(\mathbf{x}, S^E) \frac{\partial \xi_{ik}(y_i^E, S_i^E)}{\partial y_i}.$$

Thus, the optimal tax paid by sector i is given by the sum of Pigouvian taxes for the externalities generated by its production. Note that here, the multiple externalities are implies that taxation needs to be sector *and* resource specific.

D Local analysis for generic specifications

As discussed, assumptions 2 and 4 are helpful as they allow describing the equilibrium quantities of interest, in closed-form, as a function of the production network, the ecological network, the externality network and the network of ecosystem services. They are, however, restrictive. In this section, I show that we can recover similar expressions under the following assumption.

Assumption 5. *There exists a strictly interior steady state such that*

$$\mathbf{x}^* = \mathbf{g}(\mathbf{x}^*; \mathbf{y}^*, S^*) \equiv \mathbf{g}^*(\mathbf{x}^*; \mathbf{y}^*(\mathbf{x}^*), S^*(\mathbf{x}^*)),$$

where $\mathbf{y}^*(\mathbf{x}^*)$ *are the equilibrium input choices when the stock of resources is* $\mathbf{x}^*$ *and such that* $\mathbf{g}$*is locally differentiable around* $(\mathbf{x}^*, \mathbf{y}^*)$*. Moreover,* S^* *is locally constant around* $\mathbf{x}^*$*.*

Assumption 5 implies in particular that small changes in the stock of resources do not lead to changes in the firms' network of suppliers. Define also the mapping $\mathbf{h}(\mathbf{y}, \mathbf{x}) = \mathbf{y}^*(\mathbf{g}(\mathbf{x}; \mathbf{y}, S))$.

For Proposition 13 below, it will be convenient to make explicit the dependence of $\mathbf{h}$ and $\mathbf{g}$ on a strictly positive vector $\boldsymbol{\alpha}$:

$$\begin{bmatrix} \mathbf{y}^* \\ \mathbf{x}^* \end{bmatrix} = \begin{bmatrix} \mathbf{h}^*(\mathbf{y}^*, \mathbf{x}^*; \boldsymbol{\alpha}) \\ \mathbf{g}^*(\mathbf{x}^*, \mathbf{y}^*; \boldsymbol{\alpha}) \end{bmatrix}.$$

For example, $\boldsymbol{\alpha}$ may capture both the resources' autonomous growth rate $\mathbf{r} + \mathbf{1}$ (as in Assumption 4) and the consumer's preference weights $\mathbf{q}$ (see Assumption 2). That is $\boldsymbol{\alpha} = [(\mathbf{r} + \mathbf{1})', \mathbf{q}']'$.

Moreover, because (under Assumption 5) the steady state is assumed to be strictly interior, we can rewrite the steady state in logarithm without any loss in generality. This allows interpreting local changes as elasticities.

$$\begin{bmatrix} \ln \mathbf{y}^* \\ \ln \mathbf{x}^* \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{h}}^*(\ln \mathbf{y}^*, \ln \mathbf{x}^*; \ln \boldsymbol{\alpha}) \\ \hat{\mathbf{g}}^*(\ln \mathbf{x}^*, \ln \mathbf{y}^*; \ln \boldsymbol{\alpha}) \end{bmatrix}.$$

