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PROFIT EFFECTS OF CONSUMERS' IDENTITY MANAGEMENT: A DYNAMIC MODEL

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Abstract/Résumé

We consider a non-durable good monopoly that collects data on its customers in order to profile them and subsequently practice price discrimination on returning customers. The monopolist's price discrimination scheme is leaky, in the sense that an endogenous fraction of consumers chooses to incur a privacy cost to become "active", i.e., to be able to conceal their identity when they return in the following periods. We characterize the Markov Perfect Equilibrium of the game. We find that, regardless of the accuracy of data on their customers, managers adjust their pricing and market expansion strategies to the presence of active customers in the following way: (i) reduce the pace at which introductory price falls over time, and (ii) strategically guarantee that market expansion is incomplete. The equilibrium number of passive customers in the market is found to be increasing in the level of the privacy cost. Investigating the impact of customers' identity management on profits, we find that the monopoly profit is a U-shaped function of the privacy cost whatever the degree of the monopolist's information accuracy. Still, the profit effects of consumers' identity management choices are shown to depend on the monopolist's profiling capabilities. Two customer profiling structures are compared. In the case of full information acquisition (FIA), the firm can practice personalized pricing on returning passive customers, while in the case of purchase history information (PHI), it has only enough information for group pricing. We show that in the FIA case, the monopoly equilibrium profit is globally an increasing function of the privacy cost while in the PHI case, it is almost always a globally decreasing function of it.

Nous considérons un monopole de biens non durables qui collecte des données sur ses clients afin de les profiler et pratique ensuite une discrimination par les prix sur les clients qui reviennent. Le système de discrimination par les prix du monopoleur est fuyant, en ce sens qu'une fraction endogène de consommateurs choisit d'encourir un coût d'anonymisation pour devenir « actif », c'est-à-dire pour pouvoir dissimuler son identité à son retour dans les périodes suivantes. Nous caractérisons l'équilibre parfait de Markov du jeu. Nous constatons que, quelle que soit l'exactitude des données sur leurs clients, les gestionnaires ajustent leurs stratégies de tarification et d'expansion du marché à la présence de clients actifs de la manière suivante : (i) réduire le rythme auquel le prix de lancement baisse au fil du temps, et (ii) garantir stratégiquement que l'expansion du marché est incomplète. On constate que le nombre d'équilibre de clients passifs sur le marché est une fonction croissante du coût d'anonymisation. En étudiant l'impact de la gestion de l'identité des clients sur les profits, nous constatons que le profit du monopole est une fonction en forme de U du coût d'anonymisation, quel que soit le

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degré d'exactitude des informations du monopoleur. Pourtant, les effets sur le profit des choix de gestion de l'identité des consommateurs dépendent des capacités de profilage du monopoleur. Deux structures de profilage des clients sont comparées. Dans le cas de l'acquisition d'informations complètes (FIA), l'entreprise peut pratiquer une tarification personnalisée sur les clients passifs, tandis que dans le cas des informations sur l'historique des achats (PHI), elle ne dispose que d'informations suffisantes pour la tarification de groupe. Nous montrons que dans le cas FIA, le profit d'équilibre de monopole est globalement une fonction croissante du coût d’anonymisation alors que dans le cas PHI, il en est presque toujours une fonction globalement décroissante.

Keywords/Mots-clés:

Consumers’ Identity Management; Anonymization; Intertemporal Price Discrimination; Monopoly; Information Structures.

gestion de l'identité des consommateurs ; Anonymisation ; Discrimination des prix ; Monopole; Structures d'informations.

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the case of purchase history information (PHI), it has only enough information for group pricing. We show that in the FIA case, the monopoly equilibrium profit is globally an increasing function of the privacy cost while in the PHI case, it is almost always a globally decreasing function of it (especially for low discount factors).

1 Introduction

With the advance of information technology and the use of big data, firms are increasingly able to profile customers according to their preferences and identify them when they return to buy in the following periods: they may do so by tracking their customers' web-browsing histories, geo-location, Internet Service Providers (ISPs), individual websites, or advertising exposure, and so on. As a result of the growing digitization of the economic activity, firms have reached unprecedented big data capabilities. As an illustration, the tech reporter Kashmir Hill wrote in The New York Times[1] that: *"In the fall, I took the right of access for a test drive, asking companies in the business of profiling and scoring consumers for their files on me. One of the companies, Sift, which assesses a user's trustworthiness, sent me a 400-page file that contained years' worth of my Airbnb messages, Yelp orders and Coinbase activity."* This impressive figure is likely to be no more than a tip of an iceberg, considering that firms' ability to gather information on their customers is expected to increase exponentially in the coming years as *"internet-connect devices become more prevalent in our everyday lives — think smart TVs, smart speakers and smart refrigerators, for example — and as our reliance on smartphones increases...Because people don't realize that their car is collecting data about their location and sending it off to some server somewhere, they're less likely to think about that, and companies are less likely to be held accountable for that kind of thing."*[2]

This new reality has prompted a wide-ranging discussion on the private and social benefits of big data collection and consumer's profiling. On the one hand, data tracking technologies allow firms to engage in personalization strategies, possibly

[1] https://www.nytimes.com/2020/01/15/technology/data-privacy-law-access.html [Date of access: 19 august 2020]

[2] Source: https://www.nytimes.com/2019/11/24/smarter-living/privacy-online-how-to-stop-advertiser-tracking-opt-out.html [Access date: 19 august 2020]. Along the same lines, Shiller (2014) points out that *"location by time of day can be obtained from smartphones or cameras which automatically read license plates. Or, automatic facial recognition could allow cameras to track consumers. Facebook can already track users to some extent, through the time and geo-location data contained in uploaded photos and facial recognition software and tagged photos"*. https://theconversation.com/big-data-and-personalised-pricing-consider-yourself-gamed-25076 [Access date: 20/08/2020]

increasing their profits (e.g. by tailoring pricing policies or product specification according to consumers' profile). A good illustrative example of this reality is Uber's route based pricing, which is building *"a new fare system which charges customers what it predicts they are willing to pay"*[3].

On the other hand, there is an increasing number of technologically sophisticated customers who take advantage of digital technologies to engage in identity management practices and conceal information about themselves (e.g., by de-activating their location information on their smartphones, erasing cookies, etc.) or even refrain from buying (see, e.g., Acquisti (2008)) to avoid sharing information with firms. The growing importance of this phenomenon has been documented in Deloitte's US Consumer Data Privacy Survey.[4] Along the same lines, Harvard Business Review reported that 32% of respondents to a survey abut privacy *"said they care about privacy, are willing to act, and have done so by switching companies or providers over data or data-sharing policies".*[5]

In recent years, policy makers in many countries have also facilitated customers' engagement in identity management practices, namely through legislations aiming at the reduction of consumers' privacy costs. For example, the General Data Protection Regulation (GDPR), adopted in April 2016 in EU, required substantial changes in how firms store and process consumer data. Firms must provide consumers with additional means to control the storage of personal data. Indeed, as pointed out in Bleier et al. (2020), Europe's GDPR expanded not only the scope of privacy protection, but also the definition of personal data, which now include customers' IP addresses. Aridor et al. (2000) found evidence that thanks to GDPR's new opt-in requirement, privacy conscious consumers substitute away from less efficient privacy

[3]"Uber Starts Charging What It Thinks You're Willing to Pay", https://www.bloomberg.com/news/articles/2017-05-19/uber-s-future-may-rely-on-predicting-how-much-you-re-willing-to-pay [Access date: 16/08/2021]

[4]https://www2.deloitte.com/content/dam/Deloitte/us/Documents/consumer-business/us-retail-privacy-survey-2019.pdf
[Access date: 19/08/2020]

[5]The 32% figure refers to a 2019 survey conducted by Cisco of 2,601 adults worldwide examined the customers' attitudes and actions towards their data protection. More information may be found here: https://hbr.org/2020/01/do-you-care-about-privacy-as-much-as-your-customers-do [Access date: 20/08/2020]

protection (such as cookie deletion) to explicitly opt out.[6]

Responding to these new trends, managers are investing in the adjustment of their pricing and market expansion strategies in order to simultaneously account for: (i) the avenue of possibilities opened by increasingly sophisticated pricing strategies and (ii) the (direct and strategic) profit effects of consumers' engagement in identity management actions.

This paper enriches the literature on the price and profit effects of privacy costs and customers' endogenous anonymization decisions within an infinite-horizon dynamic model. In particular, we aim at answering the following questions: what are firms' best way of adapting their market expansion strategies to the presence of active customers (who conceal their identity)? Do best responses to the presence of active customers entail more or less aggressive prices targeted to recognizable passive customers? What are the profit effects of customers' identity management practices? Are firms always worse off when privacy costs go down or are there any circumstances in which managers may actually profit from strategically reacting to legislation designed to reduce consumers' privacy costs?

In order to address these questions, we develop a stylized infinite-horizon dynamic model in which a monopoly sells a non-durable good to customers who make repeated purchases. Customers may choose to engage in identity management practices, incurring in a privacy cost: the ones who choose to do so are called *active customers*, whereas the ones who allow the firm to collect and store information about themselves are called *passive customers*[7].

The monopolist uses data on passive customers to profile them and price discriminate accordingly, when those customers return for further purchases in later periods.

[6] The consent requirement of GDPR enables consumers to deny any data to be sent to the website. See Aridor et al. 2000, p.9, where they also discussed how this implied a different data generating process. An interesting implication, which we do not address in our paper, is that the decision by a privacy-active consumer to switch from cookie deletion to a more effective GDPR-enabled opt-out may increase the trackability of privacy-passive consumers, an externality that Aridor et al. (2000) identified.

[7] Clearly, if the firm is unable to recognize any of its previous customers, the equilibrium of the infinite horizon game between the firm and the consumers is the infinite repetition of the static monopoly equilibrium.

The monopolist price targeting policy may be more or less sophisticated depending on the firms' profiling/ pricing accuracy. In this respect, we consider two alternative settings: the Full Information Acquisition (FIA) scenario, and the Purchase History Information (PHI) scenario.

The FIA set-up corresponds to a theoretical benchmark where the firm is able to collect information about the exact willingness to pay (WTP) of each passive customer immediately after their first purchase. Thus, in each period, the firm is able to partition the set of former passive customers into a continuum of market segments, each consisting of those who have the same WTP (capturing the theoretical scenario of hyper-segmentation, which new generation digital technologies increasingly try to reach). In this scenario, the firm charges each returning (passive) customer with a personalized price equal to her WTP (i.e., first degree price discrimination).

In contrast, under the probably more realistic PHI scenario, the monopolist can only recognize the initial period at which each passive customer purchased and thus it can only partition former passive customers into a finite number of market segments, each consisting of heterogeneous passive customers. Consumers belonging to a given segment may have different WTPs but they all have made their first purchases in the same period. They are thus clustered in the same segment and they will be asked to pay a common price when they return in subsequent periods.

Accordingly, in each period, the monopolist sets (i) a new introductory price with the aim of attracting new customers (while resigning to the fact that returning active customers will take advantage of it); (ii) a discriminatory pricing scheme targeted to passive returning consumers (which depends on the profiling/ information structure setting). Consumers, aware of the firm's price discrimination capabilities, choose the timing of their of first purchase according to their WTPs. One advantage of our infinite-horizon framework (vis-à-vis alternative modelling settings with a limited number of periods) lies on the ability to provide an analytical characterization of the equilibrium market expansion strategies (and the corresponding equilibrium trajectories for introductory prices).

To be more precise, we characterize the Markov Perfect Equilibria of the resulting games between the firm and the consumers, under the two polar customer profiling structures described above (FIA and PHI).

One of our main findings is that, when privacy costs are not prohibitively high, managers should adopt their pricing and market expansion strategies to the presence of active customers (both in the FIA and in the PHI scenarios). In particular, they should (i) reduce the pace at which the introductory price falls over time, in comparison to the scenario where all customers react passively (in other words, the presence of active customers relaxes the monopolist's introductory prices); and (ii) strategically guarantee that market expansion is incomplete (leaving some low-end customers unserved). The magnitude of these effects is shown to depend of the consumer profiling structure: the proportion of passive customers who end up buying the good is larger under PHI than under FIA (where more consumers choose to be active in order to avoid revealing their WTP). In addition, the market expands more quickly under FIA than under PHI (but more low-end consumers are left out of the market in the FIA setting than in the PHI setting (as the higher proportion of active consumers in the FIA case leads to higher introductory prices than in the PHI setting).

We also investigate the effects of privacy costs on the equilibrium configuration: at equilibrium the fraction of active customers and the fraction of unserved (low-end) customers are both decreasing in the privacy cost. When the latter exceeds a threshold value (so that privacy costs become prohibitively high), all customers optimally choose to be passive and the market is asymptotically fully covered.

When investigating how privacy costs affect firms' optimal pricing decisions, we identify two opposite pricing (and profit) effects arising both in the FIA and in the PHI set-up. On the one hand, there is a direct effect of identity management which is clearly detrimental to profit: the firm can only apply targeted pricing to former passive customers (with less customers behaving passively when consumers' privacy cost goes down). On the other hand, there is a strategic effect: when the privacy cost is low, the number of active customers will be high, consequently there is little incentive to set low introductory prices to attract new passive customers.

What are then the profit effects of a fall in the privacy cost? We find that the monopolist's aggregate profit is a U-shaped function of the privacy cost whatever the degree of the monopolist's profiling accuracy. In both FIA and PHI settings, a fall in the privacy cost increases the number of active customers, exerting the above-mentioned (opposite) effects on the monopolist's profit. In both cases, for a sufficiently small range of privacy cost, the positive strategic effect dominates the direct effect, leading to increased profit as the privacy cost falls. However, this effect is stronger in the PHI case, which explains why profits are generally the greatest when hiding one's identity is costless, while in the FIA case, they are always the greatest when the privacy cost is prohibitive (and all customers behave passively).[8]

This shows that it is not always the case that firms' profit are not always hurt from consumer's identity management (or from tighter privacy regulations aiming at reducing consumers' privacy cost): The potential presence of active customers reduces the incentive for the "future selves" of the monopolist to lower prices, departing from the usual Coase conjecture outcomes (as account must be taken for the fact that in the pool of customers perceived as new, there exist active returning customers, whose WTP for the good is high but the monopolist cannot observe it). This positive strategic profit effect of identity management has an intrinsic dynamic nature as it results from the lower competitive pressure that the future selves of the monopolist exert at each moment of time. Moreover this effect is stronger in the PHI case. This explains the difference on the profit effects of privacy costs under FIA and PHI profiling settings. In the FIA case, the monopolist's aggregate profit is globally increasing in the privacy cost (when we compare profits under the extreme scenario of null privacy costs and prohibitively high privacy costs), while in the PHI case, they are *globally* decreasing in the privacy cost for most values of the discount factor (with the exception of large discount factors).

It is worthwhile to highlight the difference in results between the present paper and that of Laussel at al. (2020a). In the latter paper, all customers are exogenously assumed to behave passively, and its main result was that firms may be hurt by their ability to collect (coarse) information on customers' preferences. It was in

[8]Figures 2 and 5 provide a graphical illustration of this point (more precisely, figure 2 depicts equilibrium profits as a function of the privacy cost in the FIA setting, whereas figure 5 represents the same function in the PHI scenario).

effect a profit comparison under the assumption that all customers are passive, so that the monopolist can gradually group consumers into market segments, *versus* the case where they are all active, so that the monopolist can learn nothing about them (which enables the firm to effectively commit to a policy of repetition of static equilibria). In our present paper, we are comparing equilibria where the privacy cost is low (including the limit case of zero privacy cost) with equilibria where it is prohibitively high (near the level that would dissuade the consumers from undertaking identity management activities).[9] This allows us to show how consumers' identity management may actually favor the monopolist's profit (leading to higher introductory prices, despite the lower market coverage rates). The case of zero privacy cost is particularly interesting as it allows us to make the point that the equilibrium when we allow for the endogenous determination of active/passive status (in the present paper) does not mimic the repetition of the static equilibrium (arising when the monopolist faces an exogenously active customer base) since herein the monopolist still accounts for the strategic effects of its pricing decisions on consumers' identity management decisions.[10]

The rest of the article is organized as follows. Section 2 presents an overview of the related literature. Section 3 introduces the model. Section 4 characterizes the Markov Perfect Equilibria of the game in the two cases (FIA and PHI) and identifies the main managerial recommendations of our model. Finally, Section 5 concludes.

2 Related Literature

A large number of papers have looked at the competitive effects of endogenous information acquisition, when firms are able to learn about consumers' purchase history and/or the personal tastes of individual consumers so as to exercise subsequently first-order or third-order price discrimination with respect to their former customers. This is the so-called *behavior-based price discrimination* (BBPD) literature (e.g.,

[9] Please see Figure 2 for the FIA case and Figure 5 for the PHI case.

