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# Impact of Tax Reforms in Applied Models: Which Functional Forms Should Be Chosen for the Demand System? Theory and Application for Morocco \*

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## Abstract/Résumé

When researchers and policymakers conduct impact analyses of economic reforms, especially fiscal reforms, the specification of the household demand system becomes crucial. There is a trade-off between using demand systems simple to manipulate but less realistic and other systems that are more realistic but often more complex and difficult to estimate or calibrate. In this paper, we compare the results from two different demand systems: a simple one, the Cobb-Douglas (CD), and a more complex one, the Constant Difference Elasticity (CDE). We develop an hybrid method of estimation - calibration based on the estimation of the parameters and elasticities of a QUAIDS system and on the calibration of those of the CDE system using a cross-entropy approach. The estimates obtained are introduced into a micro-simulated partial equilibrium model to approximate the impact of the VAT reform on poverty measures in Morocco. We show that when the simulated shocks are moderate, the gain of using a CDE system instead of a CD system is marginal but, when these shocks are stronger, the differences become significant and increase. Then the use of these models can lead to different results when evaluating public policies and their impacts on poverty measures.

**Keywords/Mots-clés:** Demand Systems, Estimation-Calibration, Tax Reform, Morocco

**JEL Codes/Codes JEL:** C51, D12, I32, H31

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# 1 Introduction and objective

In a context of general economic reforms in several countries in the Middle East and North Africa (MENA), including Morocco, tax reforms are often put on the agenda. During the 3<sup>emes</sup> National Assises on Taxation held in May 2019 in Skhirat, under the theme "tax equity", a working group was formed to explore the neutrality of value added tax (VAT), make a diagnosis on the purchasing power of consumers and propose reform scenarios. Researchers, government authorities and international organizations try to approach the impacts of such reforms on the economy as a whole and on households in particular. Indeed, the latter are directly impacted following the implementation or after the changes in the VAT rates that directly or indirectly, modify the price levels of various goods and services. These reforms naturally change the levels of monetary poverty measures and household welfare measures (see for example, De Quatrebarbes et al. (2016), Ndemezo and Baye (2016) or Boeters et al. (2010)).

In terms of the economic analysis tools used to anticipate and assess these various impacts, several models are usually used. These are computable general equilibrium (CGE) models or micro-simulated calculable partial equilibrium models such as the one we propose in this article. These models use household survey data. At the level of household utility and/or demand functions used in these models, several functional forms are proposed in the literature. Each has its pros and cons. Some are rather simple to use with parameters that are easy to estimate or calibrate but often have *a priori* unrealistic properties. Others are, on the other hand, much more flexible and presumably, better approximate household behaviour but reveal parameters that are difficult to estimate and even to calibrate. The choice between these two types of specifications often raises problems for researchers, who are faced with a dilemma between facility, complexity, rigour, reliability and credibility of the results. In this paper, we propose to approach and empirically examine the gap between two demand systems. The first is the simplest and derives from the well known Cobb-Douglas direct utility function (CD). The second is more flexible but more complex; it is the one resulting from

the Constant Difference Elasticity (CDE). This demand system was introduced by Hanoch (1975).

At the methodological level, we develop an hybrid estimation-calibration approach that involves several steps to approximate the parameters of the CDE demand system based on Yu et al. (2004), Chen (2017) and van der Mensbrugghe (2020). This method consists of calibrating the elasticities and parameters of the CDE system using estimates obtained from another demand system (AIDS<sup>1</sup> or QUAIDS<sup>2</sup>, for example) and minimizing an appropriate entropy family criterion.

This article contains three contributions. The first one is theoretical. It aims to improve the estimation-calibration of the parameters of the CDE demand systems by clearly proposing for the first time to our knowledge, the necessary econometric and numerical steps to link the elasticities of AIDS or QUAIDS with those of the CDE. The second consists in estimating-calibrating the parameters and elasticities of a CDE system on Moroccan data under an appropriate aggregation using the proposed approach, which is a significant added value in itself. The third contribution is clearly in terms of economic policy approach. Indeed, our contribution allows to compare the results in terms of poverty of a tax reform (VAT reform in Morocco) under two alternative expenditure systems (CD and CDE). This third contribution is of major interest to compare and choose the specification to be retained in different economic policy models at the level of the household demand module, in microsimulated models in particular.

This article is articulated as follows. In the section 2, we begin with a brief literature review of the theoretical framework for demand systems in applied models derived. In the subsection 2.1, we briefly present the classic demand system generated from a Cobb-Douglas (CD) utility function. We then present in more detail the Constant Difference Elasticity (CDE) demand system and its properties (Subsection 2.2). As our approach is based on the estimation of the Quadratic Almost Ideal Demand System (QUAIDS) to calibrate the

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<sup>1</sup>Almost Ideal Demand System (AIDS)

<sup>2</sup>Quadratic Almost Ideal Demand System (QUAIDS)

elasticities and parameters of the CDE system, we first present this demand system in the subsection 2.2.1. We formally deduce the demand equations and then the elasticities of the latter. We also recall the major microeconomic conditions that these elasticities must respect.

In section 3, our hybrid estimation - calibration method is developed and we present the data used. We detail the methodology to proceed from the estimation of a QUAIDS demand system to the calibration of the parameters and elasticities of a CDE demand system. More specifically, we present the canonical form of the QUAIDS model and its extensions, in particular the one that integrates demographic variables. We then develop the calibration approach based on the minimization of a cross-entropy criterion. Finally, we present the algorithm we developed to calibrate the elasticities and the parameters of the CDE demand system using GAMS software. The subsection 3.2 presents the 2019 wave of the *Enquête Panel des Ménages* (EPM) of the *Observatoire national du développement humain* (ONDH) of Morocco which is the main data used for the estimations. We then dedicate the subsection 3.3 to the implementation of the proposed method by breaking it down into six steps that allow to move from theory to practice.

In the next section (Section 4), we present the twelve (12) scenarios considered for simulations according to the variation of household expenditures and taxation systems (VAT). Comparative point estimation and stochastic-dominant analyses are conducted to highlight the impact of the reforms depending on the demand systems chosen. In the last section (Section 5), we conclude and make recommendations to the researchers who use this type of model and to the policy makers who use the results produced by these researchers.

## 2 Theoretical framework : from the microeconomics to the applied models

In applied models, it is necessary to assign a value for the parameters of some behavioural functions. Some parameters are sometimes estimated on an *ad hoc* basis or can be taken from applications in other countries requiring situations to be comparable (Abdelkhalek and Dufour, 1998).

In general, parameter calibration and estimation are crucial in applied models as they drive the results. On the consumption side, the household expenditure module is critical. Its calibration is therefore fundamental for simulating the impact of fiscal and public policies. Since one wants to capture the transmission mechanisms of the effects of these policies on households, it is interesting and even crucial to introduce heterogeneity between them. This heterogeneity can be introduced based on household budget shares and assuming a rich and flexible demand system that will allow for enhanced microsimulation analysis (Savard, 2004). In practice, some of these constraints are not respected in certain demand systems. It is precisely at this level that the trade-off is made in favour of simple but probably unrealistic models. In the next section, we present the two demand systems on which we base our comparison, derived from a CD utility function and the CDE demand system.

### 2.1 Demand system based on the Cobb-Douglas utility function

The Cobb Douglas (CD) utility function is a special case of the Constant Elasticity of Substitution (CES) utility function (when elasticity of substitution equal to one) and of the Linear Expenditure System (LES) utility function (when the minima are zero) (Shoven and Whalley, 1992).

Formally, a CD utility function is as follows <sup>3</sup>:

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<sup>3</sup>The properties are presented for example in Shoven and Whalley (1992)

$$u(q) = \prod_{i=1}^k q_i^{\alpha_i} \quad (1)$$

where  $q_i$  is the consumed quantity of good  $i$  and  $\alpha_i > 0$ . Without loss of generality, we often impose  $\sum_{i=1}^k \alpha_i = 1$ .

The Marshallian demand functions are as follows<sup>4</sup>:

$$q_i = \frac{\alpha_i}{\sum_{j=1}^k \alpha_j} \cdot \frac{m}{p_i} = \frac{\alpha_i m}{p_i} \quad \forall i = 1, \dots, k \quad (2)$$

where  $m$  is the total household expenditure and  $p_i$ , the price of good  $i$ . From this demand system, several properties are derived. Those we are particularly interested in are the following:

- $\alpha_i = w_i$ , the budget share of the good  $i$  in the total expenditure and is constant for all  $m$  and  $p_i$  ;
- The elasticity of total expenditure, noted  $\eta_i$  is unitary for any  $i$  product;
- The uncompensated own price elasticity, denoted  $\sigma_{ii}^u$  is equal to  $-1$  for every product  $i$ ;
- The compensated own price elasticity, denoted  $\sigma_{ii}^c$  is equal to  $-(1 - \alpha_i)$  for every product  $i$ ;
- The demand for each good  $i$  depends only on the price of the good  $i$  and not on the price of other goods. This implies that the cross-price elasticity, denoted  $\sigma_{ij}^u$  is null  $\forall i \neq j$ .