Under Assumption 5, we can easily compute

$$\begin{bmatrix} \frac{\partial \ln \mathbf{y}^*}{\partial \ln \boldsymbol{\alpha}} \\ \frac{\partial \ln \mathbf{x}^*}{\partial \ln \boldsymbol{\alpha}} \end{bmatrix} = \begin{bmatrix} \frac{\partial \hat{\mathbf{h}}^*(\ln \mathbf{y}^*, \ln \mathbf{x}^*; \boldsymbol{\alpha})}{\partial \ln \mathbf{y}} & \frac{\partial \hat{\mathbf{h}}^*(\ln \mathbf{y}^*, \ln \mathbf{x}^*; \ln \boldsymbol{\alpha})}{\partial \ln \mathbf{x}} \\ \frac{\partial \hat{\mathbf{g}}^*(\ln \mathbf{x}^*, \ln \mathbf{y}^*; \ln \boldsymbol{\alpha})}{\partial \ln \mathbf{y}} & \frac{\partial \hat{\mathbf{g}}^*(\ln \mathbf{x}^*, \ln \mathbf{y}^*; \ln \boldsymbol{\alpha})}{\partial \ln \mathbf{x}} \end{bmatrix} \begin{bmatrix} \frac{\partial \ln \mathbf{y}^*}{\partial \ln \boldsymbol{\alpha}} \\ \frac{\partial \ln \mathbf{x}^*}{\partial \ln \boldsymbol{\alpha}} \end{bmatrix} + \begin{bmatrix} \frac{\partial \hat{\mathbf{h}}^*(\ln \mathbf{y}^*, \ln \mathbf{x}^*; \ln \boldsymbol{\alpha})}{\partial \ln \boldsymbol{\alpha}} \\ \frac{\partial \hat{\mathbf{g}}^*(\ln \mathbf{x}^*, \ln \mathbf{y}^*; \ln \boldsymbol{\alpha})}{\partial \ln \boldsymbol{\alpha}} \end{bmatrix},$$

or in matrix form:

$$\boldsymbol{\Delta}^* = \begin{bmatrix} \boldsymbol{\mathcal{W}}^* & \boldsymbol{\mathcal{B}}^* \\ \boldsymbol{\mathcal{C}}^* & \boldsymbol{\mathcal{D}}^* \end{bmatrix} \boldsymbol{\Delta}^* + \boldsymbol{\Sigma}^*,$$

where I use a similar notation as the one used for the production network, the ecological network, the externality network and the network of ecosystem services. Note, however, that contrary to the matrices defined in assumptions 2 and 4, here the matrices $\boldsymbol{\mathcal{W}}^*$, $\boldsymbol{\mathcal{B}}^*$, $\boldsymbol{\mathcal{C}}^*$ and $\boldsymbol{\mathcal{D}}^*$ only capture the local network around the steady state. Still, they capture exactly the same economic idea.

Solving, we obtain an explicit formula for the matrix of elasticities $\boldsymbol{\Delta}^*$:

$$\boldsymbol{\Delta}^* = \begin{bmatrix} \mathbf{I} - \boldsymbol{\mathcal{W}}^* & -\boldsymbol{\mathcal{B}}^* \\ -\boldsymbol{\mathcal{C}}^* & \mathbf{I} - \boldsymbol{\mathcal{D}}^* \end{bmatrix}^{-1} \boldsymbol{\Sigma}^*, \tag{26}$$

whenever the inverse exists. Proposition 13 provides more details.

Proposition 13. *Suppose that Equation (26) holds. Then, if* $(\mathbf{I} - \boldsymbol{\mathcal{D}}^*)$ *is invertible and* $\partial \hat{\mathbf{g}}^*(\ln \mathbf{x}^*, \ln \mathbf{y}^*; \ln \boldsymbol{\alpha})/\partial \ln \boldsymbol{\alpha} = \mathbf{0}$*,*

$$\frac{\partial \ln \mathbf{y}^*}{\partial \ln \boldsymbol{\alpha}} = [\mathbf{I} - \boldsymbol{\mathcal{W}}^* - \boldsymbol{\mathcal{B}}^*(\mathbf{I} - \boldsymbol{\mathcal{D}}^*)^{-1}\boldsymbol{\mathcal{C}}^*]^{-1} \frac{\partial \hat{\mathbf{h}}^*(\ln \mathbf{y}^*, \ln \mathbf{x}^*; \ln \boldsymbol{\alpha})}{\partial \ln \boldsymbol{\alpha}}, \tag{27}$$