[10] This explains why we obtain, in the PHI case, if the discount factor lies in a very small range (namely for discount factors between 0.697977 and 0.752345) the result that the firm's profit is greater when the privacy cost is dissuasive than when it is near zero.

Fudenberg and Tirole, 2000, Chen, 1997, for pioneering papers[11]; for surveys, see Fudenberg and Villas-Boas (2006), or Stole (2007)). Most of these articles focus on duopoly competition in a two-period model[12] and conclude that history-based pricing intensifies duopolist competition because it tempts each firm to poach the previous customers of its rival, and such competition erodes both firms' profits. Moreover, profit erosion is more severe under full information acquisition than under coarser information settings, such as conventional behavior-based price discrimination (see Choe et al. 2018).

Different from the duopolist competition models, a number of papers (Taylor (2004), Villas-Boas (2004), Laussel et al., (2020a, 2020b)) have dealt with the effects of endogenous information acquisition in the case of a dynamic monopoly that lacks the ability to precommit to future prices and/or quality levels. Here the competition is not between two firms, rather it is between the current and the future incarnations of the monopolist, reducing his market power. As in the oligopoly case, more accurate consumers profiling tends to result in more competition and the question is whether this negative competition effect on profit outweighs the positive direct effect of price targeting. This is of course reminiscent of the literature on durable good monopoly, starting from Coase (1972), according to which a monopolist who is unable to fully commit to future prices would lose all his monopoly power if the time interval between two consecutive price offers goes to zero. The intuition behind this result is that early customers would delay their purchase rather than accept monopoly prices because they rationally anticipate a future price decrease. The net effect of endogenous information acquisition on the firm's aggregate profit turns out to depend on the degree of coarseness of the information which it collects. Laussel et al. (2020a) show that, in a model where all consumers are exogenously assumed to behave passively, this effect is positive in the FIA case, and negative in the PHI one. This is different

[11] Chen (1997) focuses on duopolistic behavior-based price discrimination (BBPD) with consumers' switching costs. Fudenberg and Tirole (2000) analyze BBPD in a Hotelling duopoly model with two periods. Choe et al. (2018) allowed duopolists to set personalized prices for a subset of old customers. They found that there are multiple equilibria when the duopolists have different information.

[12] For additional contributions, see Chen and Pearcy (2010), Esteves (2009, 2010), Gherig et al. (2011, 2012), Carroni (2016).

from the duopoly outcome where profits' erosion occurs in both cases, though more severely under FIA.

The above-mentioned dynamic models share a common assumption: consumers are not able to guard against firms' attempt to collect information on them. However, as pointed out in the introduction, there has been an increasing concern with customers' data protection issues and more and more consumers are taking actions to conceal information on their preferences (at the same time, policy makers are, in many countries, increasing efforts to reduce consumers' privacy costs). This shift in customers' behavior may substantially affect firms' business practices as well as their equilibrium profits, as already pointed out by several works investigating the effects of consumers' anonymization practices (for example by blocking or deleting cookies, using anonymous browsing or payment tools, and so on. See Acquisti (2008), and the survey by Acquisti, Taylor and Wagman (2016)).[13] The majority of these models looks at the issue of consumers' anonymity decisions within a finite period duopoly setting (mostly a static or a two-period setting).

The present paper enriches this literature by investigating, within an infinite horizon setting, managers' best response to consumers' anonymization practices. This allows us to shed light on the dynamic (strategic) effects of consumers' endogenous identity management actions over firms' optimal market expansion, pricing trajectories, long-run market coverage and equilibrium profits. Consumers endogenous privacy decision on whether to evade (or nor) firms' exploitation of big data arises as key ingredient of our framework. In this respect, we concentrate on the case of "*ex ante identity management*" (as in Acquisti and Varian (2005) and Conitzer et al. (2012), whose contributions are described in more detail below). Like them, when looking at consumers' identity management decisions, we essentially model how a subset of consumers, based on the level of cost of identity management, choose to "anonymize", avoiding being recognized when they return.[14] These active consumers

[13]See, in particular, Acquisti et al., 2016, pp. 454-457.

[14]Ichihashi (2020) considers a multi-product setting where a monopoly firm besides using consumers' information disclosure to exercise price discrimination, also uses it to improve the quality of the recommendations made to these customers. He finds that in equilibrium the seller actually prefers to commit not to price discriminate in order to encourage consumers' information provision

remain anonymous in any circumstances[15] and accordingly they will not face personalized offers when they return buying.[16]

It is worth noting that other forms of identity management have been addressed in the literature. For example, Chen, Choe and Matsushima (2020) have analyzed the case of *ex post identity management* within a duopoly framework with price personalization (within firms' own turf). In their setting, old customers may either choose (i) to actively conceal their identity when they return buying (e.g., creating new accounts for new transactions, or maintaining several online identities) or (ii) to passively use the same identity as for their first purchase (and then be offered a personalized price). They conclude that active identity management raises firms' profits and reduces consumer surplus and social welfare (differently, in our FIA setting consumers' anonymization may be *detrimental* to the firm). Montes et al. (2019) consider a finite horizon duopoly game in which firms may also send personalized price offers to the customers whose preferences they know (as in Chen et al., 2020). They take the monopoly structure as a benchmark to find that a monopolist always benefits from higher privacy costs.[17] As referred earlier, the closer works to this paper are Acquisti and Varian (2005) and especially Conitzer et al. (2012). Using a two-period monopoly model with two types of customers, Acquisti and Varian (2005) allow the monopolist to condition pricing on purchase history of individual customers. They found that if consumers are sophisticated (non-myopic),

(which leads to higher rents in his multi-product set-up).

[15]Casadesus-Masanell and Hervas-Drane (2015) have looked into the effects of privacy on price competition when not only customers choose how much personal information they want to provide but also firms compete on information disclosure. Considering a static setting they find that there are circumstances in which privacy actually softens price competition. Those circumstances require consumers to be sufficiently heterogeneous and their WTP should not be sufficiently high.

[16]Belleflamme and Vergote (2016) look at a one-period monopoly set-up and consider the possibility of consumers using anonymizing technologies while the firm uses tracking ones. Obviously, the dynamic mechanism driving firms' optimal strategies in our setting does not arise in their static setting. Indeed, differently from us, they conclude that consumers as a whole end up being hurt by the possibility to engage in identity management practices.

[17]Although our infinite time horizon model leads to qualitatively similar predictions in the FIA case, we get contrasting results in the PHI case, where the monopolist does not necessarily benefit from consumers' passive behavior. This allows us to make the point that we need to take into account the specific information structure when we try to assess the effects of having to face an increasing fraction of active customers.

then intertemporal price discrimination is not profitable for a monopolist who can commit to prices. On the other hand, if a large fraction of consumers is myopic, then price conditioning may be profitable. Also, if the value of the second unit of consumption is more valuable than the first (because the seller can offer enhanced service to returning customers), then price conditioning may be profitable.

The two-period behavior-based model of a monopolist that cannot commit, analyzed in Conitzer et al. (2012), is closest to ours in that there is a continuum of customer types $\theta \in [0,1]$, and that consumers who want to anonymize themselves must incur a common cost $c \geq 0$. They make a special assumption: all consumers who buy the good in period 1 anonymize with the same endogenous probability $\alpha(c)$, independent of their type θ.[18]. In period two, the monopolist sells to three groups of customers: repeat customers who are identified as such, new potential customers who are offered a period 2 introductory price, and repeat customers who are not identified and can take advantage of the introductory price targeted at new customers. In the "full recognition benchmark scenario" where customers are not able to anonymize (i.e., c is prohibitively high, so that $\alpha(c) = 0$), in period 2 the firm will offer a low introductory price, $p_2^{0,FR}$ to new customers and charge returning former customers a higher price, $p_2^{1,FR}$. In contrast, if $c = 0$, then $\alpha(0) = 1$, so that the monopolist makes no new introductory offer in period 2. Between these two benchmarks, Conitzer et al. (2012) found that the firm's profit, $\Pi(c)$, is non-monotone in c: its attains its global maximum at $c = 0$,[19] in particular $\Pi(c)$ is non-decreasing over the some interval $[\widehat{c}, \overline{c}]$, with $\Pi(\widehat{c}) < \Pi(\overline{c})$, as shown in their Figure 4(a).[20]

Our paper differs from Conitzer et al. (2012) in a number of aspects. First, the firm's time horizon is infinite and the game takes place in continuous time. This allows us to characterize the dynamics of the sequence of introductory prices, and the speed of market expansion as function of the privacy cost level. (For example, as

[18]This assumption is responsible for their result that when $c = 0$ the monopoly's profit is equal to the profit obtained under the static repetition of the monopoly equilibrium. This result departs from the corresponding counterpart in the present paper.

[19]At $c = 0$, the monopolist obtains the no-recognition benchmark profit.

[20]They denote by $\widehat{c} > 0$ the threshold privacy cost level beyond which $\alpha(\widehat{c}) = 0$, and they define $\overline{c} = p_2^{1.FR} - p_2^{0,FR} > 0$.

we show in Figure 1, the smaller is the privacy cost, the higher are the introductory prices and the slower is their rate of decline over time). Second, we characterize the equilibrium "pivot" customer type, who is indifferent between anonymizing or not, taking account of the monopoly's price dynamics.[21] Third, we consider both the FIA case and the PHI case. We show that the firm's profit, $\Pi(c)$, is non-monotone in c in both the FIA case and the PHI case, but when comparing extreme values of privacy cost, $\Pi(c)$ is globally increasing in privacy cost in the FIA case but globally decreasing in the PHI case for most values of the discount factor. Fourth, we also show how the equilibrium varies with the discount factor.

3 The framework

This section summarizes the main ingredients of our modelling framework, which builds on Laussel et al. (2020a), extending their framework in order to endogenize consumers' identity management decisions. A monopolist produces a non-durable good (or a service) at a constant marginal cost, normalized to zero. Time is a continuous variable but the firm partitions the time line $[0, \infty)$ into a sequence of contracting periods of duration Δ. Period n (where $n = 0, 1, 2, 3, ...$) corresponds to the interval $[n\Delta, (n+1)\Delta)$. The duration Δ corresponds to the length of the monopolist's commitment period. It can be perceived as the time that elapses between two different contract offers: during each period of length Δ, the prices selected at the beginning of the period remain contractually fixed. This formulation is flexible enough to include situations in which prices are fixed at the instant the consumer buys the good (with $\Delta \to 0$) as well as situations in which consumers sign up contracts for an exogenously fixed period (e.g. in telecommunications or energy markets).

There is a continuum of consumers who live forever. Each consumer (she) buys and instantaneously consumes at most one unit of the good at each instant of time. We let θ be a variable denoting the consumers' type. The distribution of θ is uniform over the unit interval $[0, 1]$ and a given consumer of type θ derives θ units of utility for consuming one unit of the good per unit of time (corresponding to her WTP).

[21] When the model is solved in full, the monopoly's equilibrium path of introductory prices itself depends on the equilibrium pivot consumer type.

Thus, if she pays p for the good, her instantaneous net utility is $\theta - p$. Let $r > 0$ be the instantaneous rate of discount so that a dollar paid at the end of the period is worth only as much as a fraction $\beta = e^{-r\Delta}$ of a dollar paid at the beginning of the period. Without loss of generality we set $r = 1$ in what follows.

Customers have the option to engage in identity management practices, concealing their preferences from the firm. In order to become active and bypass price discrimination, a consumer has to incur, in the initial period, a fixed once-for-all privacy cost c. This up-front cost is assumed to be common to all consumers and it may capture different sources of identity management costs. For example, it can be interpreted as an investment in data protection apps/ software or as an investment in the time needed to understand the firm's data management policies (e.g. consumers need to spend time to read firms data management policy and get acquainted with different alternative data protection policies; after getting acquainted with the firms' data policies this "time cost" becomes negligible or even null).

Consumers rationally take their identity management policies by comparing the respective discounted intertemporal utilities from becoming active and from being passive, given their individual preferences (i.e. their type θ) and the privacy cost c. For now, we shall assume this choice is made in the initial period, $n = 0$ (later on, we show that generally, at equilibrium, there is indeed no gain for an active customer to delay this choice).[22] If consumers act passively, they don't hide their identity from the firm, which is able to profile them and price discriminate accordingly in the subsequent periods. Active consumers manage to escape the firm's recognition, which means that they are able to bypass price discrimination, i.e., they buy in all periods at the introductory price $p^I(n)$ offered to new customers (the superscript I in $p^I(n)$ standing for the "introductory" nature of the offer).

Let us provisionally suppose that, given the monopolist's market expansion and pricing strategies, consumers use a cut-off strategy in what comes to their identity management decisions. This strategy consists of choosing to hide or not their identity depending on a threshold level $\widetilde{\theta}$ which depends on the privacy cost c, such that if a

[22] As will be shown, this condition (at equilibrium there is no gain for an active customer to delay this choice) is always satisfied in the FIA case, and almost always so in the PHI case.

consumer's type belongs to the subset $\left[\widetilde{\theta},1\right]$ then she will choose to be active, while if $\theta<\widetilde{\theta}$, she will choose to be passive. Later on, we shall show this threshold strategy is indeed as an equilibrium one as we show that if a type θ-consumer optimally chooses to become active, then in equilibrium any consumer $\theta'>\theta$ optimally takes a similar decision (which is to be expected since the benefits of remaining untraceable are higher for θ' than θ). Obviously, the equilibrium cut-off $\widetilde{\theta}$ also depends on the information structure (i.e. the pivotal customer type $\widetilde{\theta}$ in the FIA case will differ from the corresponding counterpart in the PHI case).

To sum up, in any given period n, the monopolist takes all active customers $(\theta>\widetilde{\theta})$ as new ones. Among passive customers, the firm distinguishes two groups: old passive customers (those who have already bought the good in some previous period $j<n$) and new ones (who first buy the good in period n).[23] We now briefly investigate the decisions of each of the three consumer groups: returning passive customers, passive new customers and active customers. At the end of this section, we look at consumers' identity management choices.

3.1 Passive returning customers

By definition, this subset of customers cannot access the price $p^I(n)$ offered to new customers. Two cases need to be considered, depending on the consumers' profiling capabilities of the monopolist:

Case 1: The Full Information Acquisition (FIA) benchmark

In this case, the monopolist will exercise first-degree price discrimination with respect to returning passive customers, charging them a price equal to her willingness to pay. Although at the current state of technology, this remains a theoretical benchmark, it is a useful one, since new generation digital technologies are attempting to reach individual-level market segmentation.

The resulting profit (in period n) is denoted by $\Pi^{PF}_{n,FIA}$, where PF refers to passive former customers, n identifies the time period and FIA identifies the information

[23] Of course there is possibly a third group: customers who will make their first purchase in some period $i>n$ or never buy the good at all. However we only need to consider here the customers who buy at n.

structure:

$$\Pi_{n,FIA}^{PF} = (1-\beta)\int_{\theta_n}^{\widetilde{\theta}} \theta d\theta = (1-\beta)\left[\frac{\widetilde{\theta}^2 - \theta_n^2}{2}\right], \quad (1)$$

where θ_n is the lowest consumers' type who has first bought the good before period n.

Case 2: The Purchase History Information (PHI) case

In this setting, the monopolist only identifies when each returning (passive) consumer has first bought the good. At the beginning of period n, the monopolist knows that there are n market segments, corresponding to n subgroups of former passive consumers: $[\theta_n, \theta_{n-1}), [\theta_{n-1}, \theta_{n-2}), ..., [\theta_1, \theta_0]$, where $\theta_0 = \widetilde{\theta}$ and $\theta_{i+1} < \theta_i$, for $i = 0, 1, 2, ...n-1$. (later on, when investigating the optimal decision of new passive customers, we will refer to passive customers of type θ_{n+1} as *the period n's marginal passive customers*). Consumers of type-θ such that $\theta > \widetilde{\theta}$ are not considered herein as we are supposing for now that they choose to be active (which we will later show to be their optimal decision in equilibrium). The monopolist knows to which subgroup any given former passive customer belongs and it may practice third-degree price discrimination among different cohorts of consumers. In period n, a returning passive customer who made her first purchase in period $i < n$ corresponds to a "*vintage i passive customer*" and she is charged a price $p(i, n)$.[24] In view of our uniform distribution assumption, the population share of passive customers that belong to vintage i is $\theta_i - \theta_{i+1}$. The set of passive customers of vintage i is called the passive market segment i and the maximum price that can be charged to this group of former passive customers is θ_i. The monopolist's profit in period n from sales to passive market segment $i < n$ is then given by:

$$\pi(i, n) = (1-\beta)\left\{[\theta_i - \max[\theta_{i+1}, p(i, n)]\right\} \times p(i, n).$$

[24]There are also vintage i active consumers, but when they return in any period $n > i$, they are able to conceal their identity, with the firm profiling them as new customers in period n (which means they are able to purchase the good at the introductory price $p^I(n)$ that is offered to new customers).