This demand system also respects all the basic microeconomic properties that we summarize in subsection 2.2.2.

The problem with this relatively simple demand system is that it seems to be limited when assessing the impact on the demand for goods and services as a result of changes in economic

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<sup>4</sup>To simplify the notation, we have not indexed household variables and parameters with  $h$ .

policy (Yu et al., 2004). For example, with this demand system derived from CD utility function, as mentioned previously, income and price elasticities are constant and then do not depend on budget shares. Because of these characteristics, the demand system is considered unrealistic and inflexible (see for example, Sadoulet and De Janvry (1995), Rimmer and Powell (1996) and Corong et al. (2017)).

To overcome at least this limit, several other more flexible and apparently more realistic demand systems have been introduced in this literature. These are the Almost Ideal Demand System (AIDS) family proposed by Deaton and Muellbauer (1980a) and the CDE system that we discuss in the next section.

Deaton and Muellbauer (1980a) developed the AIDS demand system, based on an approximation of the first-order conditions of a demand system. Even this one has interesting properties both from a theoretical point of view and in its estimation, it does not capture the non-linearities of the Engel curves, even though they are empirically observed. By introducing a quadratic term in the AIDS specification, Banks et al. (1997) propose a more flexible demand system that respects the Engel condition and other constraints of the AIDS system.

In this literature on demand systems, already in 1975, Hanoch introduced the functional form called Constant Difference of Elasticity (CDE). It lies between the CES form and more flexible specification. The CDE demand system allows richer representation of the effects of income (Dimaranan et al., 2006). Because of its properties and the number of parameters (only three for each good), the functional form of the CDE offers many potential applications (See Surry (1993), Yu et al. (2004) and van der Mensbrugghe (2020)).

However, it seems that few researchers have directly estimated the CDE parameters because of the complexity to do so. The few examples that exist in terms of estimation are exclusively on the production side or based on cost functions (Surry, 1993). Yu et al. (2004), Chen (2017) and van der Mensbrugghe (2020) propose a hybrid estimation-calibration approach that involves several steps. This method consists of approaching the elasticities and

the parameters of the CDE demand system using estimates from another demand system (e.g. AIDS, AIDADS, QUAIDS, ...) and minimizing a criterion, respecting only some microeconomics conditions of a demand system. This is especially relevant since it has been shown that the CDE system can violate some of Engel's properties in its empirical version (Yu et al., 2004). Considering this, we improve these approaches taking into consideration a larger number of microeconomic constraints than what is done by Chen (2017), Yu et al. (2004) and van der Mensbrugghe (2020) as we will see in the next section.

## 2.2 A Constant Difference Elasticity demand system

In this article, we compare the impact of several tax reforms on poverty measures using two demand systems: the first one generated from a simple CD utility function and the second one from the more complex, CDE demand system. The calibration of the elasticities and parameters of this system necessarily requires the estimation of the parameters and elasticities of the AIDS or QUAIDS demand system.<sup>5</sup> Consequently, we describe this demand system with some detail before presenting the CDE demand system.

### 2.2.1 The Quadratic Almost Ideal Demand System - QUAIDS

Using the notation of Banks et al. (1997), the functional form of the QUAIDS for a household is as follows:

$$w_i = \alpha_i + \sum_{j=1}^k \gamma_{ij} \ln p_j + \beta_i \ln \left\{ \frac{m}{a(\mathbf{p})} \right\} + \frac{\lambda_i}{b(\mathbf{p})} \left[ \ln \left\{ \frac{m}{a(\mathbf{p})} \right\} \right]^2, \quad i = 1, \dots, k \quad (3)$$

where  $w_i$  is the budget share of the good  $i$  defined as :

$$w_i = \frac{p_i q_i}{m}, \quad (4)$$

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<sup>5</sup>At this step, other flexible demand systems could be used for the estimation of the parameters and elasticities.

$m$  the total household expenditure,  $p_i$  the price of good  $i$  and  $q_i$ , the Hicksian demand for good  $i$  by the household,  $k$  is the number of considered goods.

By definition:

$$\ln a(\mathbf{p}) = \alpha_0 + \sum_{i=1}^k \alpha_i \ln p_i + \frac{1}{2} \sum_{i=1}^k \sum_{j=1}^k \gamma_{ij} \ln p_i \ln p_j \quad (5)$$

is a translog price index and

$$b(\mathbf{p}) = \prod_{i=1}^k p_i^{\beta_i} \quad (6)$$

a price index. This system must respect a set of microeconomic conditions referring to additivity, homogeneity and symmetry that are expressed in constraints on parameters for each household ( $\alpha_i$ ,  $\beta_i$ ,  $\gamma_{ij}$  and  $\lambda_i$ ):<sup>6</sup>

$$\sum_{i=1}^k \alpha_i = 1, \quad \sum_{i=1}^k \beta_i = 0, \quad \sum_{j=1}^k \gamma_{ij} = 0, \quad \sum_{i=1}^k \lambda_i = 0, \quad \text{and} \quad \gamma_{ij} = \gamma_{ji}. \quad (7)$$

Note  $\lambda_i$ ,  $\alpha_i$ ,  $\beta_i$  and  $\gamma_{ij}$  are the parameters of the model QUAIDS that will be estimated.<sup>7</sup>

In addition, control variables relating to household characteristics are introduced to the above equations to avoid specification errors. In particular, these are demographic variables that characterize either the household or specific members of the household, such as the head of the household. The variables encountered in this context are: household size, household size squared, number of children, number of adults, gender, age of the head of household, socio-professional category, level of education. Furthermore, variables regarding the region or place of living of the household can also be introduced as explanatory variables if they were not used as stratification variables in the estimation itself.

Based on Ray (1983), Poi (2012) notes that modeling heterogeneity in this way does not mean that the heterogeneity enters the model in a linear fashion. As we saw previously, heterogeneity appears not only linearly in the intercepts, but also non-linearly in all expenditures

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<sup>6</sup>When  $\lambda_i = 0$ ,  $\forall i = 1, \dots, k$ , in Equation 3 (this can be tested after parameters estimation) the quadratic term disappears and the QUAIDS becomes AIDS system, respecting the conditions of symmetry, separability and Cournot and Engel.

<sup>7</sup>See Diagram 1.

through the first price aggregator. Following Poi (2012), the expression (3) becomes:<sup>8</sup>

$$w_i = \alpha_i + \sum_{j=1}^k \gamma_{ij} \ln p_j + (\beta_i + \nu'_i \mathbf{z}) \ln \left\{ \frac{m}{\bar{m}_0(\mathbf{z})a(\mathbf{p})} \right\} + \frac{\lambda_i}{b(\mathbf{p})c(\mathbf{p}, \mathbf{z})} \left[ \ln \left\{ \frac{m}{\bar{m}_0(\mathbf{z})a(\mathbf{p})} \right\} \right]^2 \quad (8)$$

with  $\mathbf{z}$  the vector of household socio-demographic characteristics, and

$$\bar{m}_0(\mathbf{z}) = 1 + \rho' \mathbf{z} \quad (9)$$

$$c(\mathbf{p}, \mathbf{z}) = \prod_{j=1}^k p_j^{\nu'_j \mathbf{z}} \quad (10)$$

where  $\nu_j$  represents the  $j$  th column of  $s \times k$  parameter matrix  $\nu$  with

$$\sum_{j=1}^k \nu_{rj} = 0 \text{ for } r = 1, \dots, s \quad (11)$$

the additivity condition.

Another common problem in estimating demand systems is that of endogeneity in both total expenditures and prices. As noted by Deaton and Muellbauer (1980a), Attfield (1985) and many other authors after, the simultaneous nature of the demand system is one of the reasons for this endogeneity. Indeed, since the total expenditure is defined as the sum of the expenses for each product, and since these expenditures are assumed to be endogenous, it can be assumed that the total expenditure is jointly endogenous and thus correlated with the error term. Therefore, if expenditures or prices are correlated with errors, the resulting estimators are biased and inconsistent.

For the price variables, when regionally and goods-specific prices do not exist, it is common in empirical studies to determine prices by relating the value spent on a good to the consumed quantity if available. However, differences in prices observed between households may reflect

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<sup>8</sup>Here again, if we set  $H_0 : \lambda_i = 0, \quad \forall i = 1, \dots, k$ , we are left with the AIDS model with demographics used by Ray (1983). This null hypothesis can be tested.

something other than the market price and may, for example, differ because of a difference in quality (Deaton, 1988).