$$\frac{\partial \ln \mathbf{x}^*}{\partial \ln \boldsymbol{\alpha}} = (\mathbf{I} - \boldsymbol{\mathcal{D}}^*)^{-1}\boldsymbol{\mathcal{C}}^*[\mathbf{I} - \boldsymbol{\mathcal{W}}^* - \boldsymbol{\mathcal{B}}^*(\mathbf{I} - \boldsymbol{\mathcal{D}}^*)^{-1}\boldsymbol{\mathcal{C}}^*]^{-1} \frac{\partial \hat{\mathbf{h}}^*(\ln \mathbf{y}^*, \ln \mathbf{x}^*; \ln \boldsymbol{\alpha})}{\partial \ln \boldsymbol{\alpha}}, \tag{28}$$

whereas if $(\mathbf{I} - \boldsymbol{\mathcal{W}}^*)$ *is invertible and* $\partial \hat{\mathbf{h}}^*(\ln \mathbf{y}^*, \ln \mathbf{x}^*; \ln \boldsymbol{\alpha})/\partial \ln \boldsymbol{\alpha} = \mathbf{0}$*,*

$$\frac{\partial \ln \mathbf{y}^*}{\partial \ln \boldsymbol{\alpha}} = (\mathbf{I} - \boldsymbol{\mathcal{W}}^*)^{-1}\boldsymbol{\mathcal{B}}^*[\mathbf{I} - \boldsymbol{\mathcal{D}}^* - \boldsymbol{\mathcal{C}}^*(\mathbf{I} - \boldsymbol{\mathcal{W}}^*)^{-1}\boldsymbol{\mathcal{B}}^*]^{-1} \frac{\partial \hat{\mathbf{g}}^*(\ln \mathbf{x}^*, \ln \mathbf{y}^*; \ln \boldsymbol{\alpha})}{\partial \ln \boldsymbol{\alpha}}, \tag{29}$$

$$\frac{\partial \ln \mathbf{x}^*}{\partial \ln \boldsymbol{\alpha}} = [\mathbf{I} - \boldsymbol{\mathcal{D}}^* - \boldsymbol{\mathcal{C}}^*(\mathbf{I} - \boldsymbol{\mathcal{W}}^*)^{-1}\boldsymbol{\mathcal{B}}^*]^{-1} \frac{\partial \hat{\mathbf{g}}^*(\ln \mathbf{x}^*, \ln \mathbf{y}^*; \ln \boldsymbol{\alpha})}{\partial \ln \boldsymbol{\alpha}}. \tag{30}$$

The proof follows directly from Theorem 2, Appendix A.

Equation (27) allows capturing the impact on production of a shock affecting only production, whereas Equation (30) allows capturing the impact of a shock affecting the ecosystem on the stock of resources.

Moreover, we can compute

$$\begin{bmatrix} \frac{\partial \ln \mathbf{y}^*}{\partial \ln \boldsymbol{x}} \\ \frac{\partial \ln \mathbf{x}^*}{\partial \ln \boldsymbol{x}} \end{bmatrix} = \begin{bmatrix} \frac{\partial \hat{\mathbf{h}}^*(\ln \mathbf{y}^*, \ln \mathbf{x}^*)}{\partial \ln \mathbf{y}} & \frac{\partial \hat{\mathbf{h}}^*(\ln \mathbf{y}^*, \ln \mathbf{x}^*)}{\partial \ln \mathbf{x}} \\ \frac{\partial \hat{\mathbf{g}}^*(\ln \mathbf{x}^*, \ln \mathbf{y}^*)}{\partial \ln \mathbf{y}} & \frac{\partial \hat{\mathbf{g}}^*(\ln \mathbf{x}^*, \ln \mathbf{y}^*)}{\partial \ln \mathbf{x}} \end{bmatrix} \begin{bmatrix} \frac{\partial \ln \mathbf{y}^*}{\partial \ln \boldsymbol{x}} \\ \mathbf{I} \end{bmatrix}$$