Given θ_i and $\theta_{i+1} < \theta_i$, the function $\pi(i,n)$ is maximized at

$$p^*(i,n) = \max\left\{\frac{\theta_i}{2}, \theta_{i+1}\right\}$$

As will be verified later, along the PHI equilibrium path, it holds that the condition $\theta_{i+1} \geq (1/2)\theta_i$ is indeed satisfied. Thus, the (third-degree discrimination) price for returning customers of vintage i in the PHI case is

$$p^*(i,n) = \theta_{i+1},\ i < n. \tag{2}$$

It follows from (2) that the monopolist's optimal aggregate profit in period n over all vintages of returning former passive customers in the PHI case is

$$\Pi^{PF}_{n,PHI} = (1-\beta)\sum_{i=1}^{n}\left[\theta_{i-1} - \theta_i\right]\theta_i, \tag{3}$$

where $\Pi^{PF}_{n,PHI}$ denotes the profit (in period n) obtained on Passive Former customers (PF) in the PHI scenario.

3.2 Passive new customers

We now turn to potentially new passive customers in period n who face an introductory price $p^I(n)$. By definition, their highest WTP is θ_n. If the introductory price $p^I(n)$ is greater than or equal to θ_n, such price will not induce any of them to buy the good. As we will show later, the monopolist may actually set an introductory price above that level if θ_n is lower than a critical positive threshold which we call θ_{inf} (to be defined and endogenously computed later). This means that lower-end types will remain unserved (so that the market is not fully covered in equilibrium). The rationale behind this result lies on the existence of a fraction of active consumers: it does not pay to cover the whole market when the set of active customers is non-empty as the firm anticipates that some consumers buying at the introductory price actually have the highest willingness to pay for the good (but are concealing it). Later on, we shall demonstrate that along the equilibrium path, the threshold θ_{inf} will be reached only asymptotically.

For now, let us consider that the set of new passive customers in period n has the property that their WTP belongs to some interval $[\theta_{n+1}, \theta_n)$, with $\theta_{n+1} \geq \theta_{\inf}$, and the monopolist wants to attract them. Then, obviously, $\theta_{n+1} \geq p^I(n)$, because the customer's surplus must be non-negative. Passive customers of type $\theta \in [\theta_{n+1}, \theta_n)$ are aware that once they buy the good in period n, their type will be identified by the monopolist. The latter will then charge them a targeted price in all future periods $n + j$ (where $j \geq 1$). In the FIA polar case, such price will actually coincide with their WTP, θ, leaving them with zero surplus. In the possibly more realistic PHI case, such price will depend on the customers' segment/vintage.

We will refer to passive customers of type θ_{n+1} as *the period n's marginal passive customers*. By definition, a marginal passive customer in period n is indifferent between the following two alternatives: (i) being a first-time buyer in period n (thus enjoying a positive surplus in period n and zero surplus in all subsequent periods $n+j$, $j \geq 1$)[25] and (ii) refraining to buy in period n, in order to be a first-time buyer in period $n + 1$ (in this alternative, the customer does not get any utility in period n but she will be able to subsequently get the good at a lower price $p^I(n + 1)$, thus enjoying a larger positive surplus in period $n + 1$ and non-negative surplus[26] in all subsequent periods $n + 1 + j$, $j \geq 1$). Taking into account the discount factor, in equilibrium, the two options must give the same intertemporal utility to the marginal passive customer.

Let $U_n(\theta_{n+1})$ denote the marginal passive customer's expected life-time net utility, as viewed at the beginning of period n. Then the above argument shows that $U_n(\theta_{n+1}) = (1 - \beta)\left(\theta_{n+1} - p^I(n)\right)$, both in the FIA case (where all customers are charged their WTP) and in the PHI case (where customer θ_{n+1} reveals her true WTP to the firm, since this customer is placed at the bottom of the rung of this vintage).

[25] This is because in the FIA case, the potential surplus of all returning passive customers is fully expropriated by first-degree price discrimination. In the PHI case, as shown in eq. (2), the third degree discrimination price for returning customers of a vintage n is exactly equal to θ_{n+1}.

[26] More precisely, in the future, she will get zero surplus in the FIA case, and positive surplus equal to $\theta_{n+1} - \theta_{n+2}$ in the PHI case, because this deviating customer is treated by the monopolist as a member of vintage $n + 1$ (as argued earlier, in a PHI scenario, the customers who end up profiled in this cohort/ vintage, will all pay the third degree discrimination price equal to θ_{n+2}).

The previous equality may be conveniently re-written as:

$$p^I(n) = \theta_{n+1} - \frac{1}{1-\beta} U_n(\theta_{n+1}). \tag{4}$$

Regardless of the information structure, for the periods in which the monopolist is still interested in expanding the market (by reducing its introductory price in order to sell the good to additional cohorts of passive customers of lower types than the ones served in the previous period) the profit of the monopolist from sales to new passive customers (PN) in period n is then equal to

$$\Pi_n^{PN} = ((1-\beta)) \left[(\theta_n - \theta_{n+1}) \times p^I(n) \right]. \tag{5}$$

In view of (4) and the fact that, as argued before, $U_n(\theta_{n+1}) = (1-\beta) \left(\theta_{n+1} - p^I(n) \right)$ both in the FIA case and in the PHI case, the function Π_n^{PN} may also be expressed as

$$\Pi_n^{PN} = (\theta_n - \theta_{n+1}) \left((1-\beta)\theta_{n+1} - U_n(\theta_{n+1}) \right). \tag{6}$$

3.3 Active customers

Let us finally look at the decision of active customers in period n, which is much simpler. Even if they have already bought the good in some previous period(s), they are able to conceal their identity, so that the monopolist cannot recognize them. Accordingly, their best course of action is to pretend they are new customers, as this enables them to purchase at the introductory price $p^I(n)$ that is intended to new customers only. Clearly, all active customers of type $\theta \geq p^I(n)$ will choose to buy the good in period n, claiming to be new customers. Let us denote by Π_n^A the monopolist's period n profit, obtained from these *A*ctive customers:

$$\Pi_n^A = (1-\beta) \left\{ \left[1 - \max\{\widetilde{\theta}, p^I(n)\} \right] \times p^I(n) \right\}.$$

From eq. (4), $p^I(n) \leq \widetilde{\theta}$ since $\theta_{n+1} \leq \widetilde{\theta}$ and $U_n(\theta_{n+1}) \geq 0$. Accordingly the above equation may be rewritten as:

$$\Pi_n^A = (1-\beta) \left\{ \left[1 - \widetilde{\theta} \right] \times p^I(n) \right\}. \tag{7}$$

3.4 Consumers' identity management choices

We now turn our attention to the consumers' identity management choices. To do so, we first analyze the *participation constraint* for the period n marginal passive customer. Her intertemporal net utility from buying in period n has been defined as $U_n(\theta_{n+1})$. If she chooses to deviate by delaying her first purchase to period $n+1$, what's going to be her intertemporal net utility? It's of course β times her intertemporal net utility measured at $n+1$. In turn, the latter equals the period $n+1$ marginal passive customer's intertemporal net utility $U_{n+1}(\theta_{n+2})$ plus the additional utility resulting from the information rent obtained by θ_{n+1} (who would be now buying at the top of the rung) in comparison with period $n+1'$s marginal customer (whose type is equal to θ_{t+2}), amounting to $(1-\beta)(\theta_{n+1}-\theta_{n+2})$. The total information rent will depend on the monopolist's profiling accuracy (i.e. we need to account for the different rents arising in the FIA case and in the PHI case).[27]

In the FIA case, such rent accrues only once (i.e., in period $n+1$ only), because both types will earn zero surplus in period $n+2$ and beyond (due to firm's ability to price customers according to their WTP). The corresponding participation constraint (arbitrage equation) writes as:

$$U_n(\theta_{n+1}) = \beta\left[U_{n+1}(\theta_{n+2}) + (1-\beta)(\theta_{n+1}-\theta_{n+2})\right], \tag{8}$$

implying

$$U_n(\theta_{n+1}) = (1-\beta)\sum_{j=1}^{\infty}\beta^j(\theta_{n+j}-\theta_{n+j+1}). \tag{9}$$

Differently, in the PHI case, the information rent accrues for several periods. By delaying her purchase, a deviating passive customer manipulates the firm's future profiling and therefore a deviating customer of type θ_{n+1} benefits in all periods $k \geq n+2$ of a lower (third degree discrimination) price, paying $p(n+1,k)$ instead of $p(n,k)$. In other words, the informational rent $(1-\beta)(\theta_{n+1}-\theta_{n+2})$ for being profiled in a "lower" market segment accrues repeatedly in all future periods, resulting in

[27] In both the FIA case and the PHI case, if the participation constraint of the marginal new passive customer θ_{n+1} is satisfied, then the participation constraint of all types $\theta \in [\theta_n, \theta_{n+1}]$ is automatically satisfied.

a total discounted information rent with value equal to $(\theta_{n+1} - \theta_{n+2})$. Thus, the participation constraint in this case writes as:

$$U_n(\theta_{n+1}) = \beta \left[U_{n+1}(\theta_{n+2}) + (\theta_{n+1} - \theta_{n+2}) \right]. \tag{10}$$

or equivalently

$$U_n(\theta_{n+1}) = \sum_{j=1}^{\infty} \beta^j (\theta_{n+j} - \theta_{n+j+1}). \tag{11}$$

We are now ready to investigate the consumers' initial decision to act actively or passively. In our framework, this amounts to search for the pivot consumer type $\widetilde{\theta}$, who is indifferent between (i) being passive or (ii) making efforts to become active. If a type θ consumer decides to become active, she has to incur the privacy cost c in the initial period, buying the good at the introductory prices $p^I(n)$ in all periods $n = 0, 1, 2, ..\infty$. The intertemporal discounted utility of an active customer of type θ is denoted by $u^A(\theta)$ and is equal to:

$$u^A(\theta) = (\theta - c) - \sum_{n=0}^{\infty} (1 - \beta) p^I(n) \beta^n, \tag{12}$$

regardless of the consumers' profiling regime. Of course, the monopolist's price sequence $\left\{ p^I(n) \right\}$ in the FIA case is not the same as in the PHI case, and therefore the intertemporal utility $u^A(\theta)$ in the FIA case is not equal to $u^A(\theta)$ in the PHI case. Nonetheless, it is worth noting that in both cases, we have $\frac{du^A(\theta)}{d\theta} = 1$.

Lemma 1. *In a MPE, customers with type $\theta > \widetilde{\theta}$ are better off incurring the privacy cost to conceal their identity (bypassing price discrimination), while customers with type $\theta < \widetilde{\theta}$ obtain a greater intertemporal utility by acting passively.*

Proof. See the Online Appendix.■

In the next Section, we investigate how consumers' identity management affects the properties of the Markov-Perfect pricing and market expansion equilibrium under the FIA and the PHI scenarios. All players are assumed to rationally expect future outcomes.

4 Markov-Perfect Equilibria

Here, we suppose that the monopolist is not able to commit to any predetermined path of prices and we consider the Markov Perfect Equilibrium (MPE) of the dynamic game played between the consumers and the monopolist. The monopolist collects information on the preferences of returning passive customers and price discriminates accordingly (first-degree price discrimination in the case of FIA and third-degree price discrimination in the case of PHI). Active customers remain untraceable.

For these dynamic problems, we use as the state variable in period n the smallest WTP of all previous passive customers, namely θ_n. It is therefore related to the fraction X_n of the total passive population that has purchased the good prior to that period in a simple way: $X_n = \widetilde{\theta} - \theta_n$. Notice that $\theta_0 = \widetilde{\theta}$.

In a MPE, the consumers have a Markovian expectations rule that, given the state variable θ_n, provides the correct forecast of the intertemporal net utility of period n marginal passive customer, $U_n(\theta_{n+1})$. Let us denote by $\Psi(.)$ the consumers' Markovian expectations rule, where, in equilibrium, $\Psi(\theta_n) = U_n(\theta_{n+1})$, i.e., expectations are rational. Consumers decide whether or not to be active based on the (correctly) anticipated monopolist market expansion strategy, to be defined below, and their own expectations. The monopolist takes the consumers expectations rule $\Psi(.)$ as given as well as the decision of consumers belonging to the set $\left[\widetilde{\theta}, 1\right]$ to incur the privacy cost in order to bypass price discrimination. We now need to study the properties of the MPE under FIA and PHI profiling settings.

Since in both cases the firm's optimal pricing for returning former passive customers is already solved (see Section 3.1), the dynamic optimization problem facing the monopolist reduces to determining, in each period n, the optimal market expansion decision (resulting from the corresponding monopolist's introductory pricing policy). In other words, the monopolist's problem may be viewed as the dynamic decision of choosing the size of the new market segment, $\theta_n - \theta_{n+1}$, given the state variable θ_n. Along an optimal path with continual market expansion, at the beginning of any period n, given θ_n, the firm simply has to choose the optimal value for $\theta_{n+1} < \theta_n$, so that the market is expanded gradually by serving consumers with lower and lower willingness to pay, until $\theta_{\inf}$ is eventually reached. Accordingly, it follows that the firm's Markovian strategy consists of a cut-off rule, $\Phi(.)$, which is a decreasing function such that $\theta_{n+1} = \Phi(\theta_n) \leq \theta_n$.

Formally a Markov Perfect Equilibrium (MPE) with endogenous privacy decisions consists of a Markovian cut-off rule Φ, a Markovian expectations rule Ψ, and consumers' privacy decisions (represented by the endogenous threshold $\widetilde{\theta}$), such that Φ is a best reply to Ψ and $\widetilde{\theta}$, the consumers' expectations are rational, and $\widetilde{\theta}$ reflects consumers' optimal privacy decisions given their correct anticipation of Φ and Ψ.

In light of the structure of the problem, it is natural to search for a quasi-linear cut-off rule such that

$$\theta_{n+1} = K + \gamma\theta_n,$$

where $0 \leq \gamma < 1$ and K are to be determined. Note that continual market expansion means that $\theta_{n+1} < \theta_n$. This implies that $(1-\gamma)\theta_n > K$ for all finite n. In addition, as n tends to infinity, θ_n tends towards a limiting value $\theta_{\inf}$ such that $\theta_{\inf} = K/(1-\gamma)$, where $0 \leq \theta_{\inf} < 1$. In order to compute the MPE, we start with the conjecture that the equilibrium consumers expectation rule is also quasi-linear,

$$\Psi(\theta_n) = C + \lambda\theta_n \tag{13}$$

where C and λ are to be determined. We expect that $\lambda > 0$, which means that along the market expansion path, as the measure of potential new passive customers gets smaller and smaller, the utility of the next period marginal customer also gets smaller and smaller. In addition, we expect that the last type of customer to be served, $\theta_{\inf}$, must have zero surplus:

$$\lim_{n\to\infty} \Psi(\theta_n) = 0. \tag{14}$$

This boundary condition implies that, if $\theta_{\inf} > 0$ then C must be negative.

The monopolist's period n profit is the sum of the period n sales to (i) returning former passive customers, (ii) new passive customers (paying the introductory price $p^I(n)$), and (iii) all active customers whose WTP is greater than or equal to $p^I(n)$:

$$\pi_n(\theta_n, \theta_{n+1}) \equiv \Pi_n^{PF} + \Pi_n^{PN} + \Pi_n^{A}$$

where Π_n^A is given by eq. (7), Π_n^{PN} is given by (6), while Π_n^{PF} is defined by (1) in the FIA case and by (3) in the PHI case. It is important to stress that in the expressions for Π_n^{PN} and Π_n^{PF}, we must replace $U_n(\theta_{n+1})$ with the Markovian expectations function $\Psi(\theta_n)$, because when looking for the monopolist's best reply to the customers' strategy, we must take into consideration that the monopolist takes as given the consumers' expectations.

The Bellman equation for the monopolist is then

$$V(\theta_n) = \max_{\theta_{n+1}} \left\{ \Pi_n^{PF}(\theta_n) + \Pi_n^{PN}(\theta_n, \Psi(\theta_n), \theta_{n+1}) + \Pi_n^A(\Psi(\theta_n), \theta_{n+1}) + \beta V(\theta_{n+1}) \right\}. \tag{15}$$

It is worth noting that the the expression of Π_n^{PF} for the FIA case and that for the PHI case differ from each other and therefore the value functions also differ (even if, for the sake of notation simplicity, we do not make that explicit in equation (15)). For the sake of simplicity, we denote by (Φ^*, Ψ^*) the equilibrium strategy profile for the FIA case and by (Φ^{**}, Ψ^{**}) the one for the PHI case. Even though both are expected to be quasi-linear, we expect them to have different characteristics.