As usual, this problem of endogeneity can be addressed by using instrumental variables. However, if the demand system is large with a large number of goods, we will need at least as many instruments as endogenous variables. This is one of the limitations of this approach in practice. This is, moreover, the main reason why many authors do not take into account the problem of endogeneity, particularly concerning prices in the demand system estimation context.

In this article, as explained in the data section 3.2, we sought to have prices differentiated by region and location to attempt to avoid endogeneity by determining prices directly from the information in the survey. Furthermore, we tested for the exogeneity of the total expenditure but not for the exogeneity of the prices which were constructed as explained in section 3.2.

### 2.2.2 The Constant Difference Elasticity - CDE

In this literature on demand systems, Hanoch (1975) introduced the functional form called Constant Difference Elasticity (CDE). Following her and Chen (2017), let  $M$  be an expenditure function with a price vector  $p$  associated to a Hicksian demand system noted  $q$ . Formally,  $m_0 = M(p_0, u) \equiv \{\min p_0 q_0 : f(q_0) \geq u\}$  where the 0 index refers to the base situation. By normalization, we postulate  $M\left(\frac{p}{m_0}, u\right) = \sum_i \beta_i u^{e_i(1-\alpha_i)} \left(\frac{p_i}{m_0}\right)^{1-\alpha_i} \equiv 1$ .

Using Roy's identity and the implicit function theorem, they deduce the associated Hicksian demand functions for a CDE demand system:

$$q_i = \frac{\beta_i u^{e_i(1-\alpha_i)} (1 - \alpha_i) \left(\frac{p_i}{m_0}\right)^{-\alpha_i}}{\sum_j \beta_j u^{e_j(1-\alpha_j)} (1 - \alpha_j) \left(\frac{p_j}{m_0}\right)^{1-\alpha_j}} \quad (12)$$

with  $q_i$  the Hicksian demand for good  $i$  by the household,  $p_i$  the price,  $\alpha_i$  a substitution parameter of the household expenditure function,  $\beta_i$  a scale parameter and  $e_i$  an expansion

parameter with  $\beta_i > 0$ ,  $e_i > 0$  and  $0 < \alpha_i < 1$ .<sup>9</sup>

Let  $\sigma_{ij}$  be Allen-Uzawa elasticity of substitution (AUES) obtained from the CDE demand system<sup>10</sup>:

$$\sigma_{ij} = \alpha_i + \alpha_j - \sum_i w_i \alpha_i - \frac{\delta_{ij} \alpha_i}{w_i} \quad (13)$$

with  $w_i$  the share of household expenditure in good  $i$ ,  $\delta_{i,j} = 1$  when  $i = j$  and 0 else. Thus, the own-AUES for the good  $i$  is equal to:

$$\sigma_{ii} = 2\alpha_i - \sum_i w_i \alpha_i - \frac{\alpha_i}{w_i} \text{ when } i = j \quad (14)$$

and the cross-AUES:

$$\sigma_{ij} = \alpha_i + \alpha_j - \sum_i w_i \alpha_i \text{ when } i \neq j. \quad (15)$$

Following Hanoch (1975), the uncompensated demand elasticities for the CDE demand system are<sup>11</sup>:

$$\sigma_{ij}^u = w_j \left[ \alpha_j - \frac{e_i(1 - \alpha_i)}{\sum_i w_i e_i} - \frac{\sum_i w_i e_i \alpha_i}{\sum_i w_i e_i} \right] - \delta_{ij} \alpha_j \quad (16)$$

where  $\delta_{i,j} = 1$  when  $i = j$  and 0 else such that:

$$\sigma_{ii}^u = w_i \left[ \alpha_i - \frac{e_i(1 - \alpha_i)}{\sum_i w_i e_i} - \frac{\sum_i w_i e_i \alpha_i}{\sum_i w_i e_i} \right] - \alpha_i \text{ when } i = j \quad (17)$$

and

$$\sigma_{ij}^u = w_j \left[ \alpha_j - \frac{e_i(1 - \alpha_i)}{\sum_i w_i e_i} - \frac{\sum_i w_i e_i \alpha_i}{\sum_i w_i e_i} \right] \text{ when } i \neq j. \quad (18)$$

Furthermore, following Hanoch (1975)<sup>12</sup>, the household expenditure elasticities are equal to:

$$\eta_i = \left( \sum_i w_i e_i \right)^{-1} \left[ e_i (1 - \alpha_i) + \sum_i w_i e_i \alpha_i \right] + \left( \alpha_i - \sum_i w_i \alpha_i \right). \quad (19)$$

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<sup>9</sup>For details and notations see Hanoch (1975) and Chen (2017).

<sup>10</sup>See equation 3 page 409 in Hanoch (1975).

<sup>11</sup>See page 414 in Hanoch (1975).

<sup>12</sup>See equations 3.21 and 3.22 page 413 in Hanoch (1975).

Note that  $w_i$ ,  $\eta_i$ ,  $\sigma_{ij}^u$  and  $\sigma_{ij}^c$  respectively represent the budget shares and the expenditure elasticity of products  $i$ , the uncompensated and compensated own-prices and cross-prices elasticities between products  $i$  and  $j$ . The elasticities of the CDE demand system must imperatively satisfy a set of microeconomic conditions<sup>13</sup>:

- The Engel aggregation condition:

$$\sum_i w_i \eta_i = 1 \quad (20)$$

- The Cournot condition:

$$\sum_i w_i \sigma_{ij}^u = -w_j \quad (21)$$

- The Slutsky's equation:

$$\sigma_{ij}^c = \sigma_{ij}^u + \eta_i w_j \quad (22)$$

- The symmetry condition:

$$\sigma_{ij}^u = \frac{w_j}{w_i} \sigma_{ji}^u + w_j (\eta_j - \eta_i) \quad \forall i \neq j \quad (23)$$

To these microeconomic conditions, the condition  $\sum_i w_i = 1$  is obviously imposed.

### 3 Method, data and implementation

To use these two specifications CD and CDE, in applied models, the parameters of the demand system must be estimated or calibrated. In the context of microsimulated model, the CD demand system does not require any econometric estimates. Its parameters are, in fact, budget shares and can be directly calibrated using a household survey. On the other hand, in

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<sup>13</sup>To establish the complete conditions of the system (24), we relied on Bieri and Janvry (1972), Hanoch (1975), Aasness (1990), Deaton and Muellbauer (1980b), Tarr (1990), Sadoulet and De Janvry (1995) and Chen (2017).

our approach, calibrating the elasticities and parameters of a CDE demand system requires going through the parameter and elasticity estimation of the QUAIDS demand system as we proposed in the previous section. In the following section, we detail the implementation of that approach.

### 3.1 From QUAIDS estimation to CDE calibration

To implement our approach, we proceed as follows. In a first step, we estimate the parameters  $\lambda_i$ ,  $\alpha_i$ ,  $\beta_i$  and  $\gamma_{ij}$  of a QUAIDS demand system, from the data of a household survey with a numerically supported aggregation of products. The resulting estimates are used to compute the different elasticities, always for the QUAIDS system.<sup>14</sup>

At the end of this step, we retain the QUAIDS estimates of the elasticities  $\eta_i$ ,  $\sigma_{ij}^c$  and  $\sigma_{ij}^u$  that become the initial values for the calibration of the elasticities and the parameters of the CDE demand system. In all these steps, it is necessary to respect both the microeconomic and econometric properties of the two demand systems (QUAIDS and CDE).

To do that, Chen (2017) proposes two calibration methods. The first one is a three-step sequential type. It consists of calibrating in a sequence  $\alpha_i$ ,  $e_i$  and  $\beta_i$ , from the elasticities estimated in the previous step for a QUAIDS type demand system.<sup>15</sup> The second method is a kind of entropy method for which the calibrated values of the substitution ( $\alpha_i$ ) and expansion ( $e_i$ ) parameters are obtained simultaneously by optimizing an objective function (or criterion) with the estimation obtained always in the previous step, for these elasticities as initial values.

In this article, we improve and systematize the method proposed by Chen (2017). For this purpose, we draw on the approach proposed by Golan et al. (1994) who have proposed an optimization method to find unknown values of a matrix from economic data and respecting many constraints or conditions.

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<sup>14</sup>There are at least two STATA commands to estimate the parameters of a QUAIDS system. The first one (*quaid*s) was developed by Poi (2002) and updated in Poi (2012) and Poi (2013). The second command (*aidsills*) was developed by Lecocq and Robin (2015).

<sup>15</sup>The GAMS program of the sequential approach is in appendix of the article in Chen (2017).

