

2012s-10

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Série Scientifique
Scientific Series

Montréal
Mai 2012

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ISSN 1198-8177

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Substitution between bio-fuels and fossil fuels: is there a Green Paradox?

Quentin Grafton^{*}, Tom Kompas[†], Ngo Van Long[‡]

Résumé / Abstract

Nous montrons que (i) la subvention de la production d'énergie renouvelable avec l'intention d'encourager la substitution des combustibles fossiles pourrait accentuer les dommages du changement climatique en accélérant l'extraction des combustibles fossiles, et que (ii) ce résultat est renversé dans certaines conditions spécifiées. Nous nous concentrons sur le cas des subventions pour des ressources renouvelables produites dans le cadre des coûts marginaux croissants, et supposons que les ressources renouvelables et les carburants fossiles sont actuellement en cours d'utilisation. Ces subventions ont un effet direct et un effet indirect dans des directions opposées. L'effet direct est la réduction de la demande de combustibles fossiles à un prix donné. L'effet indirect est la réduction du prix d'équilibre actuelle de carburants fossiles, ce qui tend à augmenter la demande de combustibles fossiles. La somme des deux effets peut avancer ou retarder la date de l'épuisement du stock de ressources non-renouvelables selon la courbure de la courbe de la demande d'énergie et de la courbe d'offre pour le substitut renouvelable.

Mots clés : Subvention des ressources renouvelables; le Paradoxe Vert ; changements climatiques.

We show that (i) subsidies for renewable energy policies with the intention of encouraging substitution away from fossil fuels may accentuate climate change damages by hastening fossil fuel extraction, and that (ii) the opposite result holds under some specified conditions. We focus on the case of subsidies for renewable resources produced under increasing marginal costs, and assume that both the renewable resources and the fossil fuels are currently in use. Such subsidies have a direct effect and an indirect effect working in opposite directions. The direct effect is the reduction in demand for fossil fuels at any given price. The indirect effect is the reduction in the current equilibrium price for fossil fuels, which tends to increase the amount of fossil fuels demanded. Whether the sum of the two effects will actually result in an earlier or later date of exhaustion of the stock of fossil fuels depends on the curvature of the demand curve for energy and of the supply curve for the renewable substitute.

Keywords: *Subsidies of renewable energy; the Green Paradox; climate change.*

Codes JEL: Q54, Q42, Q30

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1 Introduction

The need to reduce CO_2 emissions to combat climate change is widely acknowledged. According to standard economic theory the first best measure would be a carbon tax, and, as Hoel (2011) succinctly stated, “at any time, the optimal price of carbon emissions should be equal to the present value of all future climate costs caused by present emissions, often called the social cost of carbon.” In the real world, however, policy makers are faced with many constraints on the choice of policy instruments and their levels, and consequently first best policies cannot be implemented. Governments, in particular, face political opposition to the introduction of efficient carbon taxes.

A more politically acceptable time path of carbon taxes would be of the “ramp” variety: a government starts with a low tax rate, but commits to raise the tax to efficient levels in the future. As has been demonstrated (e.g. Sinn, 2008a), such time paths of carbon tax may induce profit-maximizing fossil resource owners to hasten the extraction of their resources, thus increasing carbon emissions in the near term.¹ Such an outcome has been termed a “Green Paradox” (Sinn, 2008b, 2012). Sinn’s message is that in designing policies one must take into account supply responses by owners of fossil fuels.

The link between climate change and the behavior of firms extracting fossil-fuel resources was explored by Sinclair (1992,1994), Ulph and Ulph (1994), Hoel and Kverndokk (1996), Farzin and Tahvonen (1996), and Tahvonen (1997). Sinclair (1992) was among the first economists to explicitly consider climate change policies that would influence the extraction path of fossil fuels, stressing that “the key decision of those lucky enough to own oil-wells is not so much how much to produce as when to extract it.” Strand (2007) showed that a technological agreement that makes carbon redundant in the future may increase current emissions. As pointed out by Hoel (2008), prior to 2008, there is little work making the link between climate policies and exhaustible resources when policies are non-optimal or

¹Long and Sinn (1985) offered an analysis of how extractive firms respond to an anticipated time path of varying tax rate, but they did not explicitly mention the effect on CO_2 concentration levels.

international agreements are incomplete.² Recent dynamic models that depict the adverse response of resource extracting firms to anticipation of taxes or substitute production include Sinn (2008a,b), Hoel (2008), Di Maria et al. (2008), Gerlagh and Liski (2008), Smulders et al. (2009), and Eichner and Pethig (2010). Hoel (2008) assumes that carbon resources remain cheaper than the substitute and analyses the situation where different countries have climate policies of different ambition levels. He shows that, in the absence of an efficient global climate agreement, climate costs may increase as a consequence of improved technology of substitute production. Hoel (2010, 2011) uses a two-period model where firms invest in capacity of producing a substitute. A number of key parameters are considered in his model (e.g. a parameter to capture how rapidly extraction costs increase with increasing total extraction, and a parameter affecting the time profile of the returns to investments in the substitute). Whether an investment subsidy results in greater environmental damages (a Green Paradox) depends on the relationship among these parameters.

Gerlagh (2011) considers a model where extraction cost is constant, and a backstop technology can produce unlimited amount of a renewable substitute, at a constant cost per unit. He defines a Weak Green Paradox as an increase in the current emissions in response to an improvement in the backstop technology, and a Strong Green Paradox when the net present value of damages increases as a result of an improvement in the backstop technology. Ploeg and Withagen (2010) focus on the case where marginal extraction costs of the exhaustible resource depend on the existing stock. They assume that the substitute is available in unlimited supply at a constant marginal cost. After characterizing the social optimum, they turn to the case where first-best policies are not feasible, and show that the Green Paradox prevails if the cost of backstop decreases, provided that the backstop remains expensive such that the non-renewable resource stock is eventually exhausted. By contrast, if the backstop becomes so cheap that physical exhaustion will not take place, the Green Paradox no longer holds.

²See Bohm (1994) and Hoel (1994) for static models along these lines.

The bulk of the theoretical literature on substitute production assumes that a renewable substitute can be produced under constant marginal cost. In that setting, assuming also a constant cost of extraction of fossil fuels, the Green Paradox outcome is *inevitable*. The intuition behind this inevitability of the Green Paradox is that subsidies on the backstop technology leads to a lower peak price for fossil fuels, whose fixed stock must be exhausted before the renewable is produced. This implies that the whole time path of fossil fuel price is shifted downward, resulting in greater consumption of fossil fuels earlier on.³ It is important to emphasize that in such models fuel consumption has two distinct phases: in the first phase, only the exhaustible resource is used, and in the second phase, only the renewable substitute is used. By construction, there is *no* phase where both resources are consumed simultaneously. The key assumption which is responsible for the absence of simultaneous use is that the renewable substitute is produced at *constant* unit cost.

In reality, renewable substitutes such as biofuels are produced under conditions of increasing marginal cost. As pointed out in Chakravorty et al. (2011), increased use of biofuel may, by moving up the supply curve of land, increase the unit cost of the renewable resource. Using an dynamic empirical model firmly grounded on the Hotelling framework, they estimated the effect of biofuel mandates on food prices. They did not address the issue of the Green Paradox. With a few exceptions, the bulk of the existing literature on biofuel subsidies uses static analysis. Many authors have found, in the static context, mechanisms for increased carbon emissions (a Green Paradox outcome) with biofuel subsidies.⁴ It has also been pointed out that the production process of biofuels is not “green” because it involves the use of many inputs with high carbon contents. Moreover, because the first generation of biofuels displace land for food production it may increase food prices.

Our paper complements the existing literature with a dynamic mechanism. The contri-

³See Dasgupta and Heal (1979). Pearce and Turner (1990) provide a good exposition. Earlier models of substitute production include Heal (1976) and Hoel (1978,1983), which, though not dealing with CO_2 emissions, contain all the ingredients from which one can deduce a Green Paradox result.

⁴For static analysis of energy substitution, see Hill et al. (2006), Steenblik (2007), Koplow (2007), Lapan and Moschini (2009), Bandyopadhyay et al. (2009), Moschini et al. (2010). Bahel et al. (2011) developed a dynamic model but their focus was on food prices.

bution is a much richer understanding of the Green Paradox, delimiting cases when it occurs and when it does not, and shedding light on the inter-temporal consequences of biofuels subsidies.⁵ While models without a phase of simultaneous consumption are useful devices to illustrate principles, it is also important to consider situations where there is a phase in which exhaustible resources and renewable substitutes are *simultaneously* consumed in varying proportions. Such situations occur when the renewable substitute is produced under *increasing* marginal cost. This corresponds to the case of first generation biofuels based on conventional crops such as sugar cane, corn and vegetable oils (soy, canola, etc.). This is because the gradual expansion biofuels output, at least for first generation biofuels, necessitates the increasing use of less productive land with inevitable diminishing returns.⁶

Using a framework where *both* types of fuels are simultaneously consumed in the first phase phase, we find conditions under which a Green Paradox outcome will *not* occur, as well as conditions under which it will occur. We focus on the effect of a biofuel subsidy on the time path of extraction of the fossil fuel resources. A striking result of our paper is that in the case of a linear demand for energy, together with (i) zero extraction cost for fossil fuels and (ii) an upward-sloping linear marginal cost of biofuels, a biofuel subsidy will have *no* effect on the amount of fossil fuels extracted at each point of time, even though the subsidy does result in a lowering of the price of both fuels at each point of time, and in a higher quantity of energy demanded at each point of time. Under the stated assumptions we show that the increase in quantity of fuel demanded is exactly matched by an increased output of biofuels, leaving the extraction rate unchanged. Thus, the time at which the stock of fossil fuels is exhausted is unchanged, *irrespective* of the rate of subsidy. This result stands in contrast to the inevitable Green Paradox in the model of Hoel (2008) where the supply

⁵Fischer and Salant (2012) examined the Green Paradox in the presence of a subsidy for renewable resources. However, unlike our model, they assumed that the unit cost of renewable is constant in any given period. Their formulation allows unit cost to fall over time through knowledge accumulation. They found several cases where the Green Paradox may hold.

