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Revisiting Nash Wages Negotiations in Matching Models

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Résumé/abstract

In labour economics theory, wage negotiations use to rely on a SymmetricNash Bargaining Solution. This article aims at showing that this kind of solution may be not relevant. Indeed, in a matching model framework, the comparison with the Kalai-Smorodinsky Solution suggests that a reflection should systematically be made with respect to the negotiation power of each agent.

Mots clés/keywords : Bargaining, matching, public policy.

Codes JEL/JEL Codes *:* C78, J64, J68

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1 Introduction

The search and matching model is the corner stone for the analysis of labour market. The job matching theory originating with Mortensen and Pissarides into the tradition of unemployment theory provides a benchmark model in labour economics. In fact, the equilibrium search and matching literature, coming from Diamond (1971, 1982), Mortensen (1982), and Pissarides (2000), has branched out into different research programs. The equilibrium theory of unemployment is probably the best known for the analysis of labour markets.

In the majority of papers dealing with the matching models, the Symmetric Nash Bargaining Solution is usually applied. However, this kind of solution could not be appropriated in some cases and it leads to move away from the labour market reality. Consequently, it could skew the analysis and the policy decisions.

Other solutions exist to solve bargaining problems: the Kalai-Smorodinsky solution [Kalai,1975] or even the equal-loss solution [Chun, 1988]. According to the selected solution, the interpretation can differ. Few authors applied the KS-solution on the labour market. Gerber and Upmann [2006] analyze a classic bargaining problem between a labour union and an employers' federation through the Nash and KS solutions. Notably, they point out the effect of the reservation wage on the employment and on the wage determination. Indeed they conclude that a higher reservation wage leads to a lower employment level with the Nash Solution, whereas the KS-solution leads up to an ambiguity. Laroque and Salanié [2004] point out the effect of the minimum wage on the employment in the case of wage bargaining between firms and workers. They prove that the KS solution is better than the Nash solution through an econometric test.

2 The Model

Here we consider a matching model with standard hypothesis ??? an economy composed of a large exogenous number of workers and a large endogenous number of firms. Firms are supposed to be identical and each offers a single job. The hypothesis of firm free-entry enables to maintain a fixed number of firms at the stationary state. All agents, which are risk neutral, discount the future with the same rate of time preference denoted by r. The exogenous job destruction rate is s.

2.1 The Hiring Process

Frictions are present in the labour market. It takes time for firms with a vacant job to find a worker. Such frictions are represented by a constant-returns matching function $m(V, U)$, where U is the number of employable unemployed workers and V is the number of vacants jobs. This matching function (Pissarides, 2000) is an homogenous function of degree 1, increasing in V and U. Instantaneous matching depends on the market tightness, noted $\theta = V/U$.

The probability for a firm to meet an employable worker is given by:

$$q = \frac{m(V,U)}{V} = m(1, \frac{1}{\theta}) = q(\theta) \tag{1}$$

This probability is a decreasing function of θ. A rise in the number of vacancies leads to a negative impact on the rate to fill a job due to the congestion effect.

The probability for an employable worker to find a job is given by:

$$p = \frac{m(V,U)}{U} = \theta q(\theta) = p(\theta) \tag{2}$$

This hiring probability is increasing in θ. Indeed, a rise of vacancies supplies more opportunities for workers to find a job.

2.2 Intertemporal Utilities and Profits

Here we consider the expected lifetime utility of workers and firms. For an employed workers, his utility depends on his hiring. Indeed, if this worker fills a job, its net present value, denoted U_1, depends on his wage w and on the destruction rate s.

$$rU_1(w, \theta) = w - s(U_1 - d_1) \tag{3}$$

Concerning an unemployed worker, his expected lifetime utility, noted d_1, depends on his income and on the hiring probability $p(\theta)$. We suppose that this income is only composed of unemployed benefits b.

$$rd_1(w, \theta) = b + p(U_1 - d_1) \tag{4}$$

Differentiation of worker's utility with respect to w and θ, holding the level of U_1 constant at, say, u_1, shows that the worker's indifference curves are downward sloping:

$$\left.\frac{dw}{d\theta}\right|_{U_1=u_1} = \frac{p'_\theta s(b-w)}{(r+p(\theta))(r+p(\theta)+s)} < 0 \tag{5}$$