Indeed, we apply the cross-entropy method based on Golan et al. (1994) by minimizing an accurate criterion to deduce the calibrated values of the elasticities of a CDE-type function. This criterion is based on the sum of the weighted "differences" between the logarithms of the unknown values of the elasticities (the CDE ones) and the values estimated from the QUAIDS demand system.

The associated minimization program respects the definitions of the elasticities of the CDE demand system (14, 15, 17, 18 and 19), the previously presented microeconomic constraints (20, 21, 22 and 23) and those on the signs of the elasticities to be obtained. The solution of this minimization program are the calibrated values of the parameters  $e_i$  and  $\alpha_i$  and the elasticities of the CDE demand system  $(\eta_i, \sigma_{ij}^u, \sigma_{ij}^c, \sigma_{ij})$ . The complete minimization program is presented in detail in System 24. This program is naturally solved for each household in the survey. In our opinion, this is a major contribution in the literature on the calibration of the parameters of a CDE demand system and in its use to approach the impact of public policies in a microsimulation model.

$$\left\{ \begin{array}{l}
\text{Min}_{\alpha_i, e_i} \left[ \sum_i \eta_i \ln \left( \frac{\eta_i}{\eta_i^t} \right) \right] + \left[ \sum_i \sum_j \sigma_{ij}^c \ln \left( \frac{\sigma_{ij}^c}{\sigma_{ij}^{ct}} \right) \right] + \left[ \sum_i \sum_j \sigma_{ij}^u \ln \left( \frac{\sigma_{ij}^u}{\sigma_{ij}^{ut}} \right) \right] + \left[ \sum_i \sum_j \sigma_{ij} \ln \left( \frac{\sigma_{ij}}{\sigma_{ij}^t} \right) \right] \\
\text{under constraints } \forall i, j = 1, \dots, k: \\
\sigma_{ij} = \alpha_i + \alpha_j - \sum_i w_i \alpha_i - \frac{\delta_{ij} \alpha_i}{w_i} \\
\sigma_{ij}^u = w_j \left[ \alpha_j - \frac{e_i(1-\alpha_i)}{\sum_i w_i e_i} - \frac{\sum_i w_i e_i \alpha_i}{\sum_i w_i e_i} \right] - \delta_{ij} \alpha_j \quad \text{where } \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{else} \end{cases} \\
\eta_i = (\sum_i w_i e_i)^{-1} [e_i(1 - \alpha_i) + \sum_i w_i e_i \alpha_i] + (\alpha_i - \sum_i w_i \alpha_i) \\
\sigma_{ij}^c = \sigma_{ij}^u + \eta_i w_j \\
\sum_i w_i \eta_i = 1 \\
\sum_i w_i \sigma_{ij}^u = -w_j \\
\sigma_{ij}^u = \frac{w_j}{w_i} \sigma_{ji}^u + w_j (\eta_j - \eta_i) \quad \forall i \neq j \\
(\eta_i - 1) \cdot (\eta_i^t - 1) > 0 \\
\eta_i \cdot \eta_i^t > 0 \\
\sigma_{ij}^c \cdot \sigma_{ij}^{ct} > 0
\end{array} \right. \tag{24}$$

where  $\eta_i^t$ ,  $\sigma_{ij}^{ut}$  and  $\sigma_{ij}^{ct}$  are respectively the expenditure elasticity of products  $i$ , the uncompensated and compensated own-prices and cross-prices elasticities between products  $i$  and  $j$ , estimated on the QUAIDS. It is important to note that  $\sigma_{ij}^t$  does not come directly from the QUAIDS system estimation. It is deduced in one step of the resolution for System 24 as explained in Section 3.3.

To complete the calibration of the CDE demand system, the  $\beta_i$  parameter still needs to be calibrated. This one is obtained by calculation from the demand function of the good  $i$  in the sequential approach. Following Chen (2017) and van der Mensbrugghe (2020), we use the calibrated value of  $\alpha_i$  and normalizing  $u = 1$ ,  $p_{0i} = 1$  and  $q_{0i} = w_i$  from (12) :

$$\beta_i = \frac{w_i}{1 - \alpha_i} / \sum_i \frac{w_i}{1 - \alpha_i}. \tag{25}$$

In this article, the resolution of the system has been carried out using GAMS software after importing the elasticities estimated from a QUAIDS function in previous steps from STATA. There outputs are estimated values of  $\alpha_i$ ,  $\beta_i$ ,  $e_i$ ,  $\eta_i$ ,  $\sigma_{ij}$ ,  $\sigma_{ij}^u$  and  $\sigma_{ij}^c$  for a CDE demand system for each household in the survey and products used in the estimation. Figure 1 summarizes the structure of these steps.

## 3.2 Data

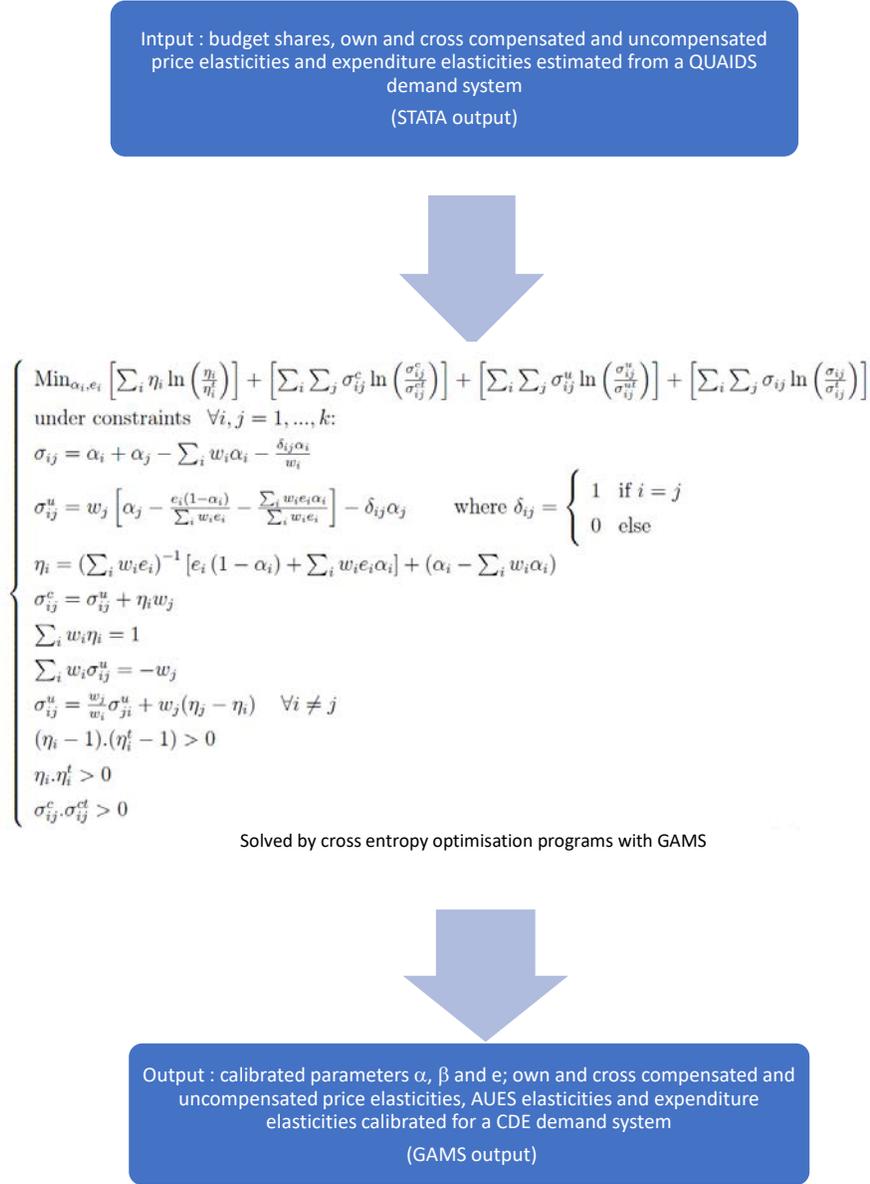
To estimate and calibrate the two considered demand systems, we need data from a household survey on expenditures by product. We also need a vector of price indices for the products considered, particularly to estimate the parameters of the QUAIDS demand system. We present the two sources of data we used in this article.

### 3.2.1 Household survey

We use data from the 2019 wave of the *Enquête Panel des Ménages* (EPM) of the *Observatoire national du développement humain* (ONDH) of Morocco. This survey was implemented for the first time in Morocco in 2012. In 2017 and 2019, the ONDH sought to ensure regional representativeness (the 12 regions of the country in addition to national representativeness and by place of residence) by increasing the sample size. The uncleaned file we ran contains 16,879 households. After clearing and discarding households with outliers or missing observations, 15,904 households were considered in our application. Table 1 presents the distribution of the ONDH wave 2019 EPM sample at the household level, as inferred from the files used.