⁶To completely replace fossil fuels, humanity must make use of the whole range of alternative clean energy sources, including biofuels, geo-thermal heat, wind, solar, etc. As a referee points out, our paper may be regarded as dealing in general with renewables produced under increasing marginal costs. It is convenient to use of the expression “biofuels” as a catch-all term.

curve of the renewable resource is horizontal. We stress that the Green Paradox does not arise in our model in the presence of a linear and increasing marginal cost of the renewable substitute to fossil fuels.

A second and important result of our paper is that when the assumption of zero extraction cost of the fossil fuel is replaced by the assumption of a positive and constant marginal extraction cost, a biofuel subsidy will result in a longer time over which the fossil fuel stock is exhausted. In this second case the Green Paradox does not arise. Indeed, there is no paradox at all: the subsidy works as intended because it extends the extraction period.

A third setting to investigate the possibility of a Green Paradox is when the marginal cost of biofuels increases at an increasing, or a decreasing rate as biofuels supply increases. In particular, we consider the case where the marginal cost of biofuel production is strictly increasing and strictly concave. In this situation along the equilibrium price path, as the price of energy rises gradually along the optimal extraction path of fossil fuels, biofuel output will increase over time, but the rate of the supply increase (per dollar increase in energy price) is *greater* when the price is higher. Consequently, fossil fuel firms, in anticipation of this greater expansion of the substitute in the later stage, respond by increasing their extraction at an earlier date. In this case, a Green Paradox outcome results.

In summary there are multiple cases when the Green Paradox does, and does not hold. The purpose of our analysis is to provide necessary and sufficient conditions for the Green Paradox to hold. We decompose the effect of a biofuel subsidy and show, for the first time, a direct effect of the subsidy (which is “pro-Green”) and an indirect effect (which is “anti-Green”). Further, we show that a Green Paradox occurs *if and only if* the indirect effect outweighs the direct effect. The analytical condition is general, but it can be hard to interpret without numerical examples. For this reason, we have supplied and commented on some simulation results.

Our model is consistent with the current state of play in terms of biofuel production. The main producing countries for transport biofuels are the U.S., Brazil and the EU. Brazil

and the U.S. combined produced 55 and 35 percent, respectively, of the world's ethanol production in 2009 while the EU produced 60 percent of the total biodiesel output. The main stimulus to the use of biofuels are policies that encourage the substitution from fossil fuels, especially for road transportation. Government mandates for blending biofuels into vehicle fuels have also been enacted in at least 17 countries, and many states and provinces within these countries. Typical mandates require blending 10–15 percent ethanol with gasoline or blending 2–5 percent biodiesel with diesel fuel. Recent targets have encouraged higher levels of biofuel use in various countries (UNEP, 2009, page 15-16).

The range of policies that have stimulated biofuel demand by setting targets and blending quotas has been aided by supporting mechanisms, such as subsidies and tax exemptions. In the US, the total biofuels support encompasses the total value of all government supports to the biofuels industry, including consumption mandates, tax credits, import barriers, investment subsidies and general support to the sector such as public research investment. A report by Koplow (2007, pp. 29, 31) for the Global Subsidies Initiative indicates that the total support estimates for the US alone, in 2008, was between \$9.2 and 11.07 billion.

What do our results imply for biofuel subsidy policies? Our analysis indicates that the effects of proposed policies can only be gauged by using empirical models that are firmly grounded on dynamic theory that takes into account intertemporal behavior of resource extraction firms.

2 A general model of simultaneous use of fossil fuels and biofuels

Most of the literature on the Green Paradox that involves renewable substitutes is based on the assumption that the substitute is adopted only when the use of fossil fuels comes to an end. Thus a typical model displays two phases. In the first phase, only fossil fuels are consumed. In the second phase, the renewable substitute completely displaces the use of fossil fuels. These models are based on the assumption that the substitute is available in

unlimited supply, at a constant unit cost.

In this paper, we focus on the case where *both* types of fuels are supplied in the market. This is made possible by postulating an upward sloping supply curve for biofuels. We allow this positive slope to be constant for all output levels, or to be increasing or decreasing (while still remaining positive) with output. Our assumptions on the cost of extraction of the fossil fuels allows for the possibility that as the stock dwindles, the extraction cost rises. The special case of stock-independent marginal cost is also admitted.

On the demand side, we assume that fossil fuels and biofuels are perfect substitutes: one unit of energy can be obtained from using one unit of fossil fuels, or alternatively one unit of biofuels. We denote by P_t as the consumer's price of fuels at time t , and by E_t^d the quantity of fuel demanded. The demand function is $E_t^d = D_E(P_t)$ where $D'_E(P_t) < 0$.

We assume that biofuel firms are price takers. Suppose that for each unit of biofuel sold, the biofuel producers receive $(1 + v)P_t$ dollars, where v is the ad valorem subsidy rate. (We may interpret v as a production subsidy). Let $z = 1 + v$. We call z the "subsidy factor." Let B_t be the quantity of biofuels supplied at time t . The supply curve of biofuels is upward sloping, $B_t = S(zP_t)$, with $S'(zP) > 0$. We allow the second derivative $S''(zP)$ to be negative or positive. The special case $S''(zP) = 0$ implies that the supply curve is linear.

Concerning the extraction of fossil fuels, we assume that the fossil fuels sector consists of n identical producers, each having an initial stock R_{i0} . Let q_{it} denote producer i 's rate of extraction at time t , and R_{it} denote his remaining stock. Then $\dot{R}_{it} = -q_{it}$. Omitting the subscripts for simplicity, we write the extraction cost function for each firm as $C(q, R)$. We assume that (i) marginal extraction cost is non-negative, $C_q \geq 0$, and non-decreasing in extraction rate, $C_{qq} \geq 0$; (ii) the extraction cost is non-increasing in the size of the reserves, $C_R \leq 0$; (iii) as the stock dwindles, the marginal extraction cost rises (or remain constant), $C_{qR} \leq 0$.⁷

$$C_q \geq 0, \quad C_{qq} \geq 0, \quad C_R \leq 0, \quad C_{qR} \leq 0. \quad (1)$$

⁷See e.g. Farzin (1992).

Let Q_t be the aggregate extraction of fossil fuels at time t : $Q_t \equiv \sum_{i=1}^n q_{it}$.

By assumption fossil fuels and biofuels are perfect substitutes. Flow equilibrium in the fuel market, thus, requires that total supply (i.e., aggregate extraction Q_t plus the output of biofuel producers, B_t) equals the quantity of fuels demanded, E_t :

$$Q_t + B_t = E_t \quad (2)$$

This condition implies that

$$Q_t = D_E(P_t) - S(zP_t) \quad (3)$$

Let us define

$$D(P_t; z) \equiv D_E(P_t) - S(zP_t) \quad (4)$$

The function $D(P_t; z)$, defined as the difference between the demand for fuels and the supply of biofuels, is called the demand for fossil fuels. For any given $z > 0$, there is a unique price \bar{P} (which depends on z) at which $D(\bar{P}; z) = 0$. We call \bar{P} the “choke price” for fossil fuels, where $D_E(\bar{P}) - S(z\bar{P}) = 0$. When the price reaches \bar{P} , the entire market demand for fuels is met by biofuel producers. A higher z implies a lower choke price:

$$\frac{d\bar{P}}{dz} = \frac{S'(z\bar{P})\bar{P}}{D'_E(\bar{P}) - S'(z\bar{P})z} < 0 \quad (5)$$

An increase in z also has an effect on the demand for fossil fuels: at any price below the choke price, a higher z implies a smaller quantity demanded (for fossil fuels). Figure 1 illustrates the effect of an increase in the subsidy factor z , from z_1 to $z_2 > z_1$. We assume in the Figure that $S(0) = 0$, but our mathematical analysis does not require this assumption.

PLEASE PLACE FIGURE 1 HERE

A higher biofuel production-subsidy implies that at any given consumer’s price P , a greater quantity of biofuels is supplied. This is represented by a clockwise rotation of the biofuel supply curve in Figure 1. This rotation implies that the demand curve for fossil fuels, $D(P_t; z)$, is rotated anti-clockwise, as is clear from equation (4). In Figure 1, this rotation is

from EM_1 to EM_2 . The old choke price $\bar{P}(z_1)$ is depicted by point M_1 and the new choke price $\bar{P}(z_2)$ is depicted by point M_2 .

Figure 1 appears to indicate that a higher rate of biofuel subsidy will result in lower emissions, but this can only be true at a given price. For example, in Figure 1, at the given P_0 , the *direct effect* of a biofuel production subsidy is to increase biofuel output from point A to point B , implying that the quantity of fossil fuels demanded falls from Y_0 to Y' . However, taking into account the full response of extractive firms, the equilibrium price P_0 would also change as a result of the increase in the subsidy rate. Thus, the extraction path of fossil fuels must, in general, be adjusted, and so must P_0 and all P_t , in order to ensure that: (i) the flow equilibrium condition (2) is satisfied at all points of time; (ii) the intertemporal equilibrium condition, known as the Hotelling Rule, is satisfied along a positive extraction path; and (iii) the stock equilibrium condition holds such that the cumulative extraction up to the exhaustion time T should equal the initial stock. Consequently, the initial price must fall, for instance, from P_0 to, say, P'_0 in Figure 1.

As the initial price falls from P_0 to P'_0 , the output of biofuel moves along the biofuel supply curve $S(z_2P)$ from point B to point N , and thus the quantity demanded for fossil fuels rises. We call this the *indirect effect* of the biofuel subsidy. Figure 1 depicts a very special case where the fall in initial price is just sufficient to restore the initial quantity of fossil fuel demanded back to Y_0 (i.e. in this case the subsidy has no effect on the extraction rate of fossil fuels, because the direct effect and the indirect effect cancel each other out).

In what follows, we show how the equilibrium paths of price and quantity would adjust to a permanent increase in the subsidy factor z . We consider two cases.

Case I (stock-independent extraction cost). In this case, $C_R = 0$ for all R .

Case II (stock-dependent extraction cost). In this case, $C_{qR} < 0$.

In Case I, it is convenient to denote the marginal extraction cost by $c(q)$. Since $c(q)$ is assumed to be non-decreasing, marginal extraction cost is lowest when $q = 0$. We assume that $c(0)$ is lower than the choke price $\bar{P}(z)$, so that the firm will find it optimal to (eventually)

exhaust its stock.