The FOC corresponding to problem (15) is generally written as:

$$(1-\beta)(1-\widetilde{\theta}+\theta_n-2\theta_{n+1}) + \Psi(\theta_n) + \beta V'(\theta_{n+1}) = 0, \tag{16}$$

where we have used the expressions of $\Pi_n^{PF}(\theta_n)$, Π_n^{PN} and Π_n^A obtained in section 3 (see conditions (1) or (3), (6), and (7)), taking into consideration that $p^I(n)$ can be obtained by equation (4). From the Envelope Theorem, one obtains

$$V'(\theta_n) = \frac{\partial(\Pi_n^{PF} + \Pi_n^{PN} + \Pi_n^A)}{\partial \theta_n}. \tag{17}$$

Evaluating this at θ_{n+1} and substituting the resulting value in (16), one obtains in each case a corresponding Euler equation for a fixed value of $\widetilde{\theta}$, which will depend on the information structure we are looking at. Using this equation together with the relevant participation constraint for passive customers we obtain in each case the equilibrium expansion strategy and the equilibrium expectations rule as functions of $\widetilde{\theta}$. Finally using the relevant indifference condition which determines $\widetilde{\theta}$ as a function of the expansion strategy and the expectations rule, we are able to solve for the full equilibrium of the game.

In what follows we investigate in more detail the properties of such MPE in order to get some insights into the effects of consumer identity management on market coverage, profits and welfare, depending on the accuracy of firms' tracking capabilities (FIA versus PHI). We start with the FIA benchmark and then we look at the PHI case.

4.1 Effects of Identity Management under FIA

Lemma 2 presents an analytical characterization of the MPE when the monopolist is able to get accurate information on the individual preferences of former passive customers.

Lemma 2. *Under the full information acquisition (FIA) scenario, for a given value of $\widetilde{\theta}$, a MPE equilibrium exists, such that the coefficients λ and γ are related through the equation,*

$$\lambda = \frac{\beta(1-\beta)(\gamma - \gamma^2)}{1-\beta\gamma},$$

and the coefficients C and K are related through the equation

$$C = -\beta K \left[\frac{\gamma(1-\beta)}{1-\beta\gamma}\right].$$

Proof: Use the Bellman equation (15), together with (1) and the arbitrage equation (8). For details, please see the Online Appendix. ■

From the two equations in Lemma 2 together with condition $\theta_{n+j+1} = K + \gamma\theta_{n+j}$, we get an equilibrium expansion strategy and an equilibrium expectations rule, where $\gamma^*(\beta)$ is the solution of the polynomial (29) in the Online Appendix, and $K^*(\gamma^*(\beta), \beta, \widetilde{\theta})$ is then given by equation (30) in the Online Appendix.

Corollary 1. *In equilibrium, γ^* is a decreasing function of β, with $\gamma^* = 1/2$ when $\beta = 0$. The fraction of customers that are unserved (corresponding to the steady state, $\theta_{\inf} \equiv \frac{K}{1-\gamma}$) is itself a decreasing function of $\widetilde{\theta}$. The fraction of unserved* *customers is zero when* $\widetilde{\theta} = 1$

Proof: The result regarding monotonic effect of β on γ^* follows from equation (29) of the Online Appendix. ■

The results in Corollary 1 show that the steady state, $\theta_{\inf} \equiv \frac{K}{1-\gamma}$ is decreasing with $\widetilde{\theta}$: a greater $\widetilde{\theta}$ implies a decrease in the number of active customers, which tends to increase the firm's gain from gradually expanding the market. The fraction of unserved customers is zero when $\widetilde{\theta} = 1$ (i.e., when all customers are passive, which occurs when the privacy cost c is high enough). This means that the market is eventually fully covered at equilibrium iff the set of active customers is empty. When this is not the case, the monopolist leaves some low-end consumers unserved: active customers end up precluding the monopolist's future selves to reduce the price to such a level that everyone buys the good (as the additional profits made on lower types would not compensate for the profit losses made on active customers who are concealing their high WTP).

Lemma 3. *For a given market expansion strategy and a given expectations rule, all consumers whose types* $\theta \geq \widetilde{\theta}$ *will choose to actively manage their identity, incurring the privacy cost. Under FIA, the pivotal customer* $\widetilde{\theta}$ *is given by*

$$\widetilde{\theta}^{FIA} = \frac{(-1+\beta)(K\beta(1+\gamma)+c(1-\beta\gamma))+K\beta\lambda}{(-1+\beta)\beta(1+\gamma(\beta(-1+\gamma)-\gamma+\lambda))}. \tag{18}$$

Proof: See the Online Appendix.■

We show in Proposition 1 below that there is a threshold value of the privacy cost, $\overline{c}^{FIA} = \frac{\beta(1-\gamma)(1+(1-2\beta)\gamma)}{(1-\beta\gamma)^2}$, above which no consumer chooses to be active at equilibrium. In that case, all customers are passive and, for those limit cases, the monopolist gradually expands the market until everyone buys the good.

Proposition 1. *When the monopolist is able to get full information on its passive customers, the MPE under endogenous identity management is such that:*

(i) when the privacy cost c is below the threshold level $\overline{c}^{FIA}$, the set of active customers is non-empty and the firm finds it optimal to leave some consumers unserved. The fraction of unserved customers as well as the fraction of active customers decrease when c increases, tending to zero as c approaches $\overline{c}^{FIA}$.

(ii) when the privacy cost $c \geq \overline{c}^{FIA}$, all consumers optimally choose to be passive and the market is asymptotically fully covered.

(iii) the life-time net utility of the customers of type $\theta_{\inf}$ is zero.

Proof: See the online Appendix. ■

Proposition 1 shows that in the MPE under FIA, the firm's optimal market expansion decision is critically affected by the consumers' privacy cost. The following managerial recommendation follows: if the privacy cost is below a threshold value, the firm should not fully cover the market (not even asymptotically). The intuition behind Proposition 1 is very simple. When there exist active customers, whenever they return in subsequent periods, they conceal their identity in order to be able to buy at the price offered to new passive customers. Aware of their incentive to conceal, the monopolist responds by setting a strictly positive lower bound on the time path of introductory-offer price that is intended for new customers.[28] The effect of active identity management is very much like the effect of a limited life of durable goods, which, by inducing high valuation consumers to return to buy the good, invalidates the Coase Conjecture. In addition, it is interesting to notice that the proportion of passive customers who eventually buy the good is a linear function of the privacy cost, with

$$\widetilde{\theta}^{FIA} - \theta_{\inf} = \frac{c(1-\beta\gamma)^2}{\beta(1-\gamma)(1+(1-2\beta)\gamma)}.$$

[28] The monopolist knows that, each period, the pool of customers buying at the new (low) introductory price consists of low-end passive customers (with lower θ) and high-end active types (who get net positive benefits from identity management practices). The monopolist faces a trade-off between two conflicting incentives: setting a rather low introductory price in order to catch a large chunk of new passive customers, or setting a rather high introductory price to get more rents from active customers with high WTP. When c is not too high, more customers become active and therefore the firm ends up losing profits when it starts setting low introductory prices to attract low-end customers.

It equals zero when $c = 0$ (and all customers choose to be active) and it equals 1 when $c \geq \bar{c}^{FIA}$ (identity management is too costly, with all consumers preferring to act passively).

Corollary 2: *At the MPE with FIA no active customer may benefit from delaying the payment of the privacy cost to a period after the initial one.*

Proof: See the Online Appendix.■

We now turn to our main question: what is the effect of an increase in the privacy cost c, on the firm's profit and consumers' surplus?

Corollary 3: *In the FIA case, the monopolist's aggregate discounted profit is a U-shaped function of the privacy cost c,taking its maximum value when c attains or exceeds the threshold value $\bar{c}^{FIA}$, at which all consumers choose to be passive. In addition, the monopolist's discounted profits are globally increasing with c (i.e. $\Pi^*(c = 0) < \Pi^*(c = \bar{c}^{FIA})$).*

Corollary 3 show that, under the FIA theoretical benchmark, the firm tends to be hurt by consumers' efforts (or consumer protection policies) aiming at reducing the privacy cost c. This is to be expected since less consumers engage in active management practices as c increases (which grants the monopolist larger profits due to its ability to engage in first-degree price discrimination over returning passive customers in the FIA case).[29]

A simple result is that, at $c = 0$, the monopolist's profit for period n, $\Pi_n = \Pi_n^{PF} + \Pi_n^{PN} + \Pi_n^A$, is

$$\Pi^*(\beta, 0) = (1 - \gamma^*(\beta))\gamma^*(\beta),$$

[29] In order to rigorously compute the profit effects resulting from increased privacy costs, we must consider the monopolist aggregate profit. The profit for period n is equal to $\Pi_n = \Pi_n^{PF} + \Pi_n^{PN} + \Pi_n^A$ where the three components are given respectively by equations (1), (6) and (7). λ and C have to be replaced by their values from the corresponding equations of the online Appendix. Then we use $\theta_n = K\frac{1-\gamma^n}{1-\gamma} + \widetilde{\theta}\gamma^n$ and we substitute for K its value from the respective equation of the online Appendix. Summing across all periods, the firm's aggregate profit is then $\Pi^*(\beta, c) \equiv \sum_{i=0}^{\infty} \beta^n \Pi_n$ and it is a function of $\gamma^*(\beta)$ as follows from the online Appendix. The resulting formula is much too long and complicated to be reproduced here but allows for numerical computations.

whatever the value of β. Recall that $\gamma^* = 1/2$ if $\beta = 0$ and γ^* is decreasing in β. Note that $\beta = 0$ means that $\Delta = \infty$, i.e., the firm is committed to a constant price $p = 1/2$, that is, we have a repetition of the static equilibrium, and the monopolist's aggregate profit is $\Pi^* = 1/4$). However, if $\beta > 0$ (i.e., the length of each period of commitment is finite) then the monopolist's price is $\gamma^*(\beta) < 1/2$, which in turn implies that the monopolist's aggregate profit is smaller than 1/4. This result allows us to conclude that the possibility to engage in identity management affects equilibrium profits in important ways: unlike the result reported in the model without identity management by Laussel et al. (2020a), herein the replication of the static equilibrium price $p = 1/2$ is not an equilibrium of the dynamic model (except when $\beta = 0$) with endogenous privacy decisions.[30] The intuition for this result is the following. Suppose that the firm contemplates setting the static equilibrium price $p = 1/2$ in all periods. Then, from the condition (20), only customer types $\theta \geq 1/2$ would buy the good, and consumers whose type is below 1/2 would choose to be passive. It follows that the future incarnation of the monopolist would have an incentive to choose introductory prices smaller than 1/2 and serve passive customers. Anticipating this, more consumers than the upper half would prefer to become active in order to benefit from these future lower introductory prices, which results in lower equilibrium introductory prices herein (with $c = 0$) than the ones obtained by Laussel et al (2020a) in the FIA scenario.

More generally, our results reveal that the consumers ability to hide their identity when they return in later periods has two contrasting effects on the firm's profit. On the one hand, there is a direct effect which is clearly detrimental to profit: the firm can only apply first-degree price discrimination to former passive customers (with less customers choosing to act passively when the privacy cost goes down). On the other hand, there is a strategic effect: when the privacy cost is low, the number of active customers will be high, consequently there is little incentive to set low introductory prices to attract new passive customers. The evolution of introductory

[30] At $c = 0$, there is arguably another equilibrium where all consumers choose to be active (they are indifferent between being active or passive), and where $\theta_n = 1/2$ for all n, which corresponds to the repetition of the static monopoly equilibrium. However, this equilibrium is not robust to the introduction of an infinitesimal privacy cost.

prices is pictured in Figure 1 below for the case $\beta = 0.5$ and for several values of the privacy cost.[31] This illustrates very neatly the strategic effect: the smaller is c, the higher are the introductory prices and the slower is their rate of decline.[32].

Insert FIGURE 1 here

Since the direct and the strategic effects identified above go in opposite directions, the overall profit effects of privacy cost are ambiguous and indeed we find that equilibrium profits evolve non-monotonically with c. This is illustrated in Figure 2[33] which depicts the U-shaped relationship between aggregate profit and the privacy cost c, for three different values of β, namely $\beta = 0.2$, $\beta = 0.5$ and $\beta = 0.9$. Figure 2 also shows that for large values of the privacy cost, such that all customers choose to be passive, a greater β (i.e., a shorter commitment period Δ) entails greater profits but that, on the contrary, the reverse holds true for small values of the privacy cost.

Insert FIGURE 2 here

It is also interesting to assess whether, on average, customers may benefit or not from the option to engage in identity management practices. That is, we must investigate how the aggregate consumer surplus varies with the privacy cost c. This is most conveniently done by evaluating first the overall welfare (defined here as the sum of consumer surplus and profits) and then we residually obtain consumers' surplus.[34]

In period $n \geq 1$, the social welfare w_n is the sum of three components: (i) the aggregate measure of gross utilities of returning former passive customers (which is equal to the profits the monopolist derives from selling goods to them at their personalized prices, i.e., equation (1)), (ii) the aggregate measure of gross utilities of

[31] For the sake of simplicity the figure is drawn as if n was a continuous variable.

[32] Notice that, since $p^I(n) = \frac{1}{1-\beta\gamma}(K+\gamma(1-\beta)\theta_n)$ (see Proof of Lemma 3 in the Online Appendix) the evolution of θ_n parallels that of $p^I(n)$.

[33] The dashed parts of the curves correspond to values of $c \geq \bar{c}^{FIA}(\beta)$, leading to outcomes where all consumers are passive.

[34] Indeed, under our assumption that production cost is zero, it is clear that social welfare equals gross utility of consumers (since consumer surplus is gross utility minus their payments to the firm, and profit is equal to the payments received from customers minus production costs).

new passive customers, which equals

$$(1-\beta)\int_{\theta_{n+1}}^{\theta_n}\theta d\theta=\frac{1-\beta}{2}\left(\theta_n^2-\theta_{n+1}^2\right),$$

and (iii) the aggregate measure of gross utilities of active customers,

$$(1-\beta)\int_{\widetilde{\theta}}^{1}\theta d\theta=\frac{1-\beta}{2}\left(1-\widetilde{\theta}^2\right),$$

The sum of the three components is straightforwardly equal to

$$w_n=\frac{1-\beta}{2}\left(1-\theta_{n+1}^2\right).$$

In period 0, one has to subtract from $\frac{1-\beta}{2}\left(1-\theta_1^2\right)$ corresponding to the privacy costs $c\left(1-\widetilde{\theta}\right)$ which are incurred by the consumers who choose to be active. We then use the cut-off rule to write w_n as a function of K and γ (and of course c and β). The aggregate social welfare is simply $W=\sum_{n=0}^{\infty}\beta^n w_n$.[35]

Aggregate consumer surplus (CS) is simply the difference between social welfare and profit. For all values of β, consumer surplus is a decreasing function of the privacy cost. This is pictured in Figure 3 below.[36]

Insert FIGURE 3 here

An increase in the privacy cost exerts two opposite effects on consumers' overall welfare. The positive one is that more consumers are eventually served ($\theta_{\inf}$ is smaller) when c goes up (because the monopolist becomes interested in serving a wider set of customers, as more of them choose to be passive when c goes up). A first negative one is that active customers incur higher costs to remain anonymous (when c increases). A second negative one is that, when there are more passive customers, they

[35] Substituting for K its value from the corresponding equation of the online Appendix, we obtain W as a function of $\gamma^*(\beta),\beta,c$ and $\widetilde{\theta}$. Substituting for $\widetilde{\theta}$ its value from the corresponding equation of the online Appendix, we finally obtain W as a function of $\gamma^*(\beta),\beta$, and c. Social welfare turns out to be a hump-shaped function of the privacy cost, taking a maximum value for an intermediate value of it.

[36] Again, the dashed parts of the curves correspond to values of $c\geq\overline{c}^{FIA}(\beta)$, to situations where all consumers are passive.

tend to pay higher targeted prices. It turns out that the negative effects outweigh the positive one, leading to the decreasing functions in Figure 3.

In the following subsection, we look at a different, possibly more realistic, setting where the monopolist's profiling capabilities are very crude: the monopolist only learns the identity of the customers, storing information about the first-purchase period (PHI setting). We shall see that a number of results remain robust (namely in respect to the qualitative properties of equilibrium market expansion and pricing trajectories) but in the PHI setting less consumers will be left unserved and more consumers will behave passively (especially, when the privacy cost goes down). The profit effects are also different in the two consumer profiling set-ups: although equilibrium profits are U-shaped functions of the privacy cost in both cases, we show that they are globally increasing in the case of FIA, while they often are globally decreasing with the privacy cost (when β is not too large, or equivalently, when the commitment period Δ is large enough).