The adopted aggregation is based on the nomenclature used by the Direction de la Comptabilité Nationale (DCN) of the Haut-Commissariat au Plan (HCP). We mapped the 1,283 goods and services included in the survey (consumer expenditures) to four aggregated products for numerical and practical considerations. Indeed, firstly, the available STATA commands (*quaid*s and *aidsills*) used for estimating the parameters allow only a limited number

Figure 1: Structure of the three stages



Source: Authors

<b>Households</b>	<b>Sample Freq.</b>	<b>%</b>	<b>Pop. Freq.</b>	<b>Pop. %</b>
Urban	9 170	57.66%	4 841 027	66.29%
Rural	6 734	42.34%	2 461 643	33.71%
Morocco	15 904	100.00%	7 302 670	100.00%

Source: Authors

Table 1: Household frequencies (sample and population) of the Survey EPM 2019 by place of residence

of goods and services. Especially with a larger number of categories, the budget shares of the products for different households are too low, which generates numerical problems at different levels at all stages of the method. The selected aggregation into four product categories avoided this problem without affecting the object of our analysis.<sup>16</sup>

Secondly, Morocco has five levels of value added tax in 2019: 0%, 7%, 10%, 14% and 20%. We have considered that the number of products taxed at 14% is low. For this reason, we have chosen to aggregate these products with those taxed at 10%. Thus, all the analysis conducted in this article is made on the basis of four value-added tax rates: 0%, 7%, 10% and 20%. After mapping, each of the 1,283 products consumed or not consumed by each household is placed in only one of these four categories. On the base of this mapping, we then computed for each household the total expenditure for each of the four product categories and computed the four associated budget shares.

### 3.2.2 Construction of the price vector

The basic price indices are derived from the HCP's briefing note on the Consumer Price Index (CPI) for the year 2019.<sup>17</sup> Four price indices are computed as a weighted arithmetic mean for each household, the weights are the budget shares computed from the 1,283 products.<sup>18</sup> As mentioned earlier, to estimate the QUAIDS model on cross-section data, it is necessary

<sup>16</sup>It should be noted that this is not a limitation of the proposed method, which remains valid regardless of the number of categories of products retained. The problem encountered is then purely numerical.

<sup>17</sup>See [https://www.hcp.ma/L-indice-des-prix-a-la-consommation-IPC-de-l-annee-2019\\_a2451.html](https://www.hcp.ma/L-indice-des-prix-a-la-consommation-IPC-de-l-annee-2019_a2451.html).

<sup>18</sup>We used a preliminary aggregation into 39 products selected from an Input-Output table of the HCP.

to have variance in prices across households. Ideally, we would like to have prices of goods by household or at least by region/area/province *via* other sources than the survey. Given the available information for the Moroccan case, prices were standardized by place of residence (urban - rural) to introduce some heterogeneity between households. In fact, since the group of products with zero VAT rate contains mainly agricultural products and since these ones are generally more expensive in urban area, we have considered that the index of these products in rural area is five points of percentage lower than its level in urban area. We consider the opposite for the other three price indices. In doing so, the problem of price endogeneity and the use of the instrumental variable method is avoided.

Obviously, by doing so, we expect to have collinearity between the different price indices of the group of products. After verification, the correlation matrix shows that the linear correlation coefficients are all in absolute value, higher than 74%. This could naturally cause a problem in terms of inference but not for the parameters and elasticities estimation that concern us in this article.

### **3.3 From theory to practice: steps for implementation**

The complexity of the approach developed and the applied contribution of this article, concerns the numerical implementation of the link between the estimation of the QUAIDS demand system and the calibration of the one resulting from the CDE specification. Indeed, from the modeler's point of view, the microeconomic constraints that must be satisfied are not necessarily part of econometricians' concerns in the estimation process. In order to ensure that we have microeconomic consistency in the estimation and calibration of expenditures, own-prices and cross-prices elasticities and Allen-Uzawa elasticities of substitution, we have broken down the work into six (6) main steps. One of them was done in STATA (*quaid*s estimation) while the other five were done with the GAMS software (calibration steps, calculation of the  $\beta$  parameter). Flowchart 3 in Annex A summarizes each of the steps we are going to detail in this section.

1. In the first step, after we have prepared the database according to the approach explained in the section 3.2, we estimate the QUAIDS demand system using the STATA command of Poi (2012). The estimate is performed decile by decile in each residence area (20 estimation procedures). Furthermore, we estimated the model retaining three demographic variables: the age and the sex of the head of household and the size of the household.

We also tested whether the form of the expenditure system was AIDS or QUAIDS ( $H_0 : \lambda = 0$ ) for each decile. We conclude that the demand system is a QUAIDS type for all deciles in the both areas. It is therefore the estimates of the elasticities of a QUAIDS demand system obtained at this stage that will be used as a starting point for initialization for the subsequent stages. Thus, at the end of this first step, we obtain the estimates of the elasticities of a QUAIDS-type demand system for four aggregated products and for each household, namely  $\sigma_{ij}^{ut1}$ ,  $\sigma_{ij}^{ct1}$  and  $\eta_i^{t1}$ .

2. This step consists of importing the results obtained in STATA in step 1 into GAMS, namely matrices of two (household, product  $i$  for  $\eta_i^t$ ) and three dimensions (household, product  $i$ , product  $j$  for  $\sigma_{ij}^{ut}$ ,  $\sigma_{ij}^{ct}$ ). Before proceeding with the subsequent calibration steps, we verified the microeconomic consistency, notably the respect of four conditions (20, 21, 22 and 23) as well as the sum of the budget shares evaluated for each household equal to one.

3. In this third step, we adjust the variables  $w_i$ ,  $\eta_i^{t1}$ ,  $\sigma_{ij}^{ut1}$  and  $\sigma_{ij}^{ct1}$  for each product  $i$  and each household obtained in STATA, that do not eventually respect the microeconomic constraints by performing a first calibration step. The objective is to obtain adjusted shares using an entropy criterion so that the microeconomic consistency is respected and re-evaluate all the other variables. The initial values used in this step are those obtained at the end of the previous one. Thus, at the end of this step we have adjusted shares ( $w_i^2$ ) consistent and that we will use in all future steps without any modification.

The solution of all other the variables of interest are then re-evaluated and will serve as a starting point for the next step. We note them  $\eta_i^{t2}$ ,  $\sigma_{ij}^{ut2}$  and  $\sigma_{ij}^{ct2}$ .<sup>19</sup>

4. In this step, we start the calibration of the elasticities and parameters of the CDE demand system. Given the non-linearity and complexity of the system 24, numerical resolution turned to be difficult. The problem encountered is mainly related to the size (number of constraints to be respected and variables considered) rather than to the specification of the system itself. We have therefore chosen to follow Chen's (Chen, 2017) example by adopting a two-step sequential approach.

- From the equation 19, the expenditure elasticity ( $\eta$ ) depends only on the  $\alpha$  and  $e$  parameters.<sup>20</sup> In a first step, we calibrate  $\eta$  for each product and for each household using an entropy criterion relative only to  $\eta$  and respecting the constraints that only show  $\eta$  (i.e. 22, 23, Engel and sign conditions) and the budget shares obtained in step 3. At the end of this step, we obtain a consistent estimate of  $\eta$  and first estimates for the  $\alpha$  and  $e$  parameters of the CDE demand system. In this step, we use as an initial point the results obtained in step 3 i.e.  $w_i^2$ ,  $\eta_i^{t2}$ ,  $\sigma_{ij}^{ut2}$  and  $\sigma_{ij}^{ct2}$ . The results obtained are denoted  $\eta_i^{t3}$ ,  $\sigma_{ij}^{ut3}$ ,  $\sigma_{ij}^{ct3}$ ,  $\alpha_i^{t3}$  and  $e_i^{t3}$ . Furthermore, with the values obtained at this step, we compute for the first time an estimate of the Allen-Uzawa elasticities compatible with the CDE system (equation 13) noted  $\sigma_{ij}^{t3}$ .
- Then, we calibrate the compensated and uncompensated own and cross-prices elasticities and the Allen-Uzawa elasticities of substitution using a new entropy criterion and the constraints associated with prices elasticities (Cournot, 22, 23 and the sign condition) and taking the values ( $w_i^2$ ,  $\sigma_{ij}^{t3}$ ,  $\sigma_{ij}^{ut3}$ ,  $\sigma_{ij}^{ct3}$  and  $\eta_i^{t3}$ ) as an initial point.

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<sup>19</sup>Note that at the end of this step, we have solved the estimation-calibration link per household for a QUAIDS-type application system that can be used in a microsimulated model.