In Case II, $C_q(0, 0)$ is called the “marginal cost of extracting the last drop of oil.” We distinguish two sub-cases. In sub-case II-A, $C_q(0, 0) \leq \bar{P}(z)$, which implies that eventual exhaustion is optimal. In sub-case II-B, $C_q(0, 0) > \bar{P}(z)$, the marginal cost of extracting the last drop of oil is high, and physical exhaustion will not take place. In sub-case II-B there exists a positive reserve level R_L such that $C_q(0, R_L) = \bar{P}(z)$, and the firm will abandon its stock when the reserve level R reaches R_L . This case involves economic exhaustion rather than physical exhaustion.

We now consider the optimization problem of the representative extractive firm. We assume it has perfect foresight of the price path, which it takes as given. It chooses the extraction path q_t , the terminal time T and the terminal stock level R_T to maximize

$$\int_0^T e^{-rt} (q_t P_t - C(q_t, R_t)) dt$$

where $r > 0$ is the rate of interest. The necessary conditions yield the Hotelling equation

$$\frac{d}{dt} [P_t - C_q(q_t, R_t)] = r(P_t - C_q) + C_R \text{ for } 0 \leq t \leq T \quad (6)$$

with $q_T = 0$, and the transversality condition takes the form

$$P_T - C_q(0, R_T) \geq 0, \quad R_T \geq 0, \quad [P_T - C_q(0, R_T)] R_T = 0 \quad (7)$$

The transversality condition (7) is intuitively plausible. If it is optimal to leave some stock R_T unexploited, it must be the case that at the terminal time T the marginal cost of extraction equals the choke price. By contrast, if resource exhaustion is optimal, the terminal marginal extraction cost must generally be lower than the terminal price.

Given that all firms are identical, we focus on the symmetric equilibrium where $nq_t = Q_t = D(P_t; z)$. Differentiation with respect to time yields $\dot{Q}_t = D_P(P_t; z)\dot{P}_t$. Substituting these two equations into the Hotelling Rule (6) gives us the evolution of the equilibrium price path:

$$\dot{P} = r \left[P - C_q \left(\frac{D(P; z)}{n}, R \right) \right] + C_R + C_{qq} \left(\frac{D(P; z)}{n}, R \right) \frac{D_P(P; z)\dot{P}}{n} - C_{qR} \left(\frac{D(P; z)}{n}, R \right) \frac{D(P; z)}{n} \quad (8)$$

For given initial stock size R_0 and given z , this differential equation, together with $\dot{R} = -\frac{D(P; z)}{n}$ and condition (7), can be solved to determine the equilibrium paths of price, output, and the equilibrium terminal time T at which $P_T = \bar{P}(z)$.

At the level of generality displayed by equation (8), it is not possible to determine the effect of an increase in the biofuel subsidy factor z on the extraction path and the exhaustion time T . In the next section, we consider the case of stock-independent marginal cost, in which equation (8) reduces to a simpler form.

3 The case of stock-independent marginal extraction cost

Consider the special case where the marginal extraction cost is stock-independent and is a constant $c \geq 0$. Then $C(q, R) = cq$, and $C_q = c$. Assume $c < \bar{P}$, so that it is profitable to extract and to eventually exhaust the resource stock, i.e. $R_T = 0$. Condition (8) reduces to $\dot{P} = r(P - c)$. This equation implies the following Hotelling price path for all $t \in [0, T]$:

$$P_t = c + (\bar{P}(z) - c)e^{r(t-T)} \equiv \phi(\bar{P}(z), t, T). \quad (9)$$

This equation shows that the price at time t is functionally related to the exhaustion time T as well as the choke price \bar{P} . Note that ϕ is decreasing in T and increasing in \bar{P} .

From equation (9) and the differential equation $\dot{R}_t = -D(P_t; z)/n$ we obtain the stock equilibrium condition that accumulated extraction (from time zero to the exhaustion time) equals the initial reserves:

$$\int_0^T D(c + (\bar{P}(z) - c)e^{r(t-T)}; z) dt = - \int_0^T n \dot{R}(t) dt = nR_0 \quad (10)$$

Given R_0 and z , this equation uniquely determines the equilibrium exhaustion time T . If an increase in z brings the exhaustion time T closer to the present, we say that we obtain a *Green Paradox*. It is clear that if T is brought closer to the present, the present value of the

flow of damages will be greater, even though we have not explicitly modelled the relationship between the time path of damages and the time path of cumulative emissions.⁸

What are the forces that determine the net effect of an increase in the biofuel subsidy factor z on the time of exhaustion T ? We know from equation (5) that an increase in z will lower the fossil fuel choke price \bar{P} . At the same time, an increase in z rotates the demand curve for fossil fuel in the anti-clockwise direction, as shown in Figure 1. Does an increase in z generate, on average, greater demand for fossil fuels over the time interval T ? To answer this question, it is useful to decompose the change in demand into a direct effect and an indirect effect.

Direct Effect: The anti-clockwise rotation of the demand curve for fossil fuels implies that any given price P_t , the quantity demanded is smaller than before. This direct effect is captured by the term $D_z < 0$. The direct effect is “pro-Green”: an increase in biofuel subsidy reduces demand for fossil fuels, at any given price P_t .

Indirect Effect: The increase in z (biofuel subsidy) lowers the fossil fuel choke price \bar{P} . This implies that, holding T constant, the price P_t must fall, see equation (9). A fall in P_t increases the quantity demanded. The indirect effect, captured by the term $\left(\frac{\partial D}{\partial P_t}\right)\left(\frac{\partial P_t}{\partial z}\right)$, is positive, i.e. it is “anti-Green”.

The total effect on fossil fuels consumption at any time t is the sum of the direct effect and the indirect effect at that time. In general, the total effect can be positive at some points of time and negative at some other points of time. Therefore, to find the effect of biofuel subsidy, one has to compute the cumulative total effect, over the interval $[0, T]$. If this cumulative total effect is positive, it means that the exhaustion time must be brought closer to the present, which is a Green Paradox outcome. A more formal analysis follows.

To find the net effect of a change in z on the exhaustion time T , let us write

$$G(T, z) = \int_0^T D(c + (\bar{P}(z) - c)e^{r(t-T)}; z)dt - nR_0 = 0$$

⁸For the importance of cumulative emissions, see Allen et al. (2009). For additional considerations such as the feedback effects of temperature increases on GHGs emissions, see Winter (2011), among others.

Then

$$\frac{dT}{dz} = -\frac{G_z}{G_T} = \frac{-\int_0^T [D_P \frac{\partial P_t}{\partial z} + D_z] dt}{D(\bar{P}(z); z) + \int_0^T [D_P \frac{\partial P_t}{\partial T}] dt} \quad (11)$$

where the denominator is positive because $D(\bar{P}(z); z) = 0$, $D_P < 0$ and $\partial P_t / \partial T < 0$ from equation (9). The integrand in the numerator is ambiguous in sign:

$$D_z + \left(D_P \frac{\partial P_t}{\partial z} \right) \leq 0 \quad (12)$$

The comparative static expression (11) depends on the integral of the expression (12). Even though in general the sign of (12) is uncertain, the sign of its integral may be determinate or at least calculated numerically.

Lemma 1: *An increase in the subsidy factor z will bring the resource-exhaustion date T closer to the present (i.e. will lower T) if and only if the cumulative indirect effect dominates the cumulative direct effect*

$$\int_0^T D_P \frac{\partial P_t}{\partial z} dt > \int_0^T (-D_z) dt \quad (13)$$

Since in general we cannot determine the sign of dT/dz , we consider some special cases.

3.1 Linear demand for energy and linear biofuel-supply function

Assume that both the demand function for energy $D_E(P)$ and the biofuel supply function are linear, $D_E(P) = a - bP$, and $S(zP) = \beta zP - g$, where $a > 0$, $b > 0$, $\beta > 0$ and $g \geq 0$. Then the (residual) demand function for fossil fuel is $D(P; z) = (a + g) - bP - \beta zP$. From equation (9)

$$P_t = c + (\bar{P}(z) - c)e^{r(t-T)} = c [1 - e^{r(t-T)}] + \bar{P}(z)e^{r(t-T)}$$

The direct effect is then

$$D_z(P_t; z) = -\beta P_t = -\beta \bar{P}(z)e^{r(t-T)} - \beta c [1 - e^{r(t-T)}] < 0 \quad (14)$$

To calculate the indirect effect, note that

$$\bar{P}(z) = \frac{(a + g)}{b + \beta z} \implies \frac{d\bar{P}}{dz} = -\frac{(a + g)\beta}{(b + \beta z)^2} < 0.$$

and

$$\partial P_t / \partial z = (d\bar{P}/dz)e^{r(t-T)} = -\frac{a\beta}{(b + \beta z)^2}e^{r(t-T)} = -\frac{\beta\bar{P}(z)e^{r(t-T)}}{b + \beta z} < 0$$

Then the indirect effect is

$$D_P \frac{\partial P_t}{\partial z} = [(b + \beta z)] \left[\frac{\beta\bar{P}(z)e^{r(t-T)}}{b + \beta z} \right] = \beta\bar{P}(z)e^{r(t-T)} > 0 \quad (15)$$

Comparing the direct effect (14) and the (15), we see that they cancel each other out if $c = 0$. If $c > 0$, then the direct effect dominates the indirect effect, and there is no Green Paradox. Thus, making use of Lemma 1, we can state the following result.

Proposition 1: *Assume that the demand function for fuels and the supply function of biofuels are both linear. Then under perfectly competitive extraction, an increase in the subsidy for biofuel producers will generally delay the exhaustion date, i.e. $dT/dz \geq 0$. In particular,*

(i) *Assume extraction costs are zero ($c = 0$). Then an increase in the biofuel subsidy factor z has no effect on the date of exhaustion of the resource stock. The cumulative extraction path under the biofuel subsidy is identical to the cumulative extraction path without the biofuel subsidy. (The direct effect and the indirect effect of an increase in z cancel each other out.)*

(ii) *Assume extraction costs are positive and smaller than the choke price ($\bar{P} > c > 0$). Then an increase in the biofuel subsidy factor z leads to a delay the date of exhaustion of the resource stock. The cumulative extraction path with the biofuel subsidy is uniformly lower than the cumulative extraction path without the biofuel subsidy.*

Remark 1. The intuition behind Proposition 1 is as follows. A permanent increase in biofuel subsidies at time $t = 0$ creates a direct effect: increased biofuel production at any given price (a clockwise rotation of the biofuel supply curve, see Figure 1). This increased supply of biofuels implies that if the price path for fuels were unchanged, less fossil fuels would be demanded at each point in time. Hence, if the price path were unchanged, the fossil fuel stock would not be exhausted by the time the price reaches the new (and lower)

choke price, M_2 in Figure 1. But from our assumption that $c < M_2$ non-exhaustion cannot be an equilibrium. So the price at all t must fall to stimulate demand for fuels (and hence for fossil fuels) sufficiently to clear the market. The fall in P_0 tends to reduce the amount of biofuels supplied (an induced effect, a movement along the already rotated biofuel supply curve, from point B to point N in Figure 1). At the lower price, the demand for energy is higher than in the no-subsidy scenario, and while some of this demand is met by biofuel production, the remaining demand is met by fossil fuel extraction. When $c = 0$ and the demand and supply are linear, the fall in the initial price (from P_0 to P'_0) is just sufficient for the flow demand for fossil fuels to be restored to the level that prevails in the no-subsidy scenario (point Y_0 in Figure 1). When $c > 0$, the fall in initial price is smaller (above the point P'_0 depicted in Figure 1), so the amount of fossil fuels demanded at time $t = 0$ will be smaller than Y_0 in Figure 1.