The expected utility for firms depends if the job is filled or not. For a filled job, the net present value is composed of the net instantaneous income $(y - w)$ and on the future profits considering the destruction rate s.

$$rU_2(w, \theta) = y - w - s(U_2 - d_2) \tag{6}$$

Concerning a vacant job, as long as this job is unfilled, firms have to invest c corresponding to the job creation and to the search of a worker. The expected value for a vacant job d_2 is given by:

$$rd_2(w, \theta) = -c + q(\theta)(U_2 - d_2) \tag{7}$$

The free-entry hypothesis implies that new jobs will be created until the optimal value of a vacant job be equal to zero.

Differentiation of firm's profit with respect to w and θ, holding the level of U_2 constant at, say, u_2, shows that the firm's indifference curves are downward sloping:

$$\left.\frac{dw}{d\theta}\right|_{U_2=u_2} = \frac{q'_\theta s(y - w + c)}{(r + q(\theta))(r + q(\theta) + s)} < 0 \tag{8}$$

The Pareto-curve is defined as the set of all pair (w, θ) such that $U(w, \theta)$ is Pareto efficient. Hence, it is the set of all (w, θ) for which worker's and firm's indifference curves are tangent to each other, *i.e* which satisfies:

$$\left.\frac{dw}{d\theta}\right|_{U_1=u_1} = \left.\frac{dw}{d\theta}\right|_{U_2=u_2} \iff \frac{p'_\theta s(b - w)}{(r + p(\theta))(r + p(\theta) + s)} = \frac{q'_\theta s(y - w + c)}{(r + q(\theta))(r + q(\theta) + s)} \tag{9}$$

The differentiation with respect to θ and w points out that the wage w is decreasing with θ along the Pareto curve ($\delta w/\delta\theta < 0$).

2.3 Wage Bargaining and Surplus Sharing

Before determining the bargaining solutions, we have to present the axioms which define them. We denote by S the set of the payoffs in the bargaining set, u_1 and u_2 the utility function for each agent, d the disagreement point (d_1 for agent 1 and d_2 for the second) and u_1^*, u_2^* the solutions. The set S is bounded, convex and closed. The solution is an application ϕ which combines a payoff vector $\phi(S, d) = (\phi_1(S, d), \phi_2(S, d)) = (u_1^*, u_2^*)$ with each bargaining problem (S,d).

- *(A1)* Individual rationality (IR): $u_1^* \geq d_1$ and $u_2^* \geq d_2$, i.e. $\phi(S, d) \geq d$.
- *(A2)* Pareto optimality (PO): For $u^* \in S$ and $\forall \hat{u} \in S$, if $\hat{u} \geq u^*$, then $\hat{u} = u^*$.
- *(A3)* Symmetry (SYM): If $d_1 = d_2$ and if $\{(u, v) : (v, u) \in S\} = S$, then $u^* = v^*$ if (S,d) is symmetric.
- *(A4)* Invariance with respect to linear utility transformations (ILUT): If T is obtained from S by a linear transformation, then the solution (u_1^*, u_2^*) will be transformed by the same function. If $T = \{(\alpha_1 u_1 + \beta_1, \alpha_2 u_2 + \beta_2) : (u_1, u_2) \in S\}$ and $h = (\alpha_1 d_1 + \beta_1, \alpha_2 d_2 + \beta_2)$, then $\phi(T, h) = (\alpha_1 \phi_1(S, d) + \beta_1, \alpha_2 \phi_2(S, d) + \beta_2)$.

- *(A5)* Independence of irrelevant alternatives (IIA): For all closed and convex set $T \subset S$, if $\phi(S,d) \in T$, then $\phi(T,d) = \phi(S,d)$.

The optimization program is given by:

$$\max_{p}(U_1 - d_1)(U_2 - d_2) \tag{10}$$

The difference between the Nash and the Kalai-Smorodinsky solutions concerns the fifth axiom: the Independence of irrelevant alternatives. This one is replaced by the monotonicity axiom.

- $(A5')$ Individual monotonicity (IM): considering two sets S and T with $S \subseteq T$ and the disagreement point of the two sets d, if (u_1^*, u_2^*) is the solution of (S,d) and if $(u_1'^*, u_2'^*)$ is the solution of (T,d), then $u_1'^* \geq u_1^*$ and $u_2'^* \geq u_2^*$.