<sup>20</sup>We proceed to the inverse of Chen (2017) which starts by calibrating the elasticity of the expenditure which depends only on the single parameter  $\alpha$ .

At the end of this step, we obtain the new calibrated values of compensated and uncompensated own and cross-prices elasticities and the Allen-Uzawa elasticities of substitution  $\sigma_{ij}^{ct4}$ ,  $\sigma_{ij}^{ut4}$ ,  $\sigma_{ij}^{t4}$  and we re-evaluate the expenditure elasticities, denoted  $\eta_i^{t4}$  and the CDE parameters,  $\alpha_i^{t4}$  and  $e_i^{t4}$ . These values are used in the next step as a starting point for the last calibration step.

5. This last calibration step in our approach consists in applying an entropy criterion for all the parameters and elasticities of System 24 using as an initial point the values obtained in the previous steps. At the end of this step, we obtain the final parameters and elasticities for the CDE demand system, calibrated for four products and for each household. We denoted these by  $\eta_i^{tFinal}$ ,  $\sigma_{ij}^{tFinal}$ ,  $\sigma_{ij}^{utFinal}$ ,  $\sigma_{ij}^{ctFinal}$ ,  $\alpha_i^{Final}$  and  $e_i^{Final}$ .
6. Following Chen (2017) and van der Mensbrugge (2020), we calibrate  $\beta_i$  by a calculation directly from equation 25 and using the  $\alpha_i^{Final}$  obtained in the previous step. For each household,  $\beta_i$  has been calibrated by normalizing  $u = 1$ ,  $p_{i0} = 1$  and  $q_{i0} = w_i^2$ .<sup>21,22</sup>

At the end of this step, we get all the parameters and elasticities of a CDE-type demand system for each household. In the following section, we use these estimates to compare the results of simulations of changes in value added taxation in Morocco by considering the two specifications CD and CDE presented previously into a microsimulated partial equilibrium model.

## 4 Simulations of fiscal policies

To carry out the comparative analysis, we develop a partial equilibrium model that holds in parallel the two demand systems showing the variables for the reference situation and for all

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<sup>21</sup>Note that theoretically the calibrated  $\beta_i$  can be positive or negative depending on the product and the value of the  $\alpha_i$  (especially when  $\alpha_i > 1$ ). In our case, to respect the conditions on the parameters derived by Hanoch (1975), we have upper bounded  $\alpha_i$  to 0.99 so that  $\beta_i$  is positive.

<sup>22</sup>See for example Yu et al. (2004), footnote 10 of page 18.

the simulations considered. We begin by describing the model followed by the presentation of the simulated scenarios and finally the results obtained.

## 4.1 Partial equilibrium model

We consider a partial equilibrium model in which the prices of different goods and total expenditures are exogenous for each household. The prices of goods incorporate different VAT rates by product group  $i$ ,  $i = 1, \dots, k = 4$  as explained above. These rates are subject to exogenous variations that we will illustrate in four VAT reforms. In parallel, we consider three levels of expenditures per household, namely the *statu quo* compared to the reference situation, a uniform increase of 5% and a decrease of 5%. Overall, we simulate twelve (12) scenarios indexed ( $s$ ) and which we will detail in the following sub-section.

In the model of Table 2, the variables in *dot* represent relative changes and those indexed 0 are relative to the baseline situation. The only new variable in the model is the price index specific to each household,  $P$ .<sup>23</sup> It allows to adjust the expenditure for each scenario ( $m^{sa}$ ) to make it comparable with the reference one and a constant poverty line. It is these variables that are used in the distributional analysis.

Following Corong et al. (2017) and van der Mensbrugge (2020), the equation 26 is the relative change in the quantity demanded of each good  $i$  as a result of the relative changes in prices and total expenditure in a CDE demand system:

$$\dot{q}_i^s = \eta_i \dot{m}^s + \sum_j \sigma_{ij}^u \dot{p}_j^s. \quad (26)$$

It is interesting to note that in terms of relative variations, the CD demand system is a special case of the CDE demand system. The equation 26 simply becomes for the CD demand system  $\dot{q}_i^s = \dot{m}^s - \dot{p}_i^s$  since in this case,  $\eta_i = 1$ ,  $\forall i = 1, \dots, k$  et  $\sigma_{ij}^u = 0$  si  $i \neq j$  et

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<sup>23</sup>In this article, we have retained two formulations of this price index. The multiplicative form, presented in table 2, and an additive form  $P^{As} = \sum_{i=1}^k w_i^s \left( \frac{p_i^s}{p_i^0} \right)$  as proposed by Corong et al. (2017) and van der Mensbrugge (2020). The results obtained are approximately the same.

$$\sigma_{ii}^u = -1.$$

CD	CDE
$\dot{q}_i^s = \dot{m}^s - \dot{p}_i^s$	$\dot{q}_i^s = \eta_i \dot{m}^s + \sum_j \sigma_{ij}^u \dot{p}_j^s$
$w_i^s = \frac{p_i^s q_i^s}{m^s} = \alpha_i^0$	$w_i^s = \frac{p_i^s q_i^s}{m^s}$
$\dot{p}_i^s = \frac{dp_i^s}{p_i^s}$ $\dot{m}_i^s = \frac{dm_i^s}{m_i^s}$ $m^s = m^0(1 + \dot{m}^s)$ $p_i^s = p_i^0(1 + \dot{p}_i^s)$ $P^s = \prod_{i=1}^k \left( \frac{p_i^s}{p_i^0} \right)^{w_i^s}$ $m^{sa} = \frac{m^s}{P^s}$	
Sources: Authors	

Table 2: Partial equilibrium model

This model has been used to perform the twelve simulations presented below.

## 4.2 Presentation of scenarios

During the 1980s, Morocco undertook a major reform of its fiscal system. Its objective was to develop a modern, coherent, efficient, and more universal fiscal system. Since then, several measures have been introduced through successive finance laws. Value added tax (VAT) is a major indirect tax that came into force in Morocco in 1986. It generates the most tax revenue for the government, affecting both domestic and imported products. VAT is defined on the basis of consumption and its coverage is very broad. Nevertheless, some sectors remain outside this taxation, especially the agricultural sector. Some retail sales and services or products are exempt by law. In 2019, the VAT rates applied in Morocco are 0% for basic goods, 7%, 10%, 14%, and 20%. It is proportional and affects the entire population in the same way.

In May 2019, the Third National Assises of Fiscality were held. The two previous ones being held in 1999 and 2013. At each of these Assises, the reform of VAT was on the agenda, in particular to make it a coherent, sustainable and neutral tax. Among the recommendations, it is considered to introduce a reduced rate on consumer goods that could currently

be exempt, including agricultural products. Another plans to consider only two rates, an intermediate rate (10 - 12%) and a standard rate of 20%.

In this article, we simulate the impact on poverty of twelve scenarios by considering four changes in VAT rates that we combine with three changes in the level of household expenditure. In three cases, noted *a*, *b*, and *c*, we consider that the household expenditures remains constant with respect to the baseline situation. In the other two cases, we simulate an increase and a decrease of 5% in these expenditures respectively. For each of these three cases, we apply four changes in the VAT rate, noted as 1, 2, 3 and 4:

- Case 1: Application of three rates namely 7% for products currently exempt or taxed at 7% ; 14% for products taxed at 10% and 14% and 20% for other goods and services.
- Case 2: Application of a flat rate of 12% for all categories of goods and services.
- Case 3: Application of two rates, i.e. 10% for products currently exempt or taxed at 7%, 10% or 14% and 20% for other goods and services.
- Case 4: Application of a flat rate of 20% for all categories of goods and services.

### 4.3 Tools for distributional analysis

To analyze the impact of VAT rate reforms, combined or not with changes in per capita household expenditures on poverty, we adopte two complementary approaches, a point estimation and distributional analyse. Comparisons are made on resulsits obtained from model based on a CDE-type demand system and a CD-type system.

For the point estimation analysis in terms of monetary poverty, it is necessary to set a poverty line (noted  $z_p$ ). Since the distributions of *per capita* household expenditures have all been adjusted to be comparable with the reference distribution, the  $z_p$  line is the same regardless of the distributions.