Remark 2. We note the possibility that the direct and indirect effects perfectly offset each other. Such outcomes are also possible in different contexts, such as, in macroeconomics, with a logarithmic utility function an increase in the interest rate has no impact on savings because the income effect and the substitution effect cancel each other out.

Remark 3. Another interpretation of part (i) of Proposition 1 is as follows. Assume that the demand for an exhaustible resource is linear, $Q = a - (b + \beta z)P \equiv a - \tilde{b}P$, and that extraction cost is zero. A rotation of the linear demand curve (keeping the horizontal intercept a unchanged, while changing \tilde{b}) will leave the equilibrium extraction path and the exhaustion time unchanged, but the equilibrium price path will be changed. Figure 2 shows the time paths of price for different values of \tilde{b} , under the assumption that $c = 0$. It is important to note that while superficially the time path of price in Figure 2 looks like the standard textbook figures which treat the case of a substitute that is available at constant cost and that is supplied only after extraction of fossil fuels has ceased, the underlying story is not the same because, in our case, both biofuels and fossil fuels are consumed simultaneously at each point of time, and the increased demand for fuels associated with the lower price path

is satisfied by the expansion of biofuel output. This explains why in Figure 2 it is possible that the stock is exhausted at the same time T even though the price path is lowered as the result of a subsidy.

PLEASE PLACE FIGURE 2 HERE

3.2 Isoelastic biofuel-supply function and modified-isoelastic energy demand function

We now consider the special case where the biofuel supply function is isoelastic

$$S(zP) = \beta(zP)^\mu \text{ where } \beta > 0 \text{ and } \mu > 0 \quad (16)$$

(where μ is the elasticity of supply) and the demand function for energy is of the modified isoelastic form⁹

$$D_E(P) = (P + \delta)^{-\varepsilon} \text{ where } \delta \geq 0 \text{ and } \varepsilon > 0 \quad (17)$$

The choke price \bar{P} is the solution of the equation

$$(\bar{P} + \delta)^{-\varepsilon} - \beta(z\bar{P})^\mu = 0 \quad (18)$$

Then an increase in biofuel subsidy will lower the choke price for fossil fuels,

$$\frac{d\bar{P}}{dz} = -\frac{\beta\mu z^{\mu-1}(\bar{P})^\mu}{\varepsilon(\bar{P} + \delta)^{-\varepsilon-1} + \mu\beta z^\mu(\bar{P})^{\mu-1}} < 0.$$

From equation (11), we know that a Green Paradox outcome occurs if and only if $\int_0^T [D_P \frac{\partial P_t}{\partial z} + D_z] dt >$

0. Recalling equation (9), we obtain the indirect effect

$$D_P \frac{\partial P_t}{\partial z} = e^{r(t-T)} \left\{ \frac{\beta\mu z^{\mu-1}(\bar{P})^\mu [\varepsilon(P_t + \delta)^{-\varepsilon-1} + \mu\beta z^\mu(P_t)^{\mu-1}]}{\varepsilon(\bar{P} + \delta)^{-\varepsilon-1} + \mu\beta z^\mu(\bar{P})^{\mu-1}} \right\} > 0$$

⁹A possible interpretation of the energy demand function (17) is that it is a derived demand function. For example, suppose a manufactured good is produced under constant returns to scale using two inputs, energy and labor, in fixed proportions, normalized at unity. Then the price of a unit of the manufactured good is $P + \delta$ where P is the price of energy and δ is the wage rate. Under this interpretation, ε is the price elasticity of demand for the manufactured good.

Proposition 2: *Under the demand and supply specification (16) and (17), a Green Paradox occurs if and only if parameter values are such that the cumulative indirect effect dominates the cumulative direct effect, i.e.,*

$$\int_0^T \left\{ e^{r(t-T)} \left[\frac{\varepsilon(P_t + \delta)^{-\varepsilon-1} + \mu\beta z^\mu (P_t)^{\mu-1}}{\varepsilon(\bar{P} + \delta)^{-\varepsilon-1} + \mu\beta z^\mu (\bar{P})^{\mu-1}} \right] - \left(\frac{P_t}{\bar{P}} \right)^\mu \right\} dt > 0 \quad (19)$$

where P_t is given by (9) and \bar{P} is given by (18). This condition is satisfied under plausible specifications of parameter values.

Proof: Using Lemma 1 and our specification of demand and supply as in (16) and (17), condition (13) is satisfied if and only if

$$\int_0^T \left\{ e^{r(t-T)} \left[\frac{\beta\mu z^{\mu-1} (\bar{P})^\mu (\varepsilon(P_t + \delta)^{-\varepsilon-1} + \mu\beta z^\mu (P_t)^{\mu-1})}{\varepsilon(\bar{P} + \delta)^{-\varepsilon-1} + \mu\beta z^\mu (\bar{P})^{\mu-1}} \right] - \beta\mu z^{\mu-1} (P_t)^\mu \right\} dt > 0$$

This condition is equivalent to (19).

Numerical examples

While it is not possible to determine the sign of (19) analytically, we can compute it when parameter values are specified. First, $\bar{P}(z)$ is computed. Then P_t is computed from (9). Next, we integrate (19) numerically. Finally, we compute (11).

In our base-line scenario, the parameters are:

$$\varepsilon = \delta = \beta = \mu = 1, \quad r = 0.05, \quad c = 0 \quad (20)$$

This means that the elasticity of supply of biofuel is unity, and the (residual) demand for fossil fuels is strictly convex, with the price elasticity of demand ranging from infinity (at the choke price \bar{P}) to zero (when the price approaches zero). Let the reserve size R_0 be large enough so that under this base-line scenario, the exhaustion time is $T = 100$ years. Then we find that, starting at the initial subsidy factor $z = 1$, an increase in z leads to an earlier exhaustion time: $dT/dz = -2.09$. This result is in sharp contrast to the case where the residual demand is linear, where under zero extraction cost an increase in z will cause a fall in the initial price P_0 just sufficiently to keep the quantity demanded q_0 (on the rotated demand curve) unchanged. With a strictly convex demand function, the initial price falls

more sharply, increasing the quantity demanded q_0 . Figure 3 shows the Green Paradox in this case.

PLEASE PLACE FIGURE 3 HERE.

The above computation shows that, given our specification (20), the Weak Green Paradox holds if the reserve size is such that $T = 100$. Is this result sensitive to the size of the reserve? Let us vary the reserve size so that T varies from 50 years to 200 years. We find that dT/dz remains negative, and not far different from -2.09 . Table 1 below reports the value dT/dz for various reserve sizes (and hence various T).

We also consider different initial subsidy levels keeping $\varepsilon = \delta = \beta = \mu = 1$, $r = 0.05$, $c = 0$, $T = 100$. Table 2 reports the results and shows there is a Green Paradox for this set of parameter values, for a wide range of values of the initial subsidy factor. Next, keeping $\beta = \delta = \mu = 1$, $r = 0.05$, $c = 0$, $T = 100$, we consider different values for ε , the price elasticity of demand for the manufactured good. The results are reported in Table 3. Again, for a wide range of possible values of this elasticity, the Green Paradox holds. We find that the greater is the elasticity ε , the stronger is the effect of an increase in the subsidy on the lowering of the exhaustion time. Finally, consider different values for supply elasticity of biofuels, μ . We find that the Green Paradox holds for a wide range of μ , as reported in Table 4. As shown in this Table 4, *with the exception of the first column where the supply elasticity of biofuels is small*, an increase in the subsidy rate will hasten the exhaustion of fossil fuels.

The intuition behind the sensitivity to the supply elasticity μ can be explained by noting that when the price elasticity of supply of biofuels is high (μ large) the rising price along the Hotelling path induces very strong biofuel supply expansion. Hence, the demand for oil primarily decreases in the future rather than earlier. Consequently, oil owners have to sell more sooner and it is this outcome that works in favor of the Green Paradox.

4 The case of stock-dependent marginal extraction cost

We now turn to an investigation of the possibility of a Green Paradox outcome in the case where extraction costs are stock-dependent, as specified in (1) with $C_R < 0$, meaning that the extraction cost rises as the remaining stock falls. There are two possible scenarios. In the first scenario, the marginal cost of extracting the ‘last drop of oil’, although high, is still below the choke price for fossil fuels, and therefore all the fossil fuel stock will eventually be exhausted. In the second scenario, the last drop of oil is prohibitively expensive to extract, and therefore firms will abandon their deposits without exhausting them.¹⁰ We investigate the possibility of the Green Paradox in both cases.

It will be convenient to work with an explicit form of the function $C(q, R)$. Following Karp (1984), we postulate that $C(q_t, R_t) = [c_0 + c_1(R_0 - R_t)]q_t$, where $c_0 \geq 0$, $c_1 \geq 0$, and $R_0 - R_t \geq 0$. Then $c_0 + (R_0 - R_t)c_1$ is the marginal cost of extraction at time t . At time $t = 0$, the marginal extraction cost is at its lowest value, c_0 . As extraction proceeds, the remaining stock R_t falls, and the marginal extraction cost rises. As R_t falls to 0, the marginal extraction cost rises to $c_0 + c_1R_0$. We call $c_0 + c_1R_0$ the marginal cost of extracting the last drop of oil. The parameter c_1 represents the sensitivity of marginal cost to the remaining stock.