4.2 Effects of Identity Management under PHI

Now we turn to the "Purchase History Information" case. We continue to assume that consumers of types $\theta \leq \widetilde{\theta}$, where $\widetilde{\theta}$ is to be endogenously determined to satisfy the indifference condition (22), choose to be passive customers while the other ones are active. However in this case, the monopolist is only able to recognize the customers' vintage/ cohort and charge them a price which depends on their first date of purchase (group pricing instead of personalized pricing).

Lemma 4 shows that it is possible to find a Markov Perfect Equilibrium in which the monopolist's market expansion strategy and the consumers' expectation rules are affine functions of the state variable θ_n (reflecting the monopolist's consumer base in the preceding period). However, the equilibrium conditions here will differ from the ones we derived in Lemma 2 for the FIA scenario.

Lemma 4. *Under the PHI scenario, at any Markov Perfect Equilibrium in quasi-linear strategies, the coefficients λ and γ are related through the equation,*

$$\lambda = \frac{\beta\gamma(1-\gamma)}{(1-\beta\gamma)}$$

and the coefficients C and K are related through the equation

$$C = -\beta K\left(\frac{\gamma}{1-\beta\gamma}\right).$$

From these equations together with condition $\theta_{n+j+1} = K + \gamma\theta_{n+j}$, we get that provided $\beta \leq \overline{\beta} \simeq 0.752345$, there exists an equilibrium expansion strategy and an equilibrium expectations condition, where $\gamma^*(\beta)$ is the solution of the polynomial (37) *in the Online Appendix, and $K^*(\gamma^*(\beta), \beta, \widetilde{\theta})$ is given by equation* (38) *of the online Appendix.*

Proof: See the Online Appendix. ■

In order to guarantee that the solution described in Lemma 4 constitutes an MPE of the game, we need to impose conditions such that $\gamma^*(\beta) \in [0,1]$ and that $\theta_{\text{inf}} \in [0,1]$. The first condition, required for the stability and non-cyclicity of the market expansion process (if this condition was violated, the model would predict an explosive market expansion), is satisfied by the solution of (37). The second is required for the feasibility of the market expansion process. We show in the online Appendix that it is satisfied if $\beta \leq \overline{\beta} \simeq 0.752345$.

Comparing the results in Lemma 4 to the ones pointed out in Lemma 2 (for the FIA case), it becomes clear that when we consider a coarser customer profiling setting (as in the PHI setting), the equilibrium value of γ is smaller than the corresponding FIA counterpart (note that the value of γ in Lemma 4 is $(1-\beta)$ times the value of γ in Lemma 2). Accordingly, for a given market size θ_n, market expansion occurs faster under FIA than under PHI (as an attempt to get additional rents from first-degree price discrimination over old passive customers). The difference in the speed of market coverage turns out to be increasing with β (with the equilibrium values of γ coinciding when $\beta = 0$).

We now show in Proposition 2 below that there is a threshold value $\overline{c}^{PHI} = \frac{\beta(1-\gamma)\gamma}{(1-\beta\gamma)^2}$ of the privacy cost above which at equilibrium no consumer chooses to be active in the PHI scenario.

Proposition 2. *In the PHI setting, provided that $\beta \leq \overline{\beta} \simeq 0.752345$, there exists an MPE such that:*

(i) when the privacy cost $c < \overline{c}^{PHI}$*, the set of active customers* $\left[\widetilde{\theta}^{PHI}, 1\right]$*, where* $\widetilde{\theta}^{PHI}$ *is defined in equation* (39) *of the Online Appendix, is non-empty and it is always optimal to leave some consumers unserved. The fraction of unserved customers as well as the fraction of active customers are decreasing in* c*, tending to zero as* c *approaches* $\overline{c}^{PHI}$*.*

(ii) when the privacy cost $c \geq \overline{c}^{PHI}$*, all consumers are passive and the market is asymptotically fully covered.*

(iii) *the life-time net utility of the customer of type* $\theta_{\inf}$ *is zero.*

Proof: See the online Appendix. ■

The results are qualitatively the same as in the FIA case (stated in Proposition 1) and the intuition is also identical. For sufficiently low privacy cost, the market is never fully covered (not even asymptotically) and there are always consumers who choose to hide their identity, provided that the privacy cost is below a threshold value. The percentage of passive customers who are served is again a linear function of the privacy cost, with,

$$\widetilde{\theta}^{PHI} - \theta_{\inf} = \frac{c(1-\beta\gamma)^2}{\beta(1-\gamma)\gamma}.$$

This share equals zero (meaning that the only consumers who buy the good are active ones) when $c = 0$ and it equals 1 when $c \geq \overline{c}^{PHI}$.

Notice that, at the equilibrium, given (22), all customers with $\theta \geq \theta_1$ are indifferent between being passive or active exactly as is the marginal customer $\widetilde{\theta}^{PHI}$. However, $\widetilde{\theta}^{PHI}$ is the equilibrium marginal customer. Would indeed the marginal be some $\widetilde{\theta} > \widetilde{\theta}^{PHI}$ (respectively $< \widetilde{\theta}^{PHI}$), the introductory prices would be smaller (resp. greater) so that the advantage of becoming active would be strictly greater (resp. smaller) than the utility from being passive.

Remark 1: *Under PHI, at the MPE, no active customer could benefit from delaying the payment of the privacy cost to a period after the initial one iff* $\beta \leq \beta_{\max} \simeq 0.707107$.

Proof: See the Online Appendix.■

A comparison between equilibria in the FIA and PHI cases is instructive. It turns out that, for the same value of the privacy cost, the proportion of passive customers,$\widetilde{\theta} - \theta_{\text{inf}}$[37] is greater under PHI, as pictured below in Figure 4 (for $\beta = 0.5$).

Insert FIGURE 4 here

As shown in Figure 4 (with the exception of c belonging to a very small interval of low values, including 0), there is a smaller number of active customers under PHI than under FIA ($\widetilde{\theta}^{PHI} > \widetilde{\theta}^{FIA}$) together with a greater market coverage (a smaller value of θ_{inf}). Under PHI, contrary to FIA, most returning passive customers can secure a positive surplus since the firm does not identify their precise WTP but only the range of values to which it belongs. Accordingly they are less willing to actively invest in the ability of bypassing price-discrimination.

Let us now determine how, in the PHI case, aggregate discounted profit varies with the privacy cost. Corollary 4 sums up the main conclusions.[38]

Corollary 4: *In the PHI case, when an MPE in quasi-linear strategies exists (i.e.,* $\beta \leq \overline{\beta} \simeq 0.752345$), *the monopolist's aggregate discounted profit is a U-shaped function of the privacy cost* c.

This result is illustrated in Figure 5 which depicts the U-shaped relationship between the privacy cost and the aggregate profit[39], for three different values of β, namely $\beta = 0.2$, $\beta = 0.5$ and $\beta = 0.7$. Figure 5 also shows that for all values of the

[37] By passive customers, we mean here the consumers who choose not to incur the privacy cost *but to buy the good at some moment of time.*

[38] To prove Corollary 4, we compute the monopolist aggregate discounted profit in the same way as in the FIA case except that in period n the firm's profit Π_n^{PF} over former customers are given by equation (3) and we use the Euler equation (34) in the online Appendix. λ and C have to be replaced by their values from the corresponding equations of the online Appendix and $\gamma^*(\beta)$ follows from the corresponding equation of the online Appendix. Here again the resulting expression is much too long to be reproduced but allows for numerical computations.

[39] The dashed parts of the curves correspond to values of $c \geq \overline{c}^{PHI}(\beta)$, to situations where all consumers are passive.

privacy cost, a greater β (i.e., a shorter interval of commitment Δ) entails a smaller aggregate profit.

Insert FIGURE 5 here

The two effects of a variation of c on the monopolist's profit are qualitatively the same as in the FIA case, namely the incentive effect (a smaller privacy cost induces the firm not to lower too much its introductory prices) and a direct effect (more customers are able to bypass price discrimination). In the PHI case, the relative strength of the incentive effect versus the direct effect is clearly greater than in the FIA case, because it is more costly for the firm to have customers bypassing first-order than third-order price discrimination. It can be shown that the aggregate profit in the PHI case is globally decreasing in the level of the privacy cost if $\beta \in [0, 0.697977)$ and globally increasing if $\beta \in (0.697977, 0.752345]$.

Notice moreover that, as in the FIA case, the equilibrium when the privacy cost $c = 0$ is not identical to the repetition of the static monopoly equilibrium. This is specially obvious on Figure 5 above when $\beta = 0.7$: the profit when $c = 0$ is substantially smaller than the static equilibrium profit (1/4). This is the reason why in this case, the aggregate profit is also smaller than the one that obtains when c is so high that all consumers are passive. Although all consumers endogenously choose to behave passively (as in the model with exogenous passive consumers of Laussel and Resende, 2020a), equilibrium profits are much smaller herein (if the firm decided to set a monopoly static price equal to 1/2, customers would then become active, eroding firm's profits, which is not feasible in Laussel et al. (2020a), who exogenously assume all consumers to be passive).

Let us now turn our attention to the customers' surplus.[40] Again, period n social welfare must be equal to the sum of gross utilities of passive (former and new) and active customers, i.e;

$$w_n = \frac{1-\beta}{2}\left(1-\theta_{n+1}^2\right),$$

[40] Exactly as in the FIA case, this is most conveniently done by computing at first the social welfare, and then computing consumer surplus as the difference between total welfare and firms' aggregate profits.

from which must be subtracted the aggregate privacy costs $c\left(1-\widetilde{\theta}\right)$ which are incurred (in the initial period) by consumers who choose to be active.[41] The aggregate social welfare is $W=\sum_{n=0}^{\infty}\beta^{n}w_{n}$, which again turns out to be a hump-shaped function of the privacy cost, taking a maximum value for an intermediate value of it.

We can now compute customers' surplus, which is simply social welfare minus aggregate profit. This is pictured in Figure 6 below for several values of β.

Insert FIGURE 6 here

5 Conclusion

The purpose of our paper is to study how consumers' profiling affect firms' market expansion and (targeted) pricing policies when we allow for endogenous consumers' identity management within an infinite horizon model with a continuum of heterogeneous consumers. We also investigate how consumers' endogenous ability to successfully hide their preferences from the monopolist (at a cost) may impact monopoly profit and consumers' surplus, under two polar information/ consumer profiling structures: full information acquisition (FIA) and purchase history information (PHI). The former structure establishes a theoretical benchmark encompassing a limit form of hypersegmentation of markets, where the firm engages in individual-level market segmentation: returning passive consumers must pay a personalized price equal to their maximum WTP, leaving them with zero surplus.

In the second (maybe more realistic) case of purchase history information, *passive* customers in any period are recognized only by the period in which they make their first purchase. When a group of *passive* customers who made their first purchase in the same period returns in later periods, they are treated as a market segment and are offered a common price specific to their segment (group pricing).

In both information scenarios, the monopolist also makes in each period a new introductory-offer price, intended to attract new customers. Active customers, thanks to their identity management ability, are able to maintain their anonymity, and thus

[41] We apply mutatis mutandis the same method of the FIA case to obtain W as a function of $\gamma^{*}(\beta),\beta$, and c.

can masquerade as new customers, benefiting from the introductory offers (instead of paying the prices targeted to old customers).

We compute the MPE under FIA and PHI and we obtain an analytical characterization of firms' optimal market expansion strategy and consumers' equilibrium expectation rules and optimal privacy decisions, in each setting. The resulting managerial recommendations can be summed up as follows: when the privacy cost is small (consumers can easily hide their identity), managers should set higher introductory prices in order to extract rents from high-end customers who are able to hide their preferences from the firm. As a result, market expansion is slower when privacy costs are low. Moreover, full market coverage is never reached (not even asymptotically, unless all customers are passive). This means that, when identity management is not too costly, there will always be, regardless of the manager's ability to collect information on his customers, a fraction of low-end customers who remains unserved and a set of active customers. We also obtain that the measure of passive customers who are served is greater under PHI than under FIA (while less consumers are left unserved in the case of PHI than in the case of FIA).

A main finding, which turns out to be independent of what kind of information the monopolist is able to obtain from passive customers, is that the equilibrium profits are a U-shaped function of the privacy cost. However, the structure of information appears to matter when one compares the polar cases of zero privacy cost and of prohibitive privacy cost level: under FIA, a very low privacy cost is always detrimental to monopoly profit when compared to a prohibitive one. In contrast, under PHI, the opposite holds for most possible values of the discount factor. The intuition is that, under PHI, whenever the privacy cost is low, a large fraction of customers will choose to incur privacy cost, which in turns makes the monopolist less willing to adopt a sequence of rapidly falling introductory prices, overcoming the curse of knowledge described in Laussel et al. (2020a). Thus, firms (even monopolist ones, which benefit from the most favorable market structure) may actually benefit from government regulations on privacy protection intended to reduce the privacy cost (especially in those sectors where firms' ability to price discriminate is not perfect, as in our PHI scenario). Indeed, the results of our model point towards the ambiguous profit effects of such regulations, which highly depend on firms' information structure and on the initial level of privacy costs. Comparing equilibrium profits under FIA and PHI, our results suggest that profits are always higher under FIA than under PHI but that this difference is much smaller when privacy costs are small. This suggests that identity management may indeed hinder the monopolist's incentives to invest in improving the accuracy of its data collection procedures: as c goes down, it becomes less interesting to invest in technologies that allow it to shift from the PHI scenario to the FIA setting, since the difference in the corresponding equilibrium profits are lower when c goes down (in addition, consumers profiling technologies usually comprise fixed costs that may not be sufficient to overcome the additional variable profits obtained when the monopolist is able to engage in first-degree price discrimination _ instead of third-degree price discrimination _ within the segment of old passive customers).

Under FIA aggregate consumers surplus (CS) is a monotonically decreasing function of the privacy cost c while, under PHI, CS is an increasing function of c for low enough values of the discount factor.

Several interesting research questions remain open. Notably it would be interesting to investigate the case when the degree of accuracy of the monopolist's data collection is determined by the firm's investments in this field, such as hiring chief data officers or buying data from data providers. It would also be very interesting to understand how different market structure configurations may change our managerial recommendations: models that integrate competition and price discrimination often lead to very different results than monopoly models. In particular, competition opens the door to asymmetric situations where the old provider has information on customers tastes but the new provider does not so that small discounts by the poaching firm (even without price discrimination) may attenuate (at least partly) the strategic effect identified herein. This leads to a complex dynamic problem which opens a promising future research avenue on the dynamic competitive effects of consumers' anonymity decisions.

References

[1] ACQUISTI, A. (2008). "Identity management, privacy, and price discrimination". *IEEE Security & Privacy*, 6(2): 46-50.

[2] ACQUISTI, A., GROSS, R. (2009). "Predicting Social Security Numbers from Public Data." *Proceedings of the National Academy of Sciences*, 106 (27), pp. 10975-80.

[3] ACQUISTI, A., TAYLOR, C., & WAGMAN, L. (2016). "The economics of privacy". *Journal of Economic Literature*, 54(2), 442-92.

[4] ACQUISTI, A., & VARIAN, H. R. (2005). "Conditioning prices on purchase history". *Marketing Science*, 24(3), 367-381.

[5] AKAN, M., ATA, B., and DANA, J. D. "Revenue Management by Sequential Screening." *Journal of Economic Theory,* Vol. 159 (2015), pp. 728-774.

[6] ANDERSON, E. T., DANA, J. D. "When is Price Discrimination Profitable?" *Management Science,* Vol. 55 (2009), pp. 980-989.

[7] ANDERSON, S. P., BAIK, A. and LARSON, N. "Personalized Pricing and Advertising: An Asymmetric Equilibrium Analysis." *Games and Economic Behavior,* Vol. 92 (2015), pp. 53-73.

[8] ANDERSON, S. P., BAIK, A. and LARSON, N. "Price Discrimination in the Information Age: Prices, Poaching and Privacy with Personalized Targeted Discounts." CEPR Discussion Paper No. 13793, Centre for Economic Policy Research, 2019.

[9] ARIDOR, G., CHE, Y. K. and SALZ, T. "The Economic Consequence of Data Privacy Protection: Empirical Evidence from GDPR." *National Bureau of Economic Research*, Working Paper w26900, March 2020.

[10] ATA, B. and DANA, J. D. "Price Discrimination on Booking Time." *International Journal of Industrial Organization,* Vol. 43 (2015), pp. 175-181.

[11] BELLEFLAMME, P. and W. VERGOTE. "Monopoly price discrimination and privacy: The hidden cost of hiding". *Economics Letters*, 149 (2016): 141-144.