,

or solving

$$\frac{\partial \ln \mathbf{x}^*}{\partial \ln \mathbf{x}} = \boldsymbol{\mathcal{D}}^* + \boldsymbol{\mathcal{C}}^*(\mathbf{I} - \boldsymbol{\mathcal{W}}^*)^{-1}\boldsymbol{\mathcal{B}}^*,$$

which is the Jacobian of the dynamic evolution of resources at the steady state. In particular, the steady state is asymptotically stable whenever the spectral radius of $\boldsymbol{\mathcal{D}}^* + \boldsymbol{\mathcal{C}}^*(\mathbf{I} - \boldsymbol{\mathcal{W}}^*)^{-1}\boldsymbol{\mathcal{B}}^*$ is strictly less than 1 (see Kelley and Peterson (2001), Corollary 4.3).

A sufficient condition for the spectral radius being strictly less than one is Lyapunov Diagonal stability for discrete-time systems, as defined in Lemma 3 (Bhaya and Kaszkurewicz, 1993; Stein, 1952): $\mathbf{J}^\top \boldsymbol{P}\mathbf{J} - \boldsymbol{P}$ is negative definite for some positive diagonal (or even positive definite) matrix $\mathbf{P}$, where $\mathbf{J}$ is the Jacobian of the dynamic system, evaluated at the steady state. When, the economy is sufficiently fast, Lyapunov Diagonal stability for continuous time systems is sufficient and thus also implies that the spectral radius is smaller than 1.

Thus, for economies that are not fast, most of the analysis remains, replacing the notion of Lyapunos Diagonal stability used in Defintion 5 by assuming that $\mathbf{J}^\top \boldsymbol{P}\mathbf{J} - \boldsymbol{P}$ is negative definite. The only result that does not carry through is Corollary 2.

E Consumption value

I have assumed throughout that the utility function at time t is not a function of the stock of resources.

Let us assume that the utility of the representative consumer can be a function of $\mathbf{x}$: $u(\boldsymbol{\gamma}) \to u(\boldsymbol{\gamma}, \mathbf{x})$. Note that theorems 1 and 2 from Acemoglu and Azar (2020) still apply; therefore, at each time we have the functions $\mathbf{y}_t^* = \mathbf{y}^*(\mathbf{x}_t)$ and $S_t^* = S^*(\mathbf{x}_t)$.

As such, Proposition 2 still holds with the appropriate modification of Assumption 2. Proposition 3 also holds.

The exact expressions in Proposition 4 and Proposition 6 only hold as is if one assumes that consumption value is separable, i.e. $u(\boldsymbol{\gamma}, \mathbf{x}) = u^c(\boldsymbol{\gamma}) + u^x(\mathbf{x})$, except for the expression of the steady-state welfare value, for which one needs to add $u^x(\mathbf{x}^*)$, where $\mathbf{x}^*$ is defined in Proposition 4 and Proposition 6.

If preferences are not separable, then the exact expressions may hold; but in general, there is no closed form expression. For example, if $\mathbf{q}(\mathbf{x})$, the expressions in Proposition 5 hold, replacing $\mathbf{q}$ with $\mathbf{q}(\mathbf{x}^*)$. In general, the main argument remains: without ecosystem services, the evolution of resources is governed by a continuous function (provided preferences for resources are continuous), which may provide conditions for global stability. In the presence of ecosystem services, changes in the set of suppliers induce discontinuous changes in production, which generally only allow for local stability.