Under this specification, equation (8) reduces to $\dot{P} = r(P - (c_0 + c_1(R_0 - R)))$.

As before, the demand for fossil fuels falls to zero at the “fossil fuel choke price” \bar{P} defined by $D_E(\bar{P}) - S(z\bar{P}) = 0$. Since the representative oil firm’s marginal cost is $c_0 + c_1(R_0 - R_t)$, it is clear that:¹¹

(i) if the marginal extraction cost of the last drop of oil is low, such that $c_0 + c_1R_0 \leq \bar{P}(z)$, then the representative firm will eventually exhaust all its stock,

(ii) if the marginal extraction cost of the last drop of oil is high, such that $c_0 + c_1R_0 > \bar{P}(z)$, the representative firm will abandon its deposit when the reserve level falls to some positive

¹⁰Some authors have therefore modeled the “resource exhaustion” in the sense of an “economic abandonment” of the deposit after the profitable part has been exploited (see for example Karp, 1984).

¹¹See Karp (1984).

level R_L defined by

$$c_0 - c_1(R_0 - R_L) = \bar{P}(z), R_L > 0 \quad (21)$$

Let us consider the case where the marginal extraction cost of the last drop of oil is low, such that $c_0 + c_1 R_0 \leq \bar{P}(z)$. Then the equilibrium paths of price and quantity are obtained from solving the following system of equations: $\dot{P} = r(P - (c_0 + c_1(R_0 - R)))$ and $\dot{R} = -D(P; z)$, subject to the boundary conditions $R(0) = R_0$, $P_T = \bar{P}(z)$ and $R_T = 0$.

The case of linear demand for fossil fuels

In this case, we can show that the Green Paradox does not hold. This is stated as Proposition 3.

Proposition 3: *If the fossil fuel demand function $D(P; z)$ is linear in P , and the entire stock R_0 is exhausted at some time T , then*

(i) *an increase in the subsidy factor z will delay the exhaustion date,*

(ii) *higher sensitivity of extraction cost with respect to the remaining stock results in a later exhaustion date.*

Proof: See the Appendix.

As a numerical example, let $a = 3, b = \beta = z = 1$ and $\bar{R} = 80$. We find that the stock \bar{R} will be exhausted in 370 years. If the subsidy factor is $z = 1.1$, we find that T increases to 410 years.

The case of non-linear demand

We consider the more general case of non-linear demand and suppose that $D(P, z) = (P + \delta)^{-\varepsilon} - \beta(zP)^\mu$. Then $\bar{P}(z)$ is the solution of $(P + \delta)^{-\varepsilon} - \beta(zP)^\mu = 0$. The system to be analyzed is: $\dot{P}_t = r [P_t - (c_0 + c_1(R_0 - R_t))]$ and $\dot{R}_t = -(P + \delta)^{-\varepsilon} + \beta(zP)^\mu$, subject to three boundary conditions: R_0 given, $R_T = 0$, $P_T = \bar{P}(z)$. Unlike linear demand case, we cannot obtain an analytical solution.

Consider an example. Let $\varepsilon = \mu = \beta = \delta = 1$. Then \bar{P} is the solution of $(P + 1)^{-1} - zP = 0$. At $z = 1$, the fossil fuel choke price is 0.618. Assuming that $c_0 = 0.005$, $c_1 = 0.0001$ and $\bar{R} = 80$, we can solve for the current oil price $P(0) = 0.017$, and the exhaustion date

$T \simeq 106.9$ years. These cost and oil reserve parameters imply that the current extraction cost/price ratio is 29%. Until the exhaustion date, costs in this case increases slightly from 0.005 to 0.013, but the fuel price increases faster from 0.017 to 0.618, and thus the relevant cost/price ratio at the exhaustion date is just 2%.¹²

Does a small increase in z lead to an increase or a decrease in T ? The answer depends on various parameters, particularly cost sensitivity or c_1 . Our numerical simulations show that if extraction costs increase faster as the reserve depletes (a large value of c_1), then, starting from $z = 1$, a small increase in the subsidy will make oil reserves last longer (a larger T). On the other hand, if c_1 is small enough, then an increase in the subsidy may lead to earlier exhaustion. We illustrate this in Table 5 where the subsidy z is increased from 1 to 1.2 ($\Delta z = 0.2$). Table 5 illustrates it is possible to find a large range of cases where fossil fuel reserves are exhausted faster, and the Green Paradox holds.

The findings in Table 5, however, do not imply the Green Paradox is a general result. Indeed, if condition (21) holds, we can show that with linear demand and a marginal extraction costs that increases linearly with accumulated extraction, and without technological change, fossil fuel deposits are abandoned before exhaustion and a subsidy for biofuels production results in a smaller overall consumption of fossil fuels. In that case, we say that the Green Paradox does not hold in the long run. However, the long run is very far away (in fact, abandonment takes place at time infinity). Thus, even when the Green Paradox does not hold even in the long run, our results indicate that it can plausibly hold in the short or medium term.

We emphasize that the time period over which the Green Paradox holds is critically important. In terms of climate change and avoiding the severe impacts of climate change, what happens to cumulative emissions over the next 30 years interval is critical. It is this

¹²For fossil fuel producers to abandon extraction before reserves are exhausted, the subsidy must satisfy:

$$c_0 + c_1 R_0 = 0.013 > \bar{P}(z) \text{ or } z > 153.8.$$

period of time that is likely to be the important period in terms of biofuels subsidies as we might reasonably assume that new, carbon neutral energy technologies will become more widespread beyond 30 years. Thus, in the time periods that matter we have real cause for concern that biofuel subsidies may actually encourage larger GHG emissions from fossil fuel combustion and increase the likelihood of the more severe impacts of climate change.¹³

Another relevant issue is technological change that affects extraction costs and the net price path on non-renewables. Technological change would seem to be especially important in terms of stock-dependent extraction costs. Positive technological change should offset the effect of stock-dependent extraction, and may do so for a long time. The overall impact of technological change would depend on the relative changes of z over time, on the extent to which technological changes affect the extraction costs and the cost of biofuel production, and the nature of stock dependent costs.

5 Extensions

In this section, we outline two extensions of our model. The first extension concerns the possibility of Green Paradox when the fossil fuels are supplied by a cartel, while biofuel producers are perfectly competitive. The second extension deals with carbon leakage in a two-country world where only one country subsidizes biofuel production.

Cartel extraction

Assume that the exhaustible resource cartel takes the biofuel subsidy factor z as a given constant. How does an increase in z affect the extraction path? As before, the residual demand function facing the oil producer is $D(P; z) \equiv D_E(P) - S(zP)$. The choke price for fossil fuels is \bar{P} , where $D_E(\bar{P}) - S_B(z\bar{P}) = 0$, and \bar{P} is decreasing in z . For simplicity we only consider the case of constant marginal cost of extraction, c . We assume that the cartel's extraction matches the demand for fossil fuels, $q_t = D(P_t; z)$. Its optimization problem

¹³If one takes into account the possible feedback effect (such as additional emissions from the permafrost biomass that could result from increased GHGs concentrations in the medium run) the steeper path of cumulative fossil fuel extraction in the medium run can result in greater irreversible damages even if long run total extraction is lower due to abandonment of reserves. See Winter (2011) on this point.

consists of choosing a time path of price $P_t \in [0, \overline{P}]$ and a terminal date T to maximize the present value of its stream of discounted profit:

$$\max_{T, P_t} \int_0^T e^{-rt} [(P_t - c)D(P_t; z)] dt$$

subject to the constraint $\dot{R}_t = -D(P_t; z)$, where $R_T \geq 0$, R_0 given.

In this case, we obtain the following results.

Proposition 4: *Assume that the demand function for fuel and the supply function of biofuel are both linear. Then, under monopoly extraction,*

(i) *if extraction costs are zero ($c = 0$), an increase in the biofuel subsidy factor z will have no effect on the date of exhaustion of the resource stock R_0 ;*

(ii) *if extraction costs are positive ($c > 0$), an increase in the biofuel subsidy factor z will delay the date of exhaustion of the resource stock R_0 .*

Proposition 5: *Under non-linear demand, a Green Paradox can arise when fossil fuels are supplied by a cartel.*

Carbon leakages in a two country-model

An important policy issue is whether well-meaning environmental policies of advanced industrialized countries is likely to lead to a Green Paradox because emerging market economies are not willing to reduce their GHG emissions. Earlier works on this important topics include Bohm (1993) and Hoel (1994). Recent discussions of carbon leakage in the context of trade can be found in Copeland and Taylor (2005) and Ishikawa and Kiyono (2006). However, these authors did not address the relationship between non-optimal biofuel policies and carbon leakage effects in a dynamic setting, a connection which we now take up. Let us consider the case where there are two countries with different energy policies. For simplicity we do not consider game theoretic or food policy issues (Bandyopadhyay et al. 2009). Let us say the “home country” is the U.S. and the “foreign country” is China. There is no biofuel production in China. Assume that U.S. biofuels are not exported to China (e.g. because of high transport costs or other barriers to trade, or because the U.S. production subsidies are

only given to domestically earmarked consumption).

Let P_t be the world price of fossil fuels. Assume that China's inverse demand function for fuels is $P_t = A - E_t^c/M^c$ where E_t^c is the quantity demanded in China, i.e., China's demand for energy is $D_E^c(P_t) = M^c(A - P_t)$. Similarly, the US inverse demand function for fuels is $P_t = A - E_t^u/M^u$. The ratio M^c/M^u is a measure of China's market size relative to the U.S.'s market size. By an appropriate choice of units, we set $M^u = 1$ in what follows.

When the price is P_t , U.S. biofuel producers earn zP_t for each unit they sell domestically, where z is the subsidy factor. The biofuel supply function is $S_B = \beta(zP)^\mu$ and the U.S.'s residual demand for fossil fuels is $D^u(P; z) = D_E^u(P_t) - \beta(zP)^\mu = A - P - \beta(zP)^\mu$. Let \bar{P}_u be the solution of $A - \bar{P}_u - \beta(z\bar{P}_u)^\mu = 0$.

Given a positive constant subsidy factor $z > 0$, the world equilibrium consists of two phases. In the first phase, fossil fuels are consumed in both the U.S. and China. This phase ends at an endogenously determined time T^u , when P_t reaches the value \bar{P}_u . In the second phase, fossil fuels are only used in China, and U.S. energy demand is completely satisfied by biofuel production.¹⁴ The second phase ends at time T^w , when the world price of fossil fuels reaches the choke price A and China's demand for fossil fuels becomes zero.