Theorem 1 (Kalai Smorodinsky, 1975)**.** *The Kalai-Smorodinsky solution is the unique solution that satisfies IR, PO, SYM, ILUT and IM. The KS curve is given by the function* ϕ^{KS} *:*

$$\phi^{KS} = (U_2 - d_2)(U_1^{max} - d_1) - (U_2^{max} - d_2)(U_1 - d_1) = 0$$

KS enables to define the ideal point I corresponding to the maximum payoff (U_1^{max}, U_2^{max}) for each agent. However, this ideal point is not feasible. The negotiation process leads to a solution which goes away the least from this point.

2.3.1 The Nash solution

In accordance with usual matching models, surplus created by a firm/worker is divided between the two agents according to their respective bargaining strength. If β $(0 < \beta < 1)$ represents the workers bargaining strength, the optimization program is:

$$\max_{w,\theta}(U_1 - d_1)^{\beta}(U_2 - d_2)^{1-\beta}$$

Therefore, the global surplus, noted S, is divided between the two agents according to the Nash rule:

$$\begin{cases} U_1 - d_1 = \beta(U_1 - d_1 + U_2 - d_2) = \beta S \\ U_2 - d_2 = (1-\beta)(U_1 - d_1 + U_2 - d_2) = (1-\beta)S \end{cases}$$

$$(1-\beta)\frac{w-b}{r+s+p(\theta)} = \beta\frac{y-w+c}{r+s+q(\theta)} \iff \frac{\dfrac{w-b}{r+s+p(\theta)}}{\dfrac{y-w+c}{r+s+q(\theta)}} = \frac{\beta}{1-\beta} \tag{11}$$

By differentiating this expression with respect to θ and w, we deduce that the wage w is increasing with θ, along the Nash curve.

2.3.2 The KS solution

The theorem 1 (Kalai, 1975) define the KS curve within the framework of matching model:

KS:$(U_2 - U_2^{min})(U_1^{max} - U_1^{min}) - (U_2^{max} - U_2^{min})(U_1 - U_1^{min}) = 0$

$$\phi^{KS}(w,\theta) = \left(\frac{y-w+c}{r+s+q(\theta)}\right)\left(\frac{\hat{w}-b}{r+s+p(\hat{\theta})}\right) - \left(\frac{y-\tilde{w}+c}{r+s+q(\tilde{\theta})}\right)\left(\frac{w-b}{r+s+p(\theta)}\right) = 0 \tag{12}$$

As for the Nash curve, the KS curve gives us an increasing relation between the wage and the market tightness. The intersection with the Pareto curve leads to the following solution:

$$\frac{\dfrac{\hat{w}-b}{r+s+p(\hat{\theta})}}{\dfrac{y-\tilde{w}+c}{r+s+q(\tilde{\theta})}} = \frac{\dfrac{q\prime_\theta}{r+q(\theta)}}{-\dfrac{p\prime_\theta}{r+p(\theta)}} \tag{13}$$

Each worker and each firm has a maximal payoff represented by an ideal point I. Concerning the firm, his ideal is to have a maximum profit resulting from a minimum wage $\tilde{w}$ payed to each worker, i.e. a wage equal to the unemployed benefits b. The ideal wage $\hat{w}$ for the worker is equal to his productivity. In this case, the probability for a worker to find a job $p(\hat{\theta})$ and the probability for a firm to meet a worker $q(\tilde{\theta})$ are supposed maximal.

We suppose that:

$$\begin{cases} \tilde{w} = b \\ \hat{w} = y \end{cases} \qquad \begin{cases} p(\hat{\theta}) = 1 \\ q(\tilde{\theta}) = 1 \end{cases} \tag{14}$$

We have an equality between $\dfrac{\dfrac{q\prime_\theta}{r+q(\theta)}}{-\dfrac{p\prime_\theta}{r+p(\theta)}}$ and $\dfrac{\dfrac{w-b}{r+s+p(\theta)}}{\dfrac{y-w+c}{r+s+q(\theta)}}$, which leads to an other expression for the KS solution. It enables to compare with the Nash solution.