[12] BLEIER, A., GOLDFARB, A. and TUCKER, C. "Consumer Privacy and the Future of Data-Based Innovation and Marketing." International Journal of Re-

search in Marketing, Vol. 37 (2020), 466-480.

[13] CARRONI, E. (2016). Competitive customer poaching with asymmetric firms. International Journal of Industrial Organization, 48, 173-206.

[14] CASADESUS-MASANELL, R., and HERVAS-DRANE, A.. "Competing with privacy." *Management Science* 61, no. 1 (2015): 229-246.

[15] CHEN, Y. “Paying Customers to Switch,” *Journal of Economics and Management Strategy,* Vol. 6 (1997), 877-897.

[16] CHEN, Y. and PEARCY, J. "Dynamic Pricing: when to Entice Brand Switching and when to Reward Consumer Loyalty," *RAND Journal of Economics,* Vol. 41 (2010), pp. 674-685.

[17] CHEN, Z. CHOE, C. and MATSUSHIMA, N. "Competitive Personalized Pricing", *Management Science,* published on line (2020) https://doi.org/10.1287/mnsc.2019.3392

[18] CHOE, C., S. KING and MATSUSHIMA, N. “Pricing with Cookies: Behavior-Based Price Discrimination and Spatial Price Competition.” *Management Science,* Vol. 64 (2018), pp. 5669-5687.

[19] CONITZER, V., TAYLOR, C. R. and WAGMAN, L. “Hide and Seek: Costly Consumer Privacy in a Market with Repeat Purchases.” *Marketing Science,* Vol. 31 (2012), pp. 277-292.

[20] CORTS, K. S. "Third-degree price discrimination in oligopoly: all-out competition and strategic commitment." *The RAND Journal of Economics*, Vol. 29 (1998), pp. 306-323.

[21] EOP "Big Data and Differential Pricing," The Executive Office of the President of the United States, 2015.

[22] ESTEVES, R. B. “Price Discrimination with Partial Information: Does it Pay-off?” *Economics Letters*, 105 (2009), pp. 28-31.

[23] ESTEVES, R. B. “Pricing with Customer Recognition.” *International Journal of Industrial Organization*, Vol. 28((2010), pp. 669-681.

[24] FUDENBERG, D. and TIROLE, J. “Customer Poaching and Brand Switching.” *RAND Journal of Economics,* Vol. 31 (2000), pp. 634-657.

[25] FUDENBERG, D. and VILLAS-BOAS, J.M. "Behavior-Based Price Discrimination and Customer Recognition." In T. Hendershot, ed., *Economics and Information Systems*. Amsterdam: Elsevier, 2006.

[26] FUDENBERG, D. and VILLAS-BOAS, J.M. "Price Discrimination in the Digital Economy." In M. Peitz and J. Waldfogel, eds., *Oxford Handbook of the Digital Economy*. Oxford: Oxford University Press, 2012.

[27] GEHRIG, T., SHY, O. and STENBACKA, R. "History-Based Price Discrimination and Entry in Markets with Switching Costs." *European Economic Review* Vol. 55 (2011), pp. 732-739.

[28] GEHRIG, T., SHY, O. and STENBACKA, R. "A Welfare Evaluation of History-Based Price Discrimination." *Journal of Industry, Competition and Trade* Vol. 12 (2012),pp. 373-393.

[29] ICHIHASHI, S. (2020). Online privacy and information disclosure by consumers. *American Economic Review*, 110(2), 569-95.

[30] LAUSSEL, D., LONG, N. V. and RESENDE, J. "The Curse of Knowledge: Having Access to Customer Information Can be Detrimental to Monopoly's Profit." *RAND Journal of Economics*, Vol. 51 (2020a), pp.650-675.

[31] LAUSSEL, D., LONG, N. V. and RESENDE, J. "Quality and Price Personalization under Customer Recognition: A Dynamic Monopoly Model with Contrasting Equilibria". *Journal of Economic Dynamics and Control,* (2020b).Volume 114, May 2020, Pages 1-25. https://doi.org/10.1016/j.jedec.2020.103869.

[32] MOHAMMED, R. "How Retailers Use Personalized Prices to Test What You're Willing to Pay." *Harvard Business Review,* 2017 October 20.

[33] MONTES, R., SAND-ZANTMAN, W., & VALLETTI, T. The value of personal information in online markets with endogenous privacy. *Management Science*, Vol. 65(3) (2019), pp. 1342-1362.

[34] RAO, A. "Online Content Pricing: Purchase and rental markets." *Marketing Science,* Vol. 34 (2015), pp. 430-451.

[35] STOLE, L. A. Price discrimination and competition. Handbook of industrial organization, Vol. 3 (2007), pp. 2221-2299.

[36] TAYLOR, C. R. "Consumer privacy and the market for customer information",

RAND Journal of Economics, Vol. 35(4) (2004), pp. 631-650.

[37] THISSE, J. F. and VIVES, X. "On the Strategic Choice of Spatial Price Policy." *American Economic Review,* Vol. 78 (1988), pp.122-137.

[38] VILLAS-BOAS, M. "Dynamic Competition with Customer Recognition." *RAND Journal of Economics,* 30 (1999), pp. 604-631

[39] VILLAS-BOAS, J. M. "Price cycles in markets with customer recognition." *RAND Journal of Economics*, 35 (2004), pp. 486-501.

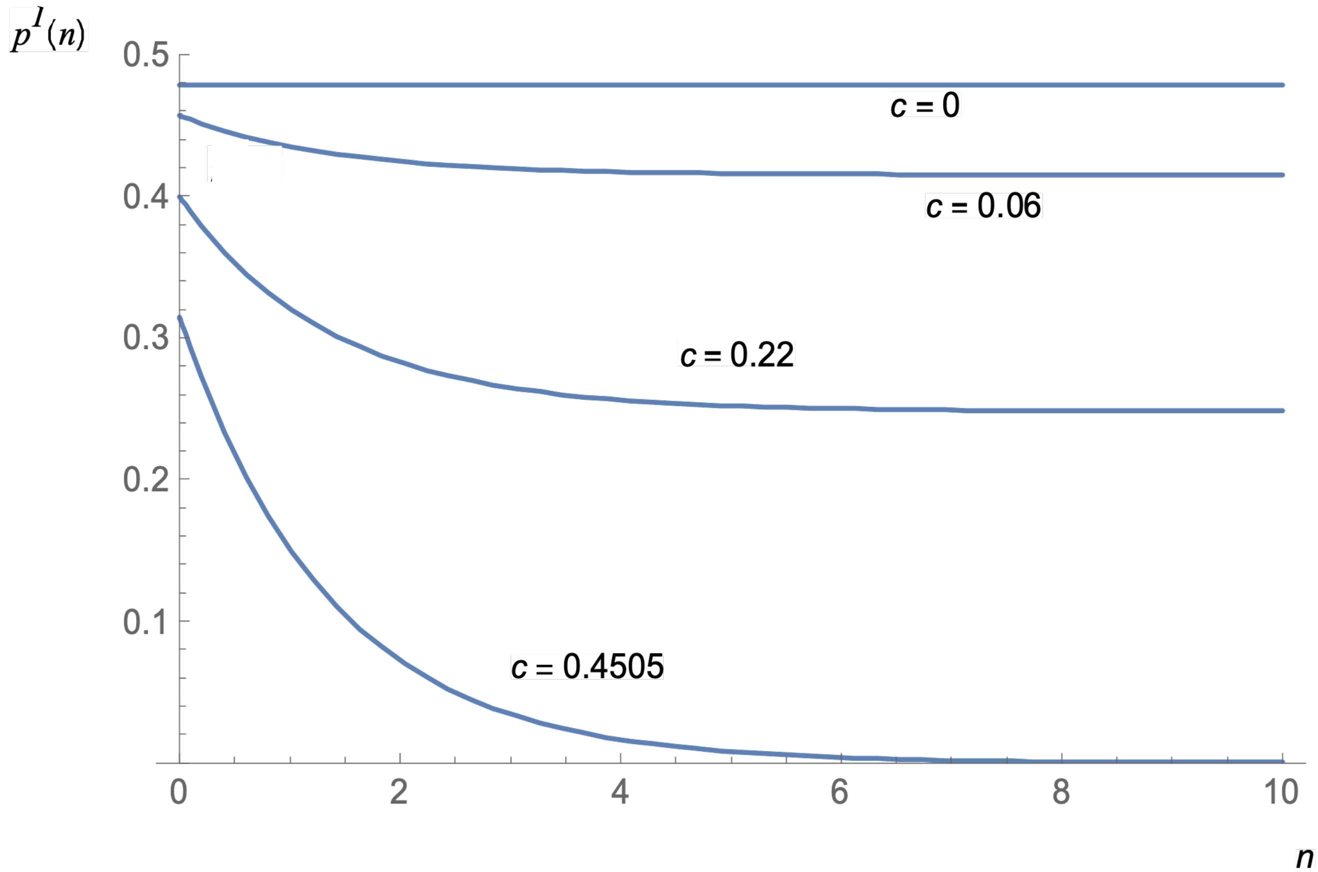

Figure 1: Evolution of introductory prices

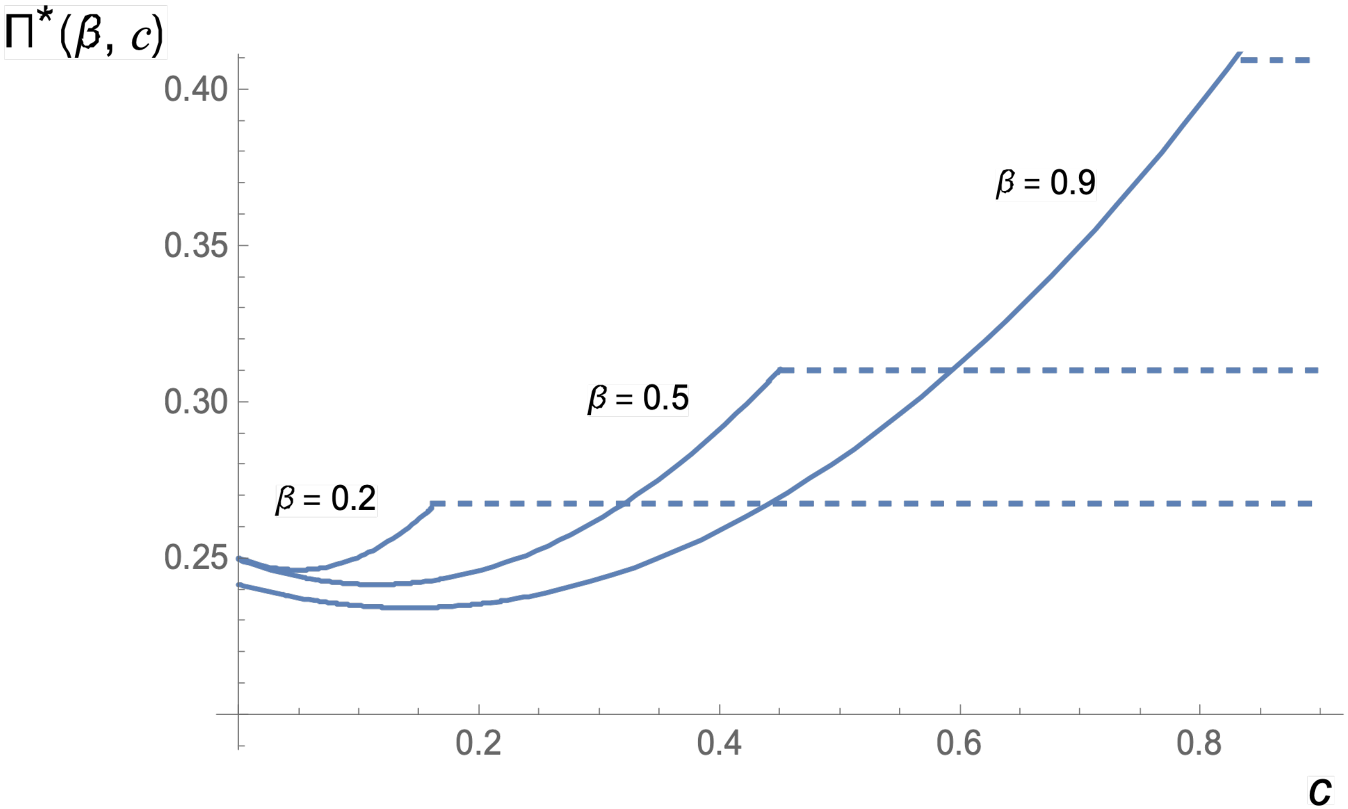

Figure 2: Profits-privacy cost relationship (FIA)