Our task is to find out how the subsidy rate z influences the two critical times T^u and T^w , and how it influences the equilibrium price path of fossil fuels, and hence the rate of emissions of CO_2 at each point of time.

Let $D^w(t)$ be the world (US and China) demand for fossil fuels during phase I, i.e. $D^w(t) = D^u(P_t; z) + D_E^c(P_t)$, for $t < T^u$. For $t \geq T^u$, let $D_E^c(t)$ be China's demand for fossil fuels in phase II. Let R_0 be the initial stock of fossil fuels. Equilibrium requires that the total use of fossil fuels equals its stock:

$$\int_0^{T^u} D^w(t)dt + \int_{T^u}^{T^w} D_E^c(t)dt = R_0 \quad (22)$$

We can prove the following Proposition.

¹⁴In this section, we use the word biofuels to represent all other renewable energy sources.

Proposition 6: *An increase in the subsidy factor z will lengthen Phase II and may shorten Phase I. The exhaustion date will be brought closer to the present if the U.S.'s choke price is highly responsive to increases in biofuel subsidies.*

6 Concluding Remarks

We examine several possible cases under which a Green Paradox may arise from a policy of biofuel subsidies whereby the supply-side response by fossil fuel producers more than offsets any gains from direct substitution to biofuels. Whether the Green Paradox holds or not depends on demand and supply elasticities, expected changes in subsidies, technological change in fossil fuel extraction and how extraction costs respond to changes in remaining reserves. We find that a Green Paradox is not a general result, but under plausible assumptions it may exist.

A key implication of our theoretical work is the need to test empirically whether a Green Paradox exists. Equally as important, our work shows the importance of considering: (1) the inter-temporal effects of biofuel subsidies on fossil fuel production; (2) the trade effects of biofuel subsidies even for countries that neither produce nor consume biofuels; and (3) the industry structure (competitive or cartel) of fossil fuel production.

The implications of our findings on energy policy are several. First, and foremost, is the importance and necessity of analyzing the impacts of biofuels policies in a dynamic context. Second, domestic policies of large biofuel producing nations must be evaluated in a welfare context that includes the effects on both biofuel and fossil fuel consumption globally. Third, we provide an additional reason why biofuel subsidies are a sub-optimal policy and, in some cases, may even generate perverse outcomes that are contrary to the stated objectives which supposedly justify their use.

Further development of our dynamic models is necessary to take into account game-theoretic issues, the effects of technological change on extraction costs, the impacts of GHG atmospheric concentrations and the rates of decay, and the direct GHG reductions from

biofuel-fossil fuel substitution at a given price. Nevertheless, our findings are sufficiently well developed to require, at the very least, that policy makers carefully evaluate the dynamic and supply-side effects of biofuel subsidies on the extraction rate of fossil fuels.

Acknowledgement An early version of this paper was presented at the SURED conference (Ascona, 2010). The final version was completed at Ifo Institute in 2011. In their research on the Green Paradox, the authors have benefited from helpful discussions with Reyer Gerlagh, Michael Hoel, Chuck Mason, Karen Pittel, Rick van der Ploeg, Hans-Werner Sinn, Sjak Smulders, and Cees Withagen.

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APPENDIX

Proof of Proposition 3

With the linear demand $D(P_t; z) = a - (b + \beta z)P_t$, the fossil fuel choke price is $\bar{P}(z) = \frac{a}{b + \beta z}$. Define $Y_t = R_0 - R_t$. Assume that the marginal cost of extracting the last drop of oil is lower than the fossil fuel choke price:

$$R_0 < \frac{a}{c_1(b + \beta z)} - \frac{c_0}{c_1}$$

Consider the system

$$\dot{P}_t = r [P_t - (c_0 + c_1 Y_t)], \quad r > 0, c_0 > 0, c_1 > 0$$

$$\dot{Y}_t = D(P_t; z) = a - (b + \beta z)P_t, \quad a > 0, b > 0, \beta > 0, z > 0$$

subject to three boundary conditions: $Y_0 = 0$, $Y_T = R_0$, $P_T = \bar{P}(z)$.

Write the system of differential equations in matrix form:

$$\begin{bmatrix} \dot{P} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} P \\ Y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

where $a_{11} = r, a_{12} = -rc_1, a_{21} = -(b + \beta z), a_{22} = 0, b_1 = -rc_0$ and $b_2 = a$. Let $\rho_1 < 0$ and $\rho_2 > 0$ be the characteristic roots. Let (\tilde{P}, \tilde{Y}) denote the stationary point. Then the general solution is

$$\begin{bmatrix} P(t) - \tilde{P} \\ Y(t) - \tilde{Y} \end{bmatrix} = \begin{bmatrix} h_1 \\ -(a_{12})^{-1} (a_{11} - \rho_1) h_1 \end{bmatrix} \exp(\rho_1 t) + \begin{bmatrix} h_2 \\ -(a_{12})^{-1} (a_{11} - \rho_2) h_2 \end{bmatrix} \exp(\rho_2 t)$$

where h_1 and h_2 are constants (to be determined using boundary conditions).

Using the boundary conditions $P_T - \tilde{P} = 0$ and $Y_T = R_0$, we can solve for h_1 and h_2 and finally we solve for the exhaustion time T as follows:

$$\frac{(\rho_2 - \rho_1) \exp(\rho_2 T)}{-\rho_1 + \rho_2 \exp[(\rho_2 - \rho_1) T]} = \frac{\tilde{Y} - R_0}{\tilde{Y}}$$

This equation has a unique solution $T > 0$ (which depends on z). Define

$$F(T, z) = \frac{(\rho_2 - \rho_1)}{\rho_2 e^{-\rho_1 T} - \rho_1 e^{-\rho_2 T}}$$

and

$$G(T, z) \equiv F(T, z) - \left(\frac{\tilde{Y}(z) - \bar{R}}{\tilde{Y}(z)} \right)$$

To prove part (i), note that the effect on T of an increase in the subsidy z is

$$\frac{dT}{dz} = - \frac{\frac{\partial G}{\partial z}}{\frac{\partial G}{\partial T}}$$

We can show that

$$\frac{\partial G}{\partial T} = - \frac{\rho_1 \rho_2 (\rho_2 - \rho_1)^2 e^{\rho_2 T}}{\{\rho_2 e^{(\rho_2 - \rho_1) T} - \rho_1\}^2} [e^{\rho_2 T} - e^{(2\rho_2 - \rho_1) T}] < 0$$

and

$$\frac{\partial G}{\partial z} = \frac{\partial F}{\partial z} - \bar{R} [\tilde{Y}(z)]^{-2} \frac{d\tilde{Y}}{dz} > 0$$

since $\frac{\partial F}{\partial z} > 0$.

To prove part (ii), we use $\frac{dT}{dc_1} = - \left(\frac{\partial G}{\partial c_1} \right) / \left(\frac{\partial G}{\partial T} \right)$ and show that $\left(\frac{\partial G}{\partial c_1} \right) > 0$. ■

Proof of Proposition 4

Denote the elasticity of the demand for fossil fuels by $\theta_t(P_t; z)$, defined by

$$\theta_t(P_t; z) \equiv -\frac{P_t}{D} \left(\frac{\partial D}{\partial P_t} \right) > 0$$

where we assume

$$\frac{\partial \theta_t(P_t; z)}{\partial P_t} \geq 0$$

Let ψ_t denote the current-value shadow price of the stock R_t and let H_t denote the current-value Hamiltonian. Then

$$H_t = (P_t - c)D(P_t; z) - \psi_t D(P_t; z)$$

The optimality conditions for the monopolist are

$$P_t \left[1 - \frac{1}{\theta_t} \right] - c - \psi_t = 0 \Leftrightarrow \frac{\partial H_t}{\partial P_t} = 0$$

$$\dot{\psi}_t = r\psi_t$$

$$\psi_T \geq 0, \quad R_T \geq 0, \quad \psi_T R_T = 0$$

$$H_T = [P_T - c - \psi_T] D(P_T; z) = 0$$

One can show that conditions imply that $\psi_T = P_T - c > 0$, $R_T = 0$, $P_T = \bar{P}$, $D(P_T; z) = 0$, and $\theta_T = \infty$. In particular, we obtain the Hotelling Rule for the monopolist: the present value of marginal profit is the same for all $t \in [0, T]$:

$$\left[\left(1 - \frac{1}{\theta_t(P_t; z)} \right) P_t - c \right] = (\bar{P} - c) e^{r(t-T)} \quad (23)$$

Equation (23) implicitly defines the monopolist's optimal price P_t^m as a function of \bar{P} , z , t and T :

$$P_t^m = \phi^m(\bar{P}(z), t, T; z) \quad (24)$$

We make use of (24) to determine the monopolist's optimal exhaustion time T . It is the value of T such that total accumulated extraction equals the initial reserve level R_0 .

$$\int_0^T D_E(\phi^m(\bar{P}(z), t, T; z); z) dt = R_0 \quad (25)$$

The following result follows immediately:

Result 1 *An increase in the subsidy factor z will bring the monopolist's resource-exhaustion date T closer to the present if and only if the indirect effect outweighs the direct effect, i.e.,*

$$\int_0^T \left[\frac{\partial D}{\partial P_t} \left(\frac{\partial P_t}{\partial z} \right) + \frac{\partial D}{\partial z} \right] dt > 0 \quad (26)$$

where

$$\frac{\partial P_t}{\partial z} \equiv \frac{\partial \phi^m}{\partial \bar{P}} \frac{d\bar{P}}{dz} + \frac{\partial \phi^m}{\partial z}.$$

We now turn to the case of linear demand:

Let $D_E(P) = a - bP$ and $S = \beta zP - g$, where $a > 0$, $b > 0$, $\beta > 0$ and $g \geq 0$. Then

$$\phi^m(\bar{P}(z), t, T; z) = \frac{1}{2} \left[(\bar{P} - c) e^{r(t-T)} + c + \frac{a}{b+z} \right]$$

and

$$D_P \left(\frac{\partial \phi^m}{\partial \bar{P}} \frac{d\bar{P}}{dz} + \frac{\partial \phi^m}{\partial z} \right) + D_z = -\frac{c}{2} (1 - e^{-r(T-t)}) \leq 0$$