<b>Scenarios</b>	<b>Simulations conducted</b>
Sim a-1	Fixed expenditures and 3 VAT rates (7%, 14% et 20%)
Sim a-2	Fixed expenditures and flat rate (12%)
Sim a-3	Fixed expenditures and 2 VAT rates (10% et 20%)
Sim a-4	Fixed expenditures and flat rate (20%)
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Sim b-1	Expenditures - 5% and 3 VAT rates (7%, 14% et 20%)
Sim b-2	Expenditures - 5% and flat rate (12%)
Sim b-3	Expenditures - 5% and 2 VAT rates (10% et 20%)
Sim b-4	Expenditures - 5% and flat rate (20%)
-----	
Sim c-1	Expenditures + 5% and 3 VAT rates (7%, 14% et 20%)
Sim c-2	Expenditures + 5% and flat rate (12%)
Sim c-3	Expenditures + 5% and 2 VAT rates (10% et 20%)
Sim c-4	Expenditures + 5% and flat rate (20%)

Sources: Authors

Table 3: Resume of scenarios

In this article, we choose a relative poverty lines. These poverty lines (distinct from one milieu to another) are computed on the distribution of per capita expenditures in households in the reference situation. More explicitly, we use a relative poverty line equal to half of the median of this distribution ( $z_p = 0.5$  median) according to place of residence (Ravallion et al., 1994).

Using these poverty lines, we computed for the reference situation and for the twelve (12) scenarios all the usual measures of monetary poverty (FGT) and inequality (Gini) at the national level and for the two aeras of residence (urban and rural).<sup>24</sup>

In addition, and in order to go beyond the setting of a poverty line in the comparisons, we have carry out a stochastic dominance analysis. Indeed, under the scenarios considered we anticipate shifts in the distributions curves. Also, first (or second) order stochastic dominance analysis is more informative in this context. Indeed, with these stochastic dominance

<sup>24</sup>Given the objective of this work, we present and analyze in this article only the variation for the poverty incidence at the national level. Results by area of residence and for depth and severity of poverty and inequality measures are not presented but are available from the authors.

tests, the comparisons are unambiguous for a wider range of poverty lines.

## 4.4 Results

Our analysis highlights several interesting results depending on the focus chosen, namely the economic policy dimension (tax reform and/or expenditure variation) and the methodological dimension that allows for a comparison of the results according to the demand system adopted. In order to conduct this comparison, we retain only what is related to the poverty incidence.

### 4.4.1 Implications of a Tax Reform in Morocco

Looking only at the results obtained with the CDE demand system, it appears that without a change in household expenditures, all of the proposed tax reforms will lead to an increase in the poverty rates. Its incidence rises from 7.06% under the baseline situation to 8.01%, 8.57%, 8.71% and 10.99% according to the selected scenario. This implies that these reforms could increase poverty. This can be explained by the fact that the products consumed mostly by poor people are subject to an increase in the VAT rate, which reduces their purchasing power. The simulations show that the increase in the poverty incidence is statistically significant for all the proposed tax reforms. Indeed, none of the four confidence intervals contains the value of the poverty incidence in the baseline situation (i.e. 7.06%). For the first scenario, which consists of introducing a 7% VAT on exempt products, the poverty incidence increases from 7.06% to 8.01% . In the Scenario 4, which puts the rates at 20% for all products including those currently exempt, the poverty incidence increases by 55.67% .

In terms of economic policy, it therefore appears that any tax reform designed to remove tax exemptions for basic consumption goods, especially consumed by the poorest households, must be done gradually so as not to have a major impact on these households. More generally, this result highlights the social implications of any tax reform.

When the tax reform scenarios are combined with a negative shock that reduces household

expenditures by 5% (pandemic, drought, etc.) (*Sim b - .*), the poverty incidence increases significantly under all four cases with, as before, stronger effects for cases 2 to 4. An opposite effect is observed for case 1 when household expenditures benefit from a positive shock that increases them by 5%. Indeed, for this reform, the poverty incidence is reduced compared to its level in the baseline situation (6.57%). However, even with this uniform positive growth (+5%) in household expenditures, under the other three cases, the poverty incidence increases, but this increase is only statistically significant when all products are taxed at 20%.

#### 4.4.2 CDE *versus* CD: is the calibration effort justified?

Table 4 has been constructed in order to compare the results of the simulations considered under each of the two systems: the first, the CDE system, supposed to be flexible but challenging in terms of estimation and calibration, and the second, the CD system, constraining but simpler to implement.

Several interesting results emerge from the analysis. For the 12 simulations considered, the poverty incidence is systematically higher under the CD system. This result can be explained by the crucial role of the values of the elasticities in this type of analysis. Indeed, with the CDE system the elasticities are different from one household to another, for each of the groups of products considered (microsimulation effect) as opposed to the CD system for which all the expenditure elasticities are constant and equal to the unit for all households and for all products.<sup>25</sup>

More generally, the results of the simulations in these models reflect the expenditure elasticities, own and cross-price elasticities estimated or calibrated for the selected demand systems. In our case, household behaviour would probably be better approached with a CDE system that captures more substitution between products and reduce the negative effects of the

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<sup>25</sup>Indeed, our results show that 17.66% of households have an expenditure elasticity greater than one for product group 1. For the other three product groups, these proportions are 12.48%, 31.78% and 86.14% respectively.

impacts of the simulated shocks.

		<b>Incidence</b>	<b>Lower bound</b>	<b>Upper bound</b>
	<b>Reference</b>	7.06	6.57	7.55
<b>No Growth (a)</b>				
<b>Case 1</b>	<b>CDE</b>	8.01	7.49	8.53
	<b>CD</b>	8.13	7.60	8.65
<b>Case 2</b>	<b>CDE</b>	8.57	8.04	9.11
	<b>CD</b>	8.86	8.31	9.40
<b>Case 3</b>	<b>CDE</b>	8.71	8.17	9.25
	<b>CD</b>	8.85	8.31	9.40
<b>Case 4</b>	<b>CDE</b>	10.99	10.39	11.60
	<b>CD</b>	11.80	11.18	12.42
<b>5% decrease in household expenditures (b)</b>				
<b>Case 1</b>	<b>CDE</b>	9.89	9.32	10.47
	<b>CD</b>	9.96	9.38	10.54
<b>Case 2</b>	<b>CDE</b>	10.72	10.13	11.32
	<b>CD</b>	10.99	10.39	11.60
<b>Case 3</b>	<b>CDE</b>	10.85	10.25	11.45
	<b>CD</b>	11.09	10.48	11.70
<b>Case 4</b>	<b>CDE</b>	13.35	12.69	14.02
	<b>CD</b>	14.19	13.52	14.87
<b>5% increase in household expenditures (c)</b>				
<b>Case 1</b>	<b>CDE</b>	6.57	6.10	7.05
	<b>CD</b>	6.61	6.14	7.09
<b>Case 2</b>	<b>CDE</b>	7.11	6.61	7.59
	<b>CD</b>	7.31	6.81	7.81
<b>Case 3</b>	<b>CDE</b>	7.20	6.70	7.69
	<b>CD</b>	7.36	6.86	7.86
<b>Case 4</b>	<b>CDE</b>	8.91	8.36	9.46
	<b>CD</b>	9.57	9.00	10.14

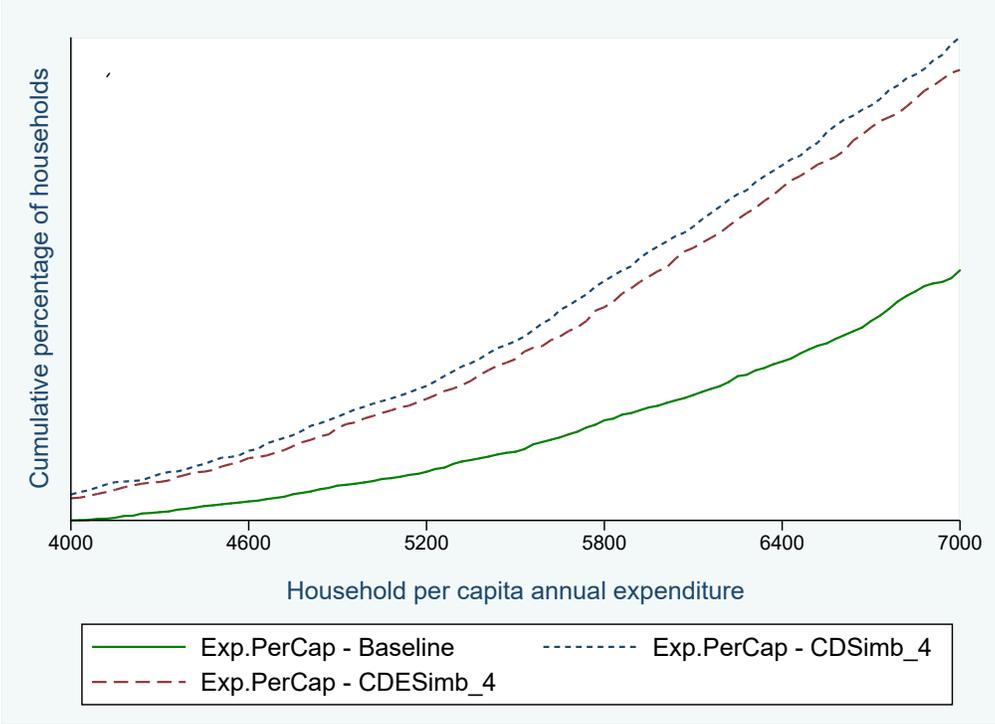
Sources: Authors

Table 4: Poverty incidence (%) and confidence intervals

The results show that the point estimates of the poverty incidence under the CDE system in three simulations conducted (Cases 1 to 3), are each time included in the confidence intervals constructed for the same measure under the CDE system and vice versa. Tests of equality between these two measures in each of the cases considered, lead to the non-rejection of the

underlying null hypotheses. This means that the differences between the two systems are not statistically significant. However, when the shock is greater (Case 4), the differences become statistically significant and the point estimates of the poverty incidence fall outside the confidence intervals of the alternative demand system. For example, in simulation Sim b-4, the estimated poverty incidence is equal to 13.35% under the CDE system and 14.19% under the CD system. The respective confidence intervals range from 12.69% to 14.02% under the CDE system and from 13.52% to 14.87% under the CD system. This underlines the fact that as soon as the simulated shocks are large, it is probably preferable to retain the CDE system. In the contrary case, the precision gain does not seem to justify the use of the CDE system in terms of point estimation regarding the cost in terms of estimation-calibration.