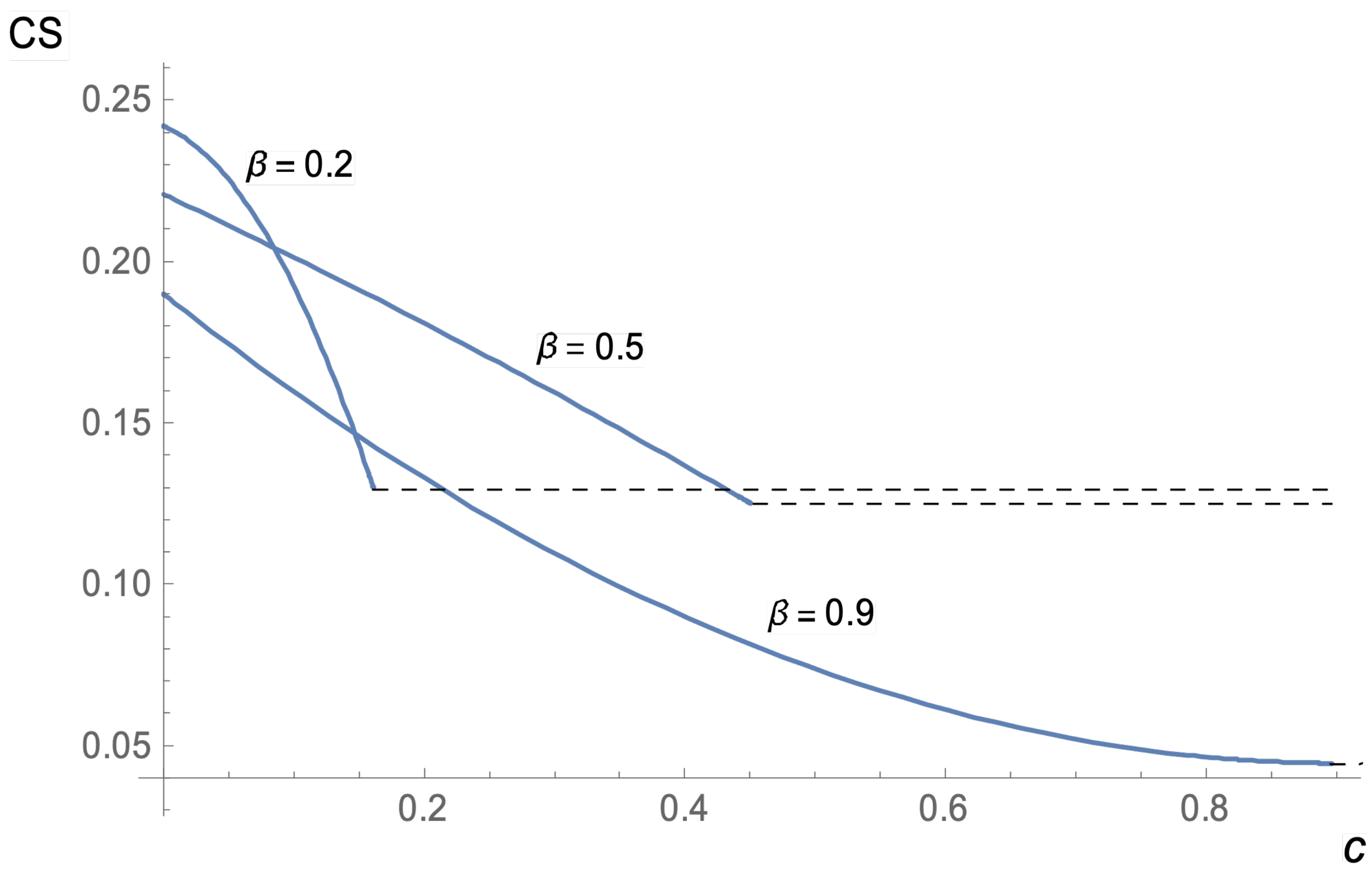

Figure 3: Consumers surplus and privacy cost

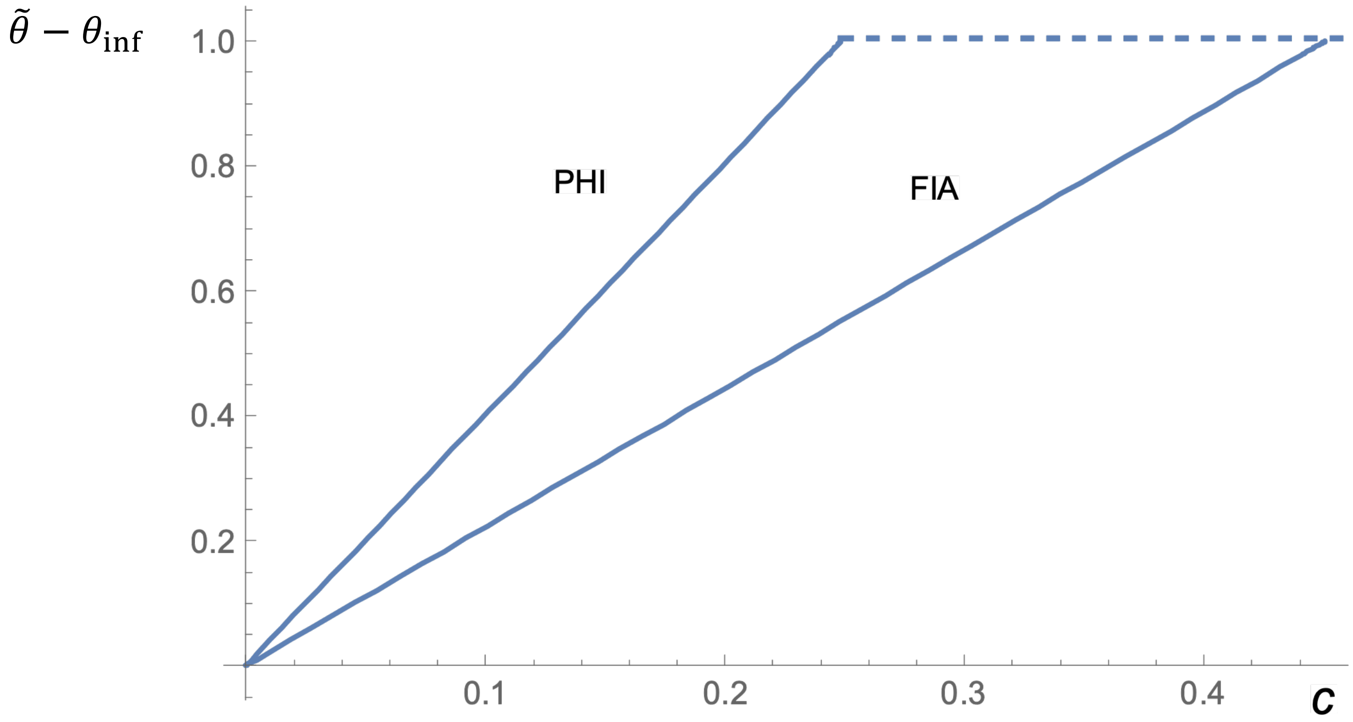

Figure 4: Proportions of passive customers as functions of the privacy cost

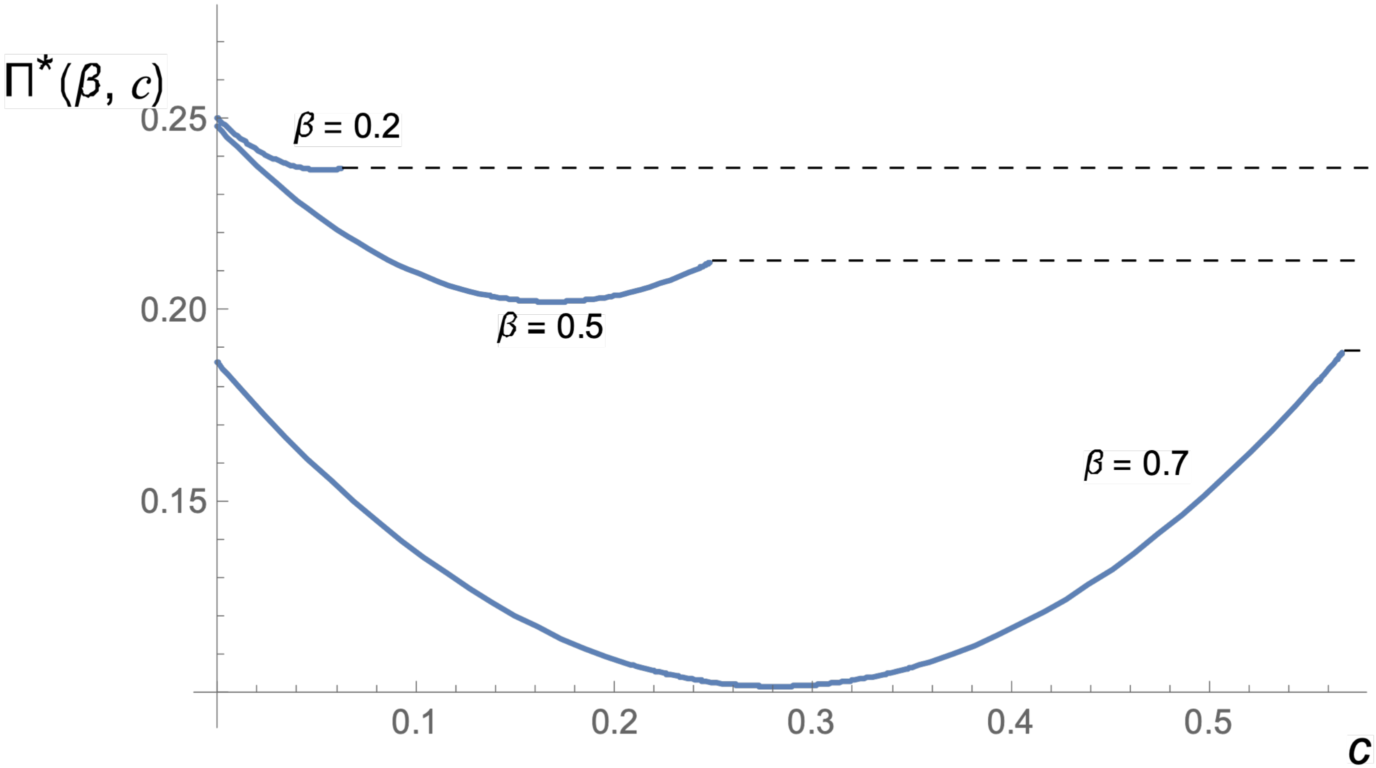

Figure 5: Profits-privacy cost relationship (PHI)

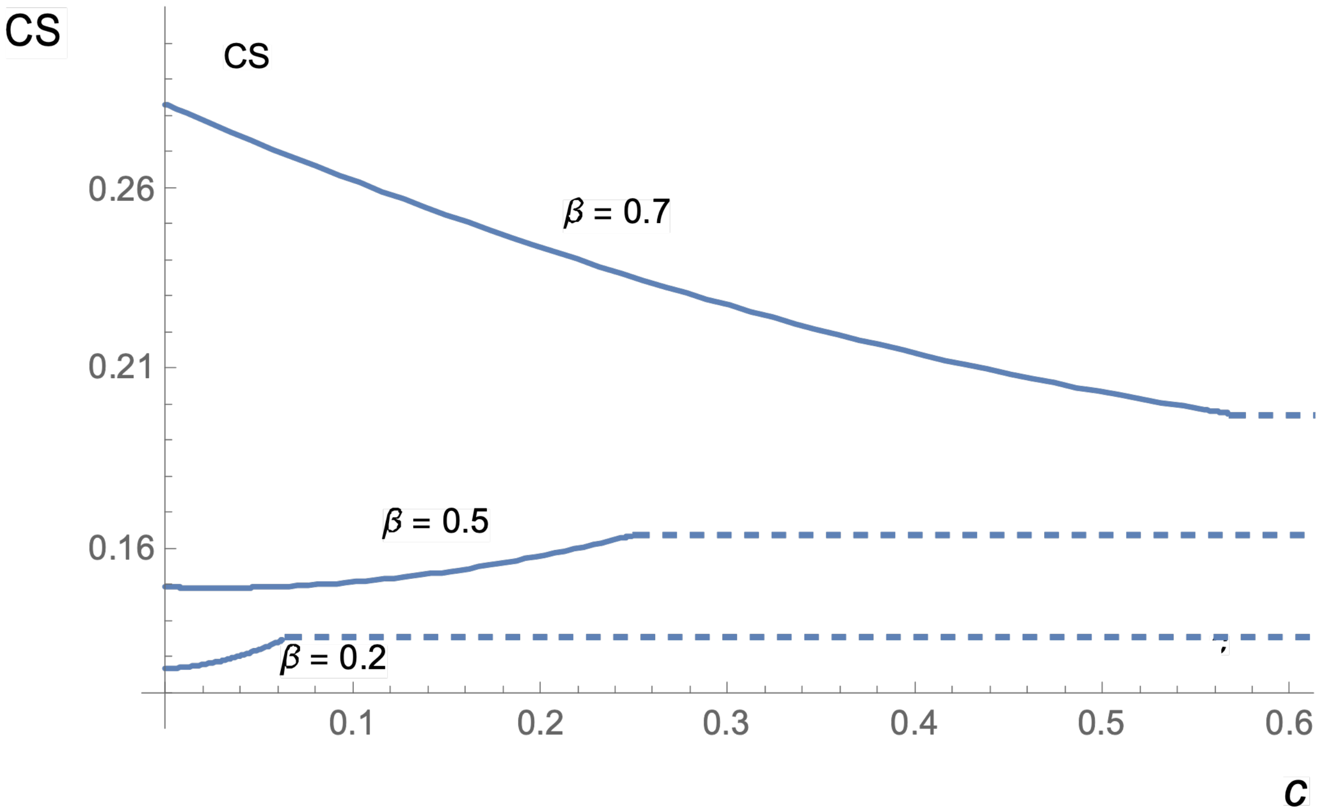

Figure 6: Consumers surplus and privacy cost (PHI)

ONLINE APPENDIX of "Profit Effects of Consumers' Identity Management: a dynamic model"[1]

Proof of Lemma 1. In the FIA case, if a consumer of type θ chooses to be passive, her intertemporal utility (discounted to the initial period) is equal to the discounted utility she obtains during her (optimally chosen) first-time purchase period N

$$u_{FIA}^{P}(\theta) = \max_{N} \beta^{N}(1-\beta)(\theta - p^{I}(N)). \tag{19}$$

Denote by $N(\theta)$ the best period for a type θ passive customer to buy the good for the first time. The pivot consumer type $\widetilde{\theta}$, if she chooses to be passive, will optimally select $N(\widetilde{\theta}) = 0$, i.e., she makes her first purchase in the initial period. The intuition follows from the fact that the pivot customer is the one with the highest θ choosing to act passively and therefore she should buy the good at the initial period (so that other lower $\theta - types$ can enter the market afterwards, as the monopolist reduces its introductory price). Being a pivot type, it also must hold that $u^{A}(\widetilde{\theta}) - u_{FIA}^{P}(\widetilde{\theta}) = 0$. Consequently, we must have

$$u^{A}(\widetilde{\theta}) - u_{FIA}^{P}(\widetilde{\theta}) = \beta\widetilde{\theta} - c - \sum_{n=1}^{\infty}(1-\beta)p^{I}(n)\beta^{n} = 0. \tag{20}$$

This equation shows that the threshold value $\widetilde{\theta}$ depends on the consumers' expectation about the monopolist's sequence of introductory prices $\left\{p^{I}(n)\right\}$ under the FIA information structure. (Notice that we should distinguish $\widetilde{\theta}_{FIA}$ from $\widetilde{\theta}_{PHI}$, but we have omitted these subscripts in the analysis above for the sake of exposition). Moreover, of course, in equilibrium the introductory price sequence $\left\{p^{I}(n)\right\}$ comes from the solution of the monopolist's dynamic optimization problem, taking $\widetilde{\theta}$ as given.

From the Envelope Theorem, $\frac{d\left(u^{A}(\theta) - u_{FIA}^{P}(\theta)\right)}{d\theta} = 1 - (1-\beta)\beta^{N(\theta)} > 0$. This implies that, given that $\widetilde{\theta}$ is the pivot type in equilibrium, we have $u^{A}(\theta) > u_{FIA}^{P}(\theta)$ for all $\theta \in \left(\widetilde{\theta}, 1\right]$. As expected, customers with type $\theta > \widetilde{\theta}$ are better off incurring the privacy cost to conceal their identity from the firm (instead of paying high prices corresponding to their WTP); while $u^{A}(\theta) < u_{FIA}^{P}(\theta)$ for all $\theta \in \left[0, \widetilde{\theta}\right)$, i.e. customers

[1] The numbering of the equations and figures follows that of the main text.

with type $\theta < \widetilde{\theta}$ obtain a greater intertemporal utility by choosing to be passive. This establishes the result in Lemma 1 for the FIA case.

In the PHI case, if a consumer of type θ chooses to be passive, her intertemporal utility is equal to the sum of her discounted utility obtained during her (optimally chosen) first-time purchase period N and the additional utility obtained in subsequent periods as the result of the informational rent obtained over that of marginal customer type of the same cohort/vintage. Accordingly:

$$u^P_{PHI}(\theta) = \max_N \beta^N \left[(1-\beta)(\theta - p^I(N)) + \beta(\theta - \theta_{N+1})\right]. \qquad (21)$$

Given that the best period for a pivot type $\widetilde{\theta}$ customer to buy the good for the first time as a passive customer is the initial period, the relevant indifference condition in the PHI case is then

$$u^A(\widetilde{\theta}) - u^P_{PHI}(\widetilde{\theta}) = \beta\theta_1 - c - \sum_{n=1}^{\infty}(1-\beta)p^I(n)\beta^n = 0, \qquad (22)$$

where $\widetilde{\theta} - \theta_1$ is the measure of passive customers that the monopolist chooses to serve in the initial period. Recall that the sequence $\{p^I(n)\}$ in equation (22) is not the same as the sequence $\{p^I(n)\}$ in equation (22) because they correspond to different information structures. In equilibrium, the sequence $\{p^I(n)\}$ in equation (22) is part of the solution of the monopolist's dynamic optimization problem, taking $\widetilde{\theta}$ as given; and θ_1 reflects the monopolist's optimal market expansion strategy, which again, is solved taking $\widetilde{\theta}$ as given. Thus, while in the right-hand side of eq. (22), the term $\widetilde{\theta}$ does not appear explicitly, it does so implicitly. In particular the value $\widetilde{\theta}$ is implicitly contained in the terms θ_1 and $p^I(n)$, which the monopolist optimally sets as response to the consumers' strategies (which include their cut-off choice $\widetilde{\theta}$), as referred in section 4 of the paper.

Notice that $\frac{d\left(u^A(\theta) - u^P_{PHI}(\theta)\right)}{d\theta} = 1 - \beta^{N(\theta)}$. The term $1 - \beta^{N(\theta)}$ is strictly positive for all θ such that $N(\theta) > 0$, i.e., for all $\theta < \theta_1$. This implies that also in the PHI case, all customers belonging to vintages greater than vintage 0 strictly prefer being passive. As for passive customers that belong to the $\left(\theta_1, \widetilde{\theta}\right)$ interval, i.e., to vintage 0, $N(\theta) = 0$, so they are indifferent between being active or passive. For this vintage, the utility differential is type independent and it reflects only future price differences.[2] This indifference may be broken by considering a small

[2]For all θ in this interval the difference $u^A(\theta) - u^P_{PHI}(\theta) = u^A(\widetilde{\theta}) - u^P_{PHI}(\widetilde{\theta})$ such as given by (22) and does not depend on θ.

perturbation in which with probability ϵ consumers expect the firm to recognize their exact WTP and considering the limit equilibrium when $\epsilon \to 0$. Anyway we shall see that there always exists an equilibrium at which consumers $\theta \in \left[\widetilde{\theta}, 1\right]$ and only them incur the privacy cost in order to conceal their identity. All the other consumers remain passive, allowing us to show the result in Lemma 1 for the PHI case. ■

Proof of Lemma 2.