Hence (i) if extraction costs are zero ($c = 0$), an increase in the biofuel subsidy factor z will have no effect on the date of exhaustion of the resource stock R_0 and (ii) if extraction costs are positive ($c > 0$), an increase in the biofuel subsidy factor z will delay the date of exhaustion of the resource stock R_0 ■

Proof of Proposition 5: Assume the functional forms $S(zP) = \beta(zP)^\mu$ where $\beta > 0$ and $\mu > 0$ and $D_E(P) = (P + \delta)^{-\varepsilon}$ where $\delta \geq 0$ and $\varepsilon > 0$. Let $\mu = 1$. Then $D = (P + \delta)^{-\varepsilon} - \beta zP$, and the elasticity of demand for fossil fuels is

$$\theta_t = \frac{[\varepsilon(P_t + \delta)^{-\varepsilon-1} + \beta z] P_t}{(P_t + \delta)^{-\varepsilon} - \beta z P_t} > 0 \text{ for } P_t < \bar{P}$$

Note that if $\beta z > 0$ and $\varepsilon \geq 1$ then $\theta_t > 1$ because

$$\theta_t > \frac{\varepsilon P_t}{P_t + \delta} \geq \varepsilon$$

Equation (23) becomes

$$\left\{ P_t - \frac{1}{\theta_t} P_t - c \right\} - (\bar{P} - c) e^{r(t-T)} = 0 \quad (27)$$

Denote the right-hand side of equation (27) by $F(P_t, \bar{P}, t, T, z)$. Equation (27) yields the implicit function $P_t = \phi^m(\bar{P}(z), t, T; z)$. Then

$$\frac{\partial \phi^m}{\partial \bar{P}} = -\frac{\partial F / \partial \bar{P}}{\partial F / \partial P_t} = \frac{e^{r(t-T)}}{\left(1 - \frac{1}{\theta_t}\right) - P_t \frac{d}{dP_t} \left(\frac{1}{\theta_t}\right)} \equiv \frac{e^{r(t-T)}}{\Omega_t} > 0$$

where $\Omega_t > 0$. Furthermore,

$$\frac{\partial \phi^m}{\partial z} = -\frac{\partial F / \partial z}{\partial F / \partial P_t}$$

where

$$\partial F / \partial z = -\frac{P_t}{\theta_t^2} \left(\frac{\partial \theta_t}{\partial z} \right) < 0$$

and

$$\frac{\partial \theta_t}{\partial z} = \frac{\beta P_t}{(D)^2} [D + P_t \varepsilon (P + \delta)^{-\varepsilon-1} + z P_t] > 0$$

(a higher biofuel subsidy increases the elasticity of demand for fossil fuels at any given price).

Consequently,

$$\frac{\partial \phi^m}{\partial z} = -\frac{\frac{\beta P_t^2}{(\theta_t D)^2} [D + P_t \varepsilon (P + \delta)^{-\varepsilon-1} + z P_t]}{\Omega_t} < 0$$

We can now compute the crucial expression in Result 1:

$$\begin{aligned} & \frac{\partial D_f}{\partial P_t} \left(\frac{\partial \phi^m}{\partial \bar{P}} \frac{d\bar{P}}{dz} + \frac{\partial \phi^m}{\partial z} \right) + \frac{\partial D}{\partial z} \\ &= \frac{\varepsilon (P_t + \delta)^{-\varepsilon-1} + \beta z}{\Omega_t} \left(\frac{\bar{P} \beta e^{r(t-T)}}{\varepsilon (\bar{P} + \delta)^{-\varepsilon-1} + \beta z} + \frac{\beta P_t^2}{(\theta_t D)^2} [D + P_t \varepsilon (P + \delta)^{-\varepsilon-1} + z P_t] \right) - \beta P_t \end{aligned}$$

Using this expression, the integral (26) can be positive or negative, depending on parameter values ■

Proof of Proposition 6

When the price is P_t , U.S. biofuel producers earn zP_t for each unit they sell domestically, where z is the subsidy factor. The biofuel supply function is $S_B = \beta (zP)^\mu$ and the U.S.'s residual demand for fossil fuels is

$$D^u(P; z) = D_E^u(P_t) - \beta (zP)^\mu = A - P - \beta (zP)^\mu \text{ since } M^u = 1.$$

Let \bar{P}_u be the solution of

$$A - \bar{P}_u - \beta (z\bar{P}_u)^\mu = 0 \quad (28)$$

Note that

$$\frac{d\bar{P}_u}{dz} = -\frac{z^{\mu-1} (\bar{P}_u)^\mu}{\frac{1}{\beta\mu} + z^\mu (\bar{P}_u)^{\mu-1}} < 0 \quad (29)$$

Given a positive constant subsidy factor $z > 0$, the world equilibrium consists of two phases. In the first phase, fossil fuels are consumed in both the U.S. and China. This phase ends at an endogenously determined time T^u , when P_t reaches the value \bar{P}_u . In the second phase, fossil fuels are only used in China, and U.S. energy demand is completely satisfied by biofuel production. The second phase ends at time T^w , when the world price of fossil fuels reaches the choke price A and China's demand for fossil fuels becomes zero.

Our task is to find out how the subsidy rate z influences the two critical times T^u and T^w , and how it influences the equilibrium price path of fossil fuels, and hence the rate of emissions of CO_2 at each point of time.

Let $D^w(t)$ be the world (US and China) demand for fossil fuels during phase I, i.e. $D^w(t) = D^u(P_t; z) + D_E^c(P_t)$, for $t < T^u$. For $t \geq T^u$, let $D_E^c(t)$ be China's demand for fossil fuels in phase II. Let R_0 be the initial stock of fossil fuels. Equilibrium requires that the total use of fossil fuels equals its stock:

$$\int_0^{T^u} D^w(t)dt + \int_{T^u}^{T^w} D_E^c(t)dt = R_0 \quad (30)$$

We begin by evaluating the second integral on the left-hand side of (30). Assume zero extraction cost and perfect competition. Hotelling's Rule gives us,

$$P_t e^{-rt} = P(T_u) e^{-rT^u} = P(T^w) e^{-rT^w}$$

Given that $P(T^w) = A$ and $P(T_u) = \bar{P}_u$,

$$T^w - T^u = \frac{1}{r} (\ln A - \ln \bar{P}_u(z)) \equiv J(z)$$

This indicates that the length of Phase II is an increasing function of the subsidy factor z .

$$\frac{dJ}{dz} = -\frac{1}{r\bar{P}_u} \left(\frac{d\bar{P}_u}{dz} \right) > 0$$

China's oil consumption during Phase II is

$$\begin{aligned}
\int_{T^u}^{T^w} D_E^c(P_t) dt &= M^c \int_{T^u}^{T^w} (A - P_t) dt = M^c \int_{T^u}^{T^w} (A - A^{-r(T^w-t)}) dt \\
&= AM^c \left\{ T^w - T^u - \frac{1}{r} [1 - e^{-r(T^w-T^u)}] \right\} \\
&= AM^c \left\{ J(z) - \frac{1}{r} [1 - e^{-rJ(z)}] \right\} \equiv K(z)
\end{aligned}$$

It is easy to verify that $K(z)$ is an increasing function: the higher is the biofuel subsidy factor in the U.S., the greater is China's total fossil fuel consumption in Phase II:

$$\frac{dK}{dz} = AM^c(1 - e^{-rJ(z)}) \frac{dJ}{dz} > 0. \quad (31)$$

The inequality (31) is our first result concerning "carbon leakage" in a two-country world.

Let us turn to Phase I, in which fossil demands are positive in both countries. The total accumulated consumption of fossil fuels in Phase I must equal $\bar{R} - K(z)$:

$$\int_0^{T^u} [D^u(P_t; z) + D_E^c(P_t)] dt = \bar{R} - K(z)$$

where

$$P_t = \bar{P}_u e^{-r(T^u-t)} \equiv \phi(\bar{P}_u(z), t, T^u)$$

Define

$$G(T^u, z, \bar{R}) \equiv \int_0^{T^u} [D^u(\phi; z) + D_E^c(\phi)] dt - \bar{R} + K(z)$$

The effect of an increase in the subsidy factor z on the time T^u at which the U.S.'s demand for fossil fuels vanishes is given by

$$\frac{dT^u}{dz} = -\frac{\partial G / \partial z}{\partial G / \partial T^u}.$$

To determine the sign of this expression, we note that

$$\partial G / \partial T^u = D_E^c(\bar{P}_u) + \int_0^{T^u} \left[\frac{\partial D^u}{\partial P_t} + \frac{\partial D_E^c}{\partial P_t} \right] \frac{\partial \phi}{\partial T^u} dt > 0$$

and

$$\partial G/\partial z = \frac{dK}{dz} + \int_0^{T^u} \left[\left(\frac{\partial D^u}{\partial P_t} + \frac{\partial D_E^c}{\partial P_t} \right) \frac{\partial \phi}{\partial \bar{P}_u} \frac{d\bar{P}_u}{dz} + \frac{\partial D^u}{\partial z} \right] dt \quad (32)$$

Using Hotelling's Rule, the integrand in (32) can be simplified to

$$z^{\mu-1} \mu \beta e^{r(t-T)} \left[\frac{M_c + \mu \beta z^\mu \left[(P_t)^{\mu-1} - (\bar{P}_u)^{\mu-1} \right]}{1 + \mu \beta z^\mu (\bar{P}_u)^{\mu-1}} \right], \quad t \leq T^u$$

which is positive if M_c is big. Thus, if China's demand is significant enough, an increase in biofuel subsidy in the U.S. will lead to an earlier beginning of Phase II.