Proposition 7 is slightly modified by the presence of preferences for resources. Indeed, the envelope condition, which gives the necessary conditions for any steady state $\mathbf{x}^E$, are modified as are equations (15) and (25). Specifically, we obtain

$$\frac{\partial v}{\partial x_k} = [\zeta_k + \frac{\partial u}{\partial x_k}] + \delta \sum_l \frac{\partial v}{\partial x_l} \frac{\partial g_l}{\partial x_k}.$$

Letting $\hat{\zeta}_k = \zeta_k + \frac{\partial u}{\partial x_k}$, equations (15) and (25) hold, replacing $\boldsymbol{\zeta}$ with $\hat{\boldsymbol{\zeta}}$. Proposition 8 can also be easily adapted similarly.

F Details on the numerical application

F.1 ATN

The procedure for generating the ATN follows GCV closely. Specifically, I implemented the following steps.

1. I generated trophic networks of 30 species using the Niche model in Williams and Martinez (2000) with a connectance of $\omega^c = 0.15$ (implying an expected density of $\omega^c = 0.15$). Species' positions n_k are drawn uniformly on $[0, 1]$ and attributed a feeding range of length $n_k\omega^R$ with $\omega^R \sim beta(1, (1-2\omega^c)/2\omega^c)$ and a center uniformly distributed between $\omega^R/2$ and the species' positions. Specie k feeds on specie l iff l is in k's feeding range. Following GCV, I only keep networks 1) that are connected, 2) for which any non-basal specie is connected to a basal specie, and 3) the realized density is within ± 0.025 of the expected density of 0.15.

2. The population dynamics follows equations (18) and (19), with parameters presented in Table F.1. Initial biomass is drawn uniformly between 0.05 and 1. Vertebrate species are selected with probability of 0.6 among species with a trophic level (Levine, 1980) of at least 3. Species with a biomass of less than 1×10^{-6} are considered extinct. I only keep networks for which the population dynamics (without externalities) leads to 1) at least one non-extinct vertebrate, 2) 20 or more non-extinct species and for which 3) the network of non-extinct species is connected and such that any non-basal specie is connected to a basal specie. The final sample comprises of 323 networks.

F.2 Stability

For the stability analysis, production network is drawn as follows, for an economy of 30 sectors. First, each entry of the production network matrix is drawn from a Pareto(0.01,1). Then, the matrix is row-normalized and multiplied by 0.75 (implying a labor share of 0.25 for all sectors). TFP levels are chosen so that the initial period average levels of catches are comparable for the specification with isolated sectors and the one with a production network, and is the same for all sectors. For isolated sectors, a is from 0.0025 to 0.02625 by increments of 0.00125. For the production sectors, a calibrated to delivers the same average productivity as for isolated sector, at time 0. The values for a range from 0.1619275 to 0.2914778 by irregular increments.

The simulation process follows GCV closely. I first generated a fishing-free ecosystem as described in Section F.1 above. Then, I simulated the two specifications described above, starting from the fishing-free steady-state. For each network (ecosystem), a maximum of 5 species can be harvested

Parameter	Description	Value
Ecological parameters		
h	Hill coefficient	1.2
ω^r	Growth rate	1
ω^K	Carrying capacity	1
ω^y	Maximum consumption rate	10
ω^e	Assimilation efficiency (herbivore, carnivore)	$(0.45, 0.85)$
ω^w	Feeding preference	Uniform over non-extinct preys
ω_0^x	Half-saturation biomass	0.2^h
ω_k^x	All. feeding rate (invertebrate)	$0.314 * 100^{-0.25\text{Trophic level}}$
ω_k^x	All. feeding rate (vertebrate)	$0.88 * 100^{-0.25\text{Trophic level}}$
c_{ki}	Externality	1 (k is harvested by i) and 0 otherwise
Economic parameters		
a_i	Total Factor Productivity	(see Appendix F.2)
$\mathbf{W}$	Production network	(see Appendix F.2)
b_{ik}	Ecosystem service	1 (i harvests k) and 0 otherwise
c_{ki}	Externality network	1 (i harvests k) and 0 otherwise
q_i	Final demand weight	$1/n$

Table F.1: Cabibrated values

and harvested species must have an initial biomass of at least 0.003.