$$\frac{y-b}{y-b+c} = \frac{\dfrac{w-b}{r+s+p(\theta)}}{\dfrac{y-w+c}{r+s+q(\theta)}} \tag{15}$$

2.3.3 Comparison of the bargaining solutions

The Nash and KS curves are increasing. For a fixed θ, we can determine the wage according to the two bargaining solutions :

We denote by $\Psi(\theta) = \dfrac{r+s+q(\theta)}{r+s+p(\theta)}$.

$$w_N = \frac{y + c + b\Psi(\theta)\left(\dfrac{1-\beta}{\beta}\right)}{1 + \Psi(\theta)\left(\dfrac{1-\beta}{\beta}\right)} \tag{16}$$

$$w_{KS} = \frac{y + c + b\Psi(\theta)\left(\dfrac{y-b+c}{y-b}\right)}{1 + \Psi(\theta)\left(\dfrac{y-b+c}{y-b}\right)} \tag{17}$$

The wage resulting from the Nash solution is higher than the one from the KS solution under a condition:

$$w_N > w_{KS} \quad if \quad \frac{y-b+c}{y-b} > \frac{1-\beta}{\beta}$$

The figure 1 gives us the position of the curves according to this two solutions. From it, we deduce that the Nash solution is preferable for the workers.

Figure 1: Nash and Kalai Smorodinsky Solutions

Proposition 1. *In the literature, the symmetric Nash solution is usually applied. The negotiation power between the firm and the worker is equal. However, the KS solution points out that this hypothesis is not viable if the cost of a vacant job is positive. The KS solution enables to determinate the negotiation power of each agent. This power is stronger for the firm to the detriment of the worker.*

Proof. The negotiation power of the Nash solution is given by $\dfrac{1-\beta}{\beta}$. In the literature, the value of β is equal to $\frac{1}{2}$, resulting in an equal negotiation power between the two agents. The KS bargaining solution leads to $\dfrac{y-b+c}{y-b}$. It is obvious that the negotiation power is unequal for a positive vacant job cost. By comparing these two expressions, we conclude that the value of $\frac{1}{2}$ is not appropriated and it brings an imbalance in the power struggle between the workers and the firms. Moreover, the two solutions coincide if $\beta = y - b$ and $1 - \beta = y - b + c$. The KS solution enables to define the negotiation power. □

2.4 Quantitative analysis

Now it would be interesting to pursue this study by focusing on the effects of the various variables on the equilibrium values. In matching models, it is the custom to use the calibration in order to determine the impacts [Table 2]. The matching function is represented by a Cobb-Douglas function: $M(V,U) = V^{0,5}U^{0,5}$, which gives us $q(\theta) = \theta^{1/2}$.

b	β	c	r	s	y
0,1	0,5	0,3	0,05	0,15	1

Table 1: Calibration of the model

Proposition 2. *Through the calibration of this model, we notice that the parameters have the same impacts on the equilibrium wage, whatever the chosen solution. However, the variation of Kalai on the wage is more accentuated than the Nash solution, except for the unemployed benefits.*

Proof. The effects of the parameters are summarized in the table 1 below, referring to the model calibration:

	b	c	r	s	y
w_N	++	-	-	-	+
w_{KS}	+	- -	- -	- -	++

Table 2: Impacts on the wage according to the bargaining solutions

□

References

[1] Binmore, K., A. Runinstein, and A. Wolinsky, *The Nash Bargaining Solution in Economic Modelling* , **Rand Journal of Economics**, 17, 176–188. 1986.

[2] Gerber, A, and T. Upmann, *Bargaining solutions at work: Qualitative differences in policy implications* , **Mathematical Social Sciences**, 52, 162–175. 2006.

[3] Kalai, E., and M. Smorodinsky, *Other Solution to Nash's Bargaining Problem*, **Econometrica**, 43, 513–518. 1975.

[4] Laroque, G., and B. Salanié, *Salaire minimum et emploi en présence de négociations salariales*, **Annales d'Économie et de Statistiques**, 73. 2004

[5] Nash, J. F. *The Bargaining Problem*, **Econometrica**, 18, 155–162. 1950.

[6] Pissarides, C., *Equilibrium Unemployment Theory*, **MIT Press**. 2000.

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