What about the effect of an increase in z on the date where all fossil fuel stocks are exhausted, T^w ? We must compute

$$\frac{dT^w}{dz} = \frac{dT^u}{dz} + \frac{dJ}{dz}$$

This expression is negative if and only if

$$(\partial G/\partial T^u) \frac{dJ}{dz} < \frac{dK}{dz} + \int_0^{T^u} \left[\left(\frac{\partial D^u}{\partial P_t} + \frac{\partial D_E^c}{\partial P_t} \right) \frac{\partial \phi}{\partial \bar{P}_u} \frac{d\bar{P}_u}{dz} + \frac{\partial D^u}{\partial z} \right] dt$$

i.e. if and only if

$$- \int_0^{T^u} \left[\left(\frac{\partial D^u}{\partial P_t} + \frac{\partial D_E^c}{\partial P_t} \right) \frac{\partial \phi}{\partial \bar{P}_u} \frac{d\bar{P}_u}{dz} + \frac{\partial D^u}{\partial z} \right] dt + \frac{\partial D^u}{\partial z} dt - \left[\frac{dJ}{dz} \right] [AM^c(1 - e^{-rJ(z)}) - (\partial G/\partial T^u)] < 0$$

We conclude that if the U.S.'s choke price is highly responsive to increases in biofuel subsidies, i.e. if $\frac{d\bar{P}_u}{dz}$ is sufficiently large in absolute value, then an increase in biofuel subsidy will bring the resource exhaustion date closer to the present ■

Elaboration on the Proof of Proposition 3

Consider the system

$$\begin{bmatrix} \dot{P} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} P \\ Y \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

where $a_{11} = r, a_{12} = -rc_1, a_{21} = -(b + \beta z), a_{22} = 0, b_1 = -rc_0$ and $b_2 = a$. In simpler notation,

$$\dot{\mathbf{w}} = \mathbf{A}\mathbf{w} + \mathbf{b}$$

where $(w_1, w_2) \equiv (P, Y)$, and $\det \mathbf{A} = a_{11}a_{22} - a_{12}a_{21} = -rc_1(b + \beta z) < 0$. Let $\tilde{\mathbf{w}}$ be the stationary point,

$$\tilde{\mathbf{w}} = -\mathbf{A}^{-1}\mathbf{b}$$

Then

$$\tilde{\mathbf{w}} = \begin{bmatrix} \frac{a}{b+\beta z} \\ \frac{a}{(b+\beta z)c_1} - \frac{c_0}{c_1} \end{bmatrix} \equiv \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \end{bmatrix}$$

Define \mathbf{x} as the deviation of \mathbf{w} from $\tilde{\mathbf{w}}$:

$$\mathbf{x} \equiv \mathbf{w} - \tilde{\mathbf{w}}$$

Let $\rho_1 < 0$ and $\rho_2 > 0$ be the characteristic roots,

$$\begin{aligned} \rho_{1,2} &= \frac{a_{11} + a_{22} \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2} \\ &= \frac{1}{2} \left(r \pm \sqrt{\Delta} \right) \quad \text{where } \Delta \equiv r^2 + 4rc_1(b + \beta z) \end{aligned}$$

Then the general solution is

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} h_1 \\ -(a_{12})^{-1}(a_{11} - \rho_1)h_1 \end{bmatrix} \exp(\rho_1 t) + \begin{bmatrix} h_2 \\ -(a_{12})^{-1}(a_{11} - \rho_2)h_2 \end{bmatrix} \exp(\rho_2 t) \quad (33)$$

where h_1 and h_2 are constants (to be determined using boundary conditions).

Define

$$v_1 \equiv -(a_{12})^{-1}(a_{11} - \rho_1) = \frac{\rho_1 - r}{-rc_1} \quad (34)$$

$$v_2 \equiv -(a_{12})^{-1}(a_{11} - \rho_2) = \frac{\rho_2 - r}{-rc_1} \quad (35)$$

Setting $t = T$ in the matrix equation (33), and noting that have $x_1(T) = P_T - \tilde{w}_1 = 0$ and $x_2(T) = Y_T - \tilde{w}_2 = \bar{R} - \tilde{w}_2 \equiv \bar{R} - \tilde{Y}(z)$, we get two equations:

$$0 = h_1 \exp(\rho_1 T) + h_2 \exp(\rho_2 T) \quad (36)$$

$$\bar{R} - \tilde{Y}(z) = v_1 h_1 \exp(\rho_1 T) + v_2 h_2 \exp(\rho_2 T) \quad (37)$$

Equation (36) gives

$$h_1 = -h_2 \exp[(\rho_2 - \rho_1)T] \quad (38)$$

Substituting into equation (37) to get

$$\bar{R} - \tilde{Y}(z) = -v_1 h_2 \exp(\rho_2 T) + v_2 h_2 \exp(\rho_2 T)$$

Therefore

$$h_2 = \frac{\bar{R} - \tilde{Y}(z)}{(v_2 - v_1) \exp(\rho_2 T)} \quad (39)$$

Setting $t = 0$ in the matrix equation (33), we get two equations

$$P_0 - \bar{P}(z) = h_1 + h_2 \quad (40)$$

$$Y_0 - \tilde{Y}(z) = v_1 h_1 + v_2 h_2 \quad (41)$$

Since $Y_0 = 0$, substituting (38) and (39) into (41) we get

$$-\tilde{Y}(z) = \frac{\bar{R} - \tilde{Y}(z)}{(v_2 - v_1) \exp(\rho_2 T)} [v_2 - v_1 \exp[(\rho_2 - \rho_1) T]]$$

or

$$\frac{(v_2 - v_1) \exp(\rho_2 T)}{v_2 - v_1 \exp[(\rho_2 - \rho_1) T]} = \frac{\tilde{Y}(z) - \bar{R}}{\tilde{Y}(z)} \quad (42)$$

Now, using (34), (35) and (42)

$$\frac{(\rho_2 - \rho_1) \exp(\rho_2 T)}{-\rho_1 + \rho_2 \exp[(\rho_2 - \rho_1) T]} = \frac{\tilde{Y}(z) - \bar{R}}{\tilde{Y}(z)}$$

Let

$$\phi(T, z) \equiv \frac{(\rho_2 - \rho_1)}{-\rho_1 \exp[-\rho_2 T] + \rho_2 \exp[-\rho_1 T]}$$

The function $\phi(T, z)$ has the following properties: $\phi(0, z) = 1$, $\partial\phi(T, z)/\partial T < 0$ for all $T > 0$,

and

$$\lim_{T \rightarrow \infty} \phi(T, z) = 0$$

It follows that the equation

$$\phi(T, z) = \frac{\tilde{Y}(z) - \bar{R}}{\tilde{Y}(z)}$$

has a unique solution $T > 0$ (which depends on z) ■

T	50	75	100	150	200
$\frac{dT}{dz}$	-1.75	-2.04	-2.09	-2.110	-2.113

Table 1: Effect of a Biofuel Subsidy on Exhaustion Date for Different Fossil Fuel Reserve Sizes

z	0.5	0.75	1	1.5	2
$\frac{dT}{dz}$	-6.6	-3.43	-2.09	-0.99	-0.56

Table 2: Effect of a Biofuel Subsidy on Exhaustion Date for Different Initial Subsidy Levels

ε	0.1	0.5	1	2	3
$\frac{dT}{dz}$	-0.35	-1.34	-2.09	-2.89	-3.32

Table 3: Effect of a Biofuel Subsidy on Exhaustion Date for Different Demand Elasticities of the Manufactured Good

μ	0.5	0.75	1	1.5	2
$\frac{dT}{dz}$	1.45	-2.07	-2.09	-3.88	-3.75

Table 4: Effect of a Biofuel Subsidy on Exhaustion Date for Different Supply Elasticities of Biofuel

Cost sensitivity parameter	Marginal extraction cost of the 'last drop'	Choke price of fuel when $z=1.2/z=1$	ΔT	$\Delta T/\Delta z$	Paradox/ No Paradox
0	0.005	0.54/0.618	-0.270	-1.35	Paradox
0.0001	0.013	0.54/0.618	-0.176	-0.88	Paradox
0.0002	0.021	0.54/0.618	-0.077	-0.39	Paradox
0.0003	0.029	0.54/0.618	0.025	+0.13	No Paradox
0.0005	0.045	0.54/0.618	0.244	+1.22	No Paradox

Table 5: Parameter Values for the Weak Green Paradox

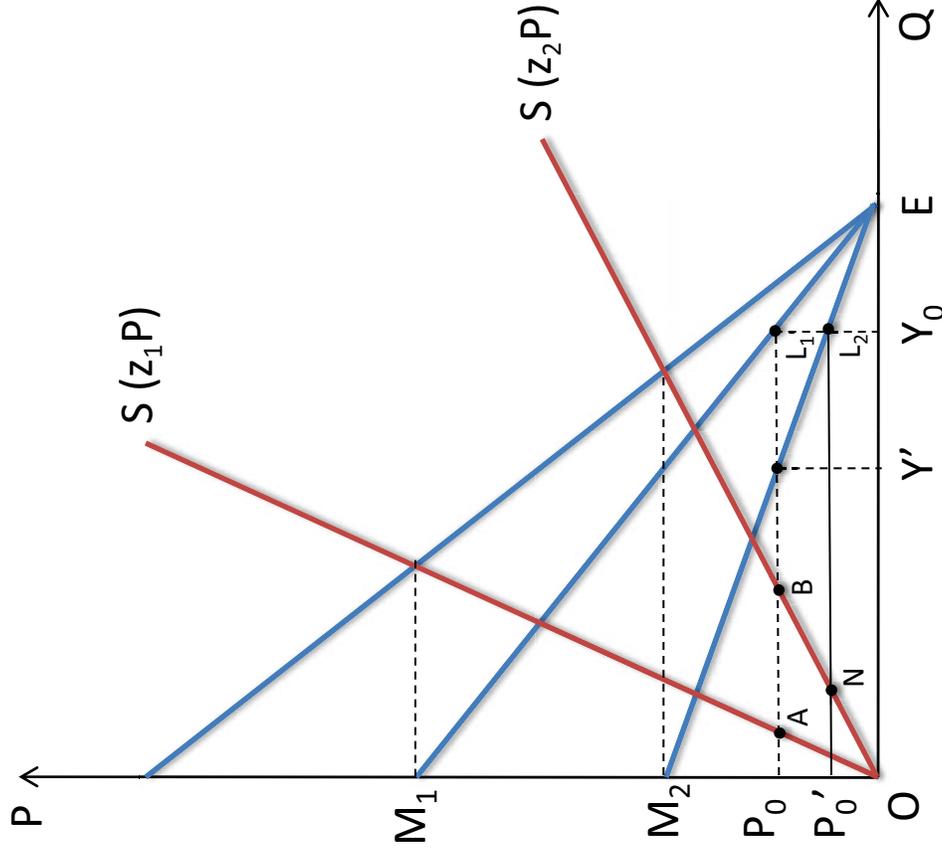


Figure 1. Direct and indirect effects of a biofuel subsidy

Figure 2 – Special case: no impact on exhaustion time and no impact on extraction path