Figure 2: First-order stochastic dominance *Sim-b-4*



Source: Authors

All of these results could be conditioned by the fact that they are associated with an incidence measure based on a poverty line. We have therefore considered a stochastic dominance

robustness analysis. For this purpose, the adjusted household *per capita* expenditure densities and distribution curves obtained from the two demand systems and compared to the baseline situation have been constructed.

Figure 2 illustrates, for example, the distribution functions of the three *per capita* expenditures of the Sim b-4 simulation featuring a uniform 5% decrease in expenditures and a 20% VAT rates for all products. It appears that whatever the value of the poverty line below *MAD* 7,000 per person and per year, poverty increases under the two demand systems considered. Moreover, the gap between the curves increases as the value of the poverty line increases and the gap between the simulated curves (CDE *versus* CD) becomes wider. This shows that the results obtained under the CD system amplify the negative impact than those obtained with a CDE system.

## 5 Conclusion and discussion

The objective of this article is to compare two demand systems, namely the CD-type system and a more flexible but more complex to estimate, the CDE demand system, in a microsimulation model. For this purpose, we developed a hybrid estimation-calibration approach to calibrate the parameters and elasticities of the CDE demand system using estimates obtained from the QUAIDS demand system and numerically minimizing an entropy criterion.

Using Moroccan ONDH data (wave 2019 of the Panel Survey), we construct a partial equilibrium microsimulation model to simulate four cases of VAT reforms with or without changes in household expenditures on the usual monetary poverty measures. We then perform point comparisons, confidence intervals and stochastic dominance analysis on the poverty measures between the two systems (CDE, CD).

It appears that when the simulated shocks are moderate, the gain of using a CDE system instead of a CD system is low. Estimates of poverty measures are statistically equivalent. On the contrary, when the simulated shocks are stronger, the differences become significant and

increase as the poverty line increases when the analysis is performed in terms of stochastic dominance.

At the theoretical level, our results show that it is clearly different to use one or other demand systems given their characteristics (parameters and elasticities). In practice, the use of these models can lead to different results when evaluating public policies and their impacts on poverty measures.

However, our conclusions are obviously conditioned by the context considered. They are inferred from aggregation and specific simulations in the framework of a partial equilibrium model in which the interactions between different prices on the one hand and between prices and demands on the other are not taken into account (no general equilibrium effects). The differences obtained could indeed be greater or smaller in different contexts.

Based on the estimation-calibration approach of a CDE demand system developed in this article, we have been able to highlight by comparison, several relevant results from both theoretical and empirical points of view. These results were not trivial *a priori*.

The approach developed in this article is numerically time-consuming and demanding. If we want our conclusions to be taken into account in a public policy simulation exercise, it is important to make it systematic by automatizing its numerical implementation. In this way, modellers and decision-makers will be better supported in their efforts. It is precisely in this sense that we are pursuing our research.

## References

- Aasness, J. (1990). Properties of demand functions for linear consumption aggregates. Technical report, Discussion Papers.
- Abdelkhalek, T. and Dufour, J.-M. (1998). Statistical inference for computable general equilibrium models, with application to a model of the moroccan economy. *Review of Economics and Statistics*, 80(4):520–534.
- Attfield, C. L. (1985). Homogeneity and endogeneity in systems of demand equations. *Journal of Econometrics*, 27(2):197–209.
- Banks, J., Blundell, R., and Lewbel, A. (1997). Quadratic engel curves and consumer demand. *Review of Economics and Statistics*, 79(4):527–539.
- Bieri, J. and Janvry, A. d. (1972). Empirical analysis of demand under consumer budgeting. Technical report.
- Boeters, S., Böhringer, C., Büttner, T., and Kraus, M. (2010). Economic effects of vat reforms in germany. *Applied Economics*, 42(17):2165–2182.
- Chen, Y.-H. H. (2017). The calibration and performance of a non-homothetic cde demand system for cge models. *Journal of Global Economic Analysis*, 2(1):166–214.
- Corong, E., T., Hertel, R., McDougall, M., T., and van der Mensbrugghe, D. (2017). The standard gtap model, version 7. *Journal of Global Economic Analysis*, 2(1):1–119.
- De Quatrebarbes, C., Boccanfuso, D., and Savard, L. (2016). Beyond representative households: The macro–micro impact analysis of vat designs applied to niger. *Economic Modelling*, 57:76–92.
- Deaton, A. (1988). Quality, quantity, and spatial variation of price. *The American Economic Review*, pages 418–430.
- Deaton, A. and Muellbauer, J. (1980a). An almost ideal demand system. *The American economic review*, 70(3):312–326.
- Deaton, A. and Muellbauer, J. (1980b). *Economics and Consumer Behavior*. Cambridge University Press.
- Dimaranan, B., McDougall, R., and Hertel, T. (2006). Behavioral parameters. *Global trade, assistance, and production: the GTAP*, 6:1–17.
- Golan, A., Judge, G., and Robinson, S. (1994). Recovering information from incomplete or partial multisectoral economic data. *The Review of Economics and Statistics*, pages 541–549.
- Hanoch, G. (1975). Production and demand models with direct or indirect implicit additivity. *Econometrica: Journal of the Econometric Society*, pages 395–419.
- Lecocq, S. and Robin, J.-M. (2015). Estimating almost-ideal demand systems with endogenous regressors. *The Stata Journal*, 15(2):554–573.
- Ndemezo, E. and Baye, F. M. (2016). Evaluation of redistributive and welfare impacts of indirect taxes reform in rwanda. In *Poverty and Well-Being in East Africa*, pages 165–188. Springer.
- Poi, B. P. (2002). From the help desk: Demand system estimation. *The Stata Journal*, 2(4):403–410.
- Poi, B. P. (2012). Easy demand-system estimation with quads. *The Stata Journal*, 12(3):433–446.
- Poi, B. P. (2013). Software updates. *The Stata Journal*, 13(3):667.

- Ravallion, M. et al. (1994). *Poverty comparisons*, volume 56. Taylor & Francis.
- Ray, R. (1983). Measuring the costs of children: an alternative approach. *Journal of Public Economics*, 22(1):89–102.
- Rimmer, M. T. and Powell, A. A. (1996). An implicitly additive demand system. *Applied Economics*, 28(12):1613–1622.
- Sadoulet, E. and De Janvry, A. (1995). *Quantitative Development Policy Analysis*. A Johns Hopkins paperback. Johns Hopkins University Press.
- Savard, L. (2004). Un système de demandes aids dans un contexte egc micro-simulation pour l’analyse de pauvreté et des inégalités. *Cahier de recherche/Working Paper*, 4:10.
- Shoven, J. B. and Whalley, J. (1992). *Applying general equilibrium*. Cambridge university press.
- Surry, Y. (1993). The constant difference of elasticities function with applications to the animal feed sector. *Journal of Agricultural Economics*, 44(1):110–125.
- Tarr, D. G. (1990). A modified cournot aggregation condition for obtaining estimates of cross-elasticities of demand. *Eastern Economic Journal*, 16(3):257–264.
- van der Mensbrugghe, D. (2020). Mitigation, adaptation and new technologies applied general equilibrium (manage) model: Version 2. Model documentation available upon request.
- Yu, W., Hertel, T. W., Preckel, P. V., and Eales, J. S. (2004). Projecting world food demand using alternative demand systems. *Economic Modelling*, 21(1):99–129.

# Appendix A. Flowchart of implementation

Figure 3: Implementation steps