In the FIA case, each former passive customer is charged a personalized price equal to her willingness to pay so that the period n monopolist's profits on former passive customers are given by equation (1). Then, under Markovian expectations, we get

$$\pi_n(\theta_n, \theta_{n+1}) = \frac{1}{b}\left[\left(\frac{\widetilde{\theta}^2 - \theta_n^2}{2}\right) + \left[1 - \widetilde{\theta} + \theta_n - \theta_{n+1}\right](\theta_{n+1} - b\Psi(\theta_n))\right]$$

where $b \equiv \frac{1}{1-\beta}$, and equation (17) becomes, in the FIA case,

$$V'(\theta_n) = (1-\beta)(\theta_{n+1} - \theta_n) - \Psi(\theta_n) - \Psi'(\theta_n)\left(1 - \widetilde{\theta} - \theta_n + \theta_{n+1}\right). \quad (23)$$

Upon substituting eq. (13) into the above equation, we get

$$V'(\theta_n) = A_0 + A_1\theta_n + A_2\theta_{n+1}, \quad (24)$$

$$V'(\theta_n) = A_0 + A_1\theta_n + A_2(K + \gamma\theta_n), \quad (25)$$

where the coefficients A_i are

$$\begin{aligned} A_0 &= -C - \lambda(1 - \widetilde{\theta}), \\ A_1 &= -(1-\beta), \\ A_2 &= (1-\beta) - \lambda. \end{aligned}$$

From (14), we get $\beta V'(\theta_{n+1}) = A_0 + A_1\theta_{n+1} + A_2\theta_{n+2}$ and upon substituting it into *(16),* we obtain the Euler equation as

$$B_0 + B_1\theta_n + B_2\theta_{n+1} + B_3\theta_{n+2} = 0, \quad (26)$$

where the coefficients B_0, B_1, B_2 and B_3 are

$$\begin{aligned} B_0 &= C(1-\beta) + ((1-\beta) - \beta\lambda)(1 - \widetilde{\theta}) \\ B_1 &= (1-\beta) + \lambda, \\ B_2 &= -(1-\beta)(2+\beta), \\ B_3 &= \beta((1-\beta) - \lambda). \end{aligned}$$

We are now ready to determine the coefficients K, γ, C and λ that make the monopolist's cut-off rule and the consumers expectation rule best reply to each other.

Using the arbitrage equation (8) we find that in the MPE equilibrium, the coefficients λ and γ are related through the equation,

$$\lambda = \frac{\beta(1-\beta)(\gamma-\gamma^2)}{1-\beta\gamma}, \tag{27}$$

and the coefficients C and K are related through the equation

$$C = -\beta K \left[\frac{\gamma(1-\beta)}{1-\beta\gamma}\right]. \tag{28}$$

Proof of Corollary 1.

Using the relationships (27) and (28), together with $\theta_{n+j+1} = K + \gamma\theta_{n+j}$, we find that the Euler equation (16) is satisfied if and only if the following two conditions are satisfied:

$$-1 + (2+\beta+\beta^2)\gamma - 2\beta(1+\beta)\gamma^2 + \beta^2\gamma^3 = 0, \tag{29}$$

$$K = -\frac{(1+\beta(-1+\beta(-1+\gamma))\gamma)(1-\widetilde{\theta})}{2+\beta(-2+\beta(-1+\gamma))\gamma}. \tag{30}$$

The solution of (29) is pictured below.

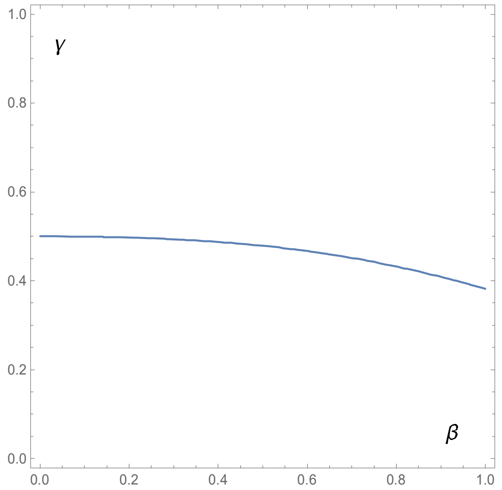

Figure 7. γ^* as function of β (FIA)

■

Proof of Lemma 3.

Starting with equation (20), we substitute for $p^I(n)$ its value from equation (4). We then use the cutoff rule $\theta_{n+1} = K + \gamma\theta_n$ and the expectations rule (14) to write $p^I(n) = K + \gamma\theta_n - \frac{1}{1-\beta}(C + \lambda\theta_n)$. Then substitute for C and λ their values from (27) and (28) to obtain $p^I(n) = \frac{1}{1-\beta\gamma}(K + \gamma(1-\beta)\theta_n)$. Finally use the market expansion equation $\theta_n = K\frac{1-\gamma^n}{1-\gamma} + \widetilde{\theta}\gamma^n$ implied by the cut-off rule and $\theta_0 = \widetilde{\theta}$. Solving for $\widetilde{\theta}$ we then obtain equation (18).■

Proof of Proposition 1.

Using equations (18) and (30), we are able to solve for the equilibrium values of K and $\widetilde{\theta}$, which are such that

$$\widetilde{\theta}^{FIA} = \min\{\gamma + Dc, 1\}, \tag{31}$$

where

$$D = \frac{(1-\beta\gamma)^2(2+\beta(-2+\beta(-1+\gamma)))\gamma}{\beta(-1+(-1+2\beta)\gamma)(-3+(2+\beta(3+\beta(-2+\gamma)(-1+\gamma)-2\gamma))\gamma)},$$

and γ stands for the solution $\gamma^*(\beta)$ of equation (29). The equilibrium value of K is obtained by substituting $\widetilde{\theta}^{FIA}$ for $\widetilde{\theta}$ in equation (30). Given that $\theta_{\inf} = \frac{K}{1-\gamma}$, it is easy to check that $\widetilde{\theta}^{FIA} - \theta_{\inf} = \min\{\frac{c(1-\beta\gamma)^2}{\beta(1-\gamma)(1+(1-2\beta)\gamma)}, 1\}$.

Straightforwardly the solution of $\frac{c(1-\beta\gamma)^2}{\beta(1-\gamma)(1+(1-2\beta)\gamma)} = 1$ is $\overline{c}^{FIA} = \frac{\beta(1-\gamma)(1+(1-2\beta)\gamma)}{(1-\beta\gamma)^2}$. One checks that $\widetilde{\theta}^{FIA} < 1$ and $\theta_{\inf} > 0$ iff $c < \overline{c}^{FIA}$.

Finally, it is easy to verify that

$$\lim_{n\to\infty} \Psi(\theta_n) = -K\left[\frac{\beta\gamma(1-\beta)}{1-\beta\gamma}\right] + \left[\frac{\beta\gamma(1-\beta)(1-\gamma)}{1-\beta\gamma}\right]\theta_{\inf} = 0.$$

■

Proof of Corollary 2.

By incurring the privacy cost only in period 1, a type θ-active customer achieves an intertemporal utility equal to

$$\beta(\theta - c) - \sum_{n=1}^{\infty}(1-\beta)\, p^I(n)\beta^n$$

whereas the intertemporal utility from beginning to consume in period 0 is given by equation (12). The difference between the latter and the former equals $(1-\beta)\left(\theta - c - p^I(0)\right)$. Accordingly, it is optimal not to delay the consumption of the good iff $\theta - c - p^I(0) \geq 0$. This condition is satisfied for any type $\theta-$active customer if it is satisfied for $\theta = \widetilde{\theta}$. Using $p^i(0) = \frac{1}{1-\beta\gamma}(K + \gamma(1-\beta)\widetilde{\theta})$ and substituting for $\widetilde{\theta}$ and K their equilibrium values, we obtain

$$\widetilde{\theta} - c - p^I(0) = c\frac{(1-\beta)(1-2\beta\gamma)}{\beta(1+(1-2\beta)\gamma)}. \tag{32}$$

We then picture in blue the area in the $(\beta, \gamma)-$ space for which the above expression is positive as well as the locus $\gamma = \gamma^*(\beta)$. This locus turns out to belong to the blue area so that we conclude that (32) is always positive at equilibrium (See Figure 8).

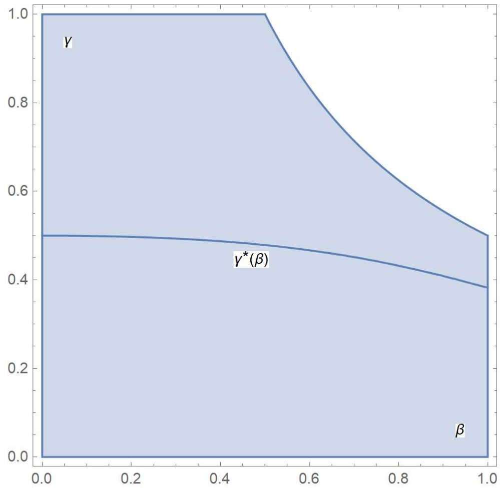

Figure 8. No delay area (FIA case)

■

Proof of Lemma 4.

In the PHI case, the monopolist will segment former passive customers according to the timing of their first purchase, engaging in third-degree price discrimination for each segment. Hence, the period n monopolist's profits on former passive customers are given by equation

$$\pi_n(\theta_n, \theta_{n+1}) = \frac{1}{b}\left[\left(\sum_{i=1}^{n}(\theta_{i-1}-\theta_i)\theta_i\right) + \left[1-\widetilde{\theta}+\theta_n-\theta_{n+1}\right](\theta_{n+1}-b\Psi(\theta_n))\right],$$

and equation (17) becomes, in the PHI case,

$$V'(\theta_n) = -\Psi'(\theta_n)\left[1-\widetilde{\theta}+\theta_n-\theta_{n+1}\right] + \frac{1}{b}(\theta_{n+1}+\theta_{n-1}-2\theta_n) - \Psi(\theta_n). \tag{33}$$

From (33), we obtain $\beta V'(\theta_{n+1})$ and upon substituting it into *(16)*, and then using the expectations rule (12), we obtain the Euler equation as

$$\begin{aligned}0 = & C(1-\beta) + (1-\beta-\lambda\beta)(1-\widetilde{\theta}) + (1-\beta^2+\lambda)\theta_n \\ & -(1-\beta^2+\beta\lambda)2\theta_{n+1} + \beta(1-\beta+\lambda)\theta_{n+2}.\end{aligned} \tag{34}$$

We are now ready to determine, for a given value of $\widetilde{\theta}$, the coefficients K, γ, C and λ that make the monopolist's cut-off rule and the consumers expectation rule best reply to each other.

Using the arbitrage equation (9) we find that in the MPE equilibrium, the coefficients λ and γ are related through the equation

$$\lambda = \frac{\beta\gamma(1-\gamma)}{(1-\beta\gamma)}, \tag{35}$$

and the coefficients C and K are related through the equation

$$C = -\beta K\left(\frac{\gamma}{1-\beta\gamma}\right). \tag{36}$$

Now, substitute for C and λ in the equation (34) their values from (36) and (35), then replace θ_{n+1} by $K+\gamma\theta_n$ and θ_{n+2} by $K+\gamma(K+\gamma\theta_n)$. The Euler equation reduces to an equation of the form $X+Y\theta_n=0$. For this to hold for all θ_n, it must be true that $Y=0$ and $X=0$. The equation $Y=0$ is a polynomial in γ with coefficients in terms of β and μ. Thus, we can solve numerically for $\gamma^*(\beta)$ for $\gamma \in (0,1)$. The equation $X=0$ is an equation in K with coefficients in terms of γ, β and $\widetilde{\theta}$.

We obtain:

$$Y = -\frac{1}{(1-\beta\gamma)}\left[C_0 + C_1\gamma + C_2\gamma^2 + C_3\gamma^3 + C_4\gamma^4\right] = 0, \tag{37}$$

where the values of the coefficients C_i $(i = 0, 1, 2, .., 4)$ are

$$\begin{aligned}C_0 &= (1-\beta)^2, \\ C_1 &= -2+2\beta^2+\beta^3, \\ C_2 &= 2\beta-3\beta^2-2\beta^3, \\ C_3 &= 2\beta^2+\beta^3, \\ C_4 &= -\beta^2.\end{aligned}$$

The equilibrium value of γ, if any, is a solution $\in [0,1]$ of a fourth order polynomial (the bracketed term above). The solution of (37) is pictured below:

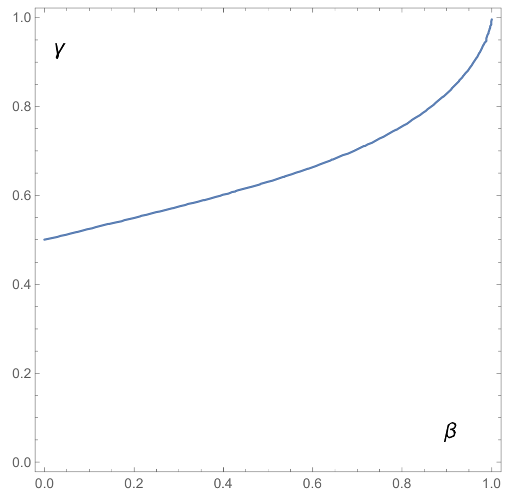

Figure 8. $\gamma^*(\beta)$ in the PHI case

From $X = 0$, one obtains the candidate equilibrium value of K as

$$K = (1-\widetilde{\theta})\left[\frac{1+\beta(-1+\gamma(-1+\beta\gamma))}{2-\beta(1+\beta+2\gamma-\beta\gamma(2+\beta-(1+\beta)\gamma+\gamma^2))}\right]. \quad (38)$$

Any meaningful solution requires that $K \geq 0$. The existence of a solution where there are active customers ($\widetilde{\theta} < 1$) supposes accordingly that the bracketed term in (38) be positive. Together with (37) it is easily checked that this requires that $\beta \leq \overline{\beta} \simeq 0.752345$.■

Proof of Proposition 2.

(i) Starting with equation (22), we substitute for $p^I(n)$ its value from equation (4). We then use the cutoff rule $\theta_{n+1} = K + \gamma\theta_n$ and the expectations rule (14) to write $p^I(n) = K + \gamma\theta_n - \frac{1}{1-\beta}(C + \lambda\theta_n)$. Then substitute for C and λ their values from (35) and (36) to obtain $p^I(n) = \frac{1}{1-\beta\gamma}(K + \gamma(1-\beta)\theta_n)$. Finally use the market expansion equation $\theta_n = K\frac{1-\gamma^n}{1-\gamma} + \widetilde{\theta}\gamma^n$ implied by the cutoff rule and $\theta_0 = \widetilde{\theta}$. Solving for $\widetilde{\theta}$ we then obtain:

$$\widetilde{\theta}^{PHI} = \frac{c(1-\beta)(1-\beta\gamma)+\beta K(\gamma-\lambda)}{(1-\beta)\beta\gamma(1-\gamma-\lambda)}. \quad (39)$$

(ii) Using equations (38) and (39), we are able to solve for the equilibrium values of K and $\widetilde{\theta}$, which are such that

$$\widetilde{\theta}^{PHI} = \min\{\frac{R+Tc}{S}, 1\} \quad (40)$$

where

$$\begin{aligned}
S &= \beta\gamma\left[2+\beta(1+\gamma)(-2+\beta\gamma)\right],\\
R &= (1-\beta)\beta\gamma-\beta^2\gamma^2+\beta^3\gamma^3,\\
T &= (1-\beta\gamma)^2\left(-2+\beta+\beta^2+\left(2\beta-2\beta^2-\beta^3\right)\gamma+\right.\\
&\quad \left.\beta^2(1+\beta)\gamma^2-\beta^2\gamma^3\right),
\end{aligned}$$

and γ stands for the solution $\gamma^*(\beta)$ of equation (37). The equilibrium value of K is obtained by substituting $\widetilde{\theta}^{PHI}$ for $\widetilde{\theta}$ in equation (38). Given that $\theta_{\text{inf}} = \frac{K}{1-\gamma}$, it is easy to check that $\widetilde{\theta}^{PHI} - \theta_{\text{inf}} = \min\{\frac{c(1-\beta\gamma)^2}{\beta(1-\gamma)\gamma}, 1\}$. Straightforwardly the solution of $\frac{c(1-\beta\gamma)^2}{\beta(1-\gamma)\gamma} = 1$ is $\overline{c}^{FIA} = \frac{\beta(1-\gamma)\gamma}{(1-\beta\gamma)^2}$. One checks that $\widetilde{\theta}^{PHI} < 1$ and $\theta_{\text{inf}} > 0$ iff $c < \overline{c}^{PHI}$.■

Proof of Remark 1.

Notice that, by the same argument as in the proof of Corollary 2, it is optimal not to delay the consumption of the good iff $\theta - c - p^I(0) \geq 0$. This condition is satisfied for any type $\theta-$active customer if it is satisfied for $\theta = \widetilde{\theta}$. Using $p^i(0) = \frac{1}{1-\beta\gamma}(K + \gamma(1-\beta)\widetilde{\theta})$ and substituting for $\widetilde{\theta}$ and K their equilibrium values, we obtain

$$\widetilde{\theta} - c - p^I(0) = c(-2 + \frac{1}{\beta\gamma}). \tag{41}$$

Not delaying is then optimal iff $-2 + \frac{1}{\beta\gamma} \geq 0$ where γ is the solution of (37). It then turns out that the no-delay condition is equivalent to $\beta \leq \beta_{\max} \simeq 0.707107$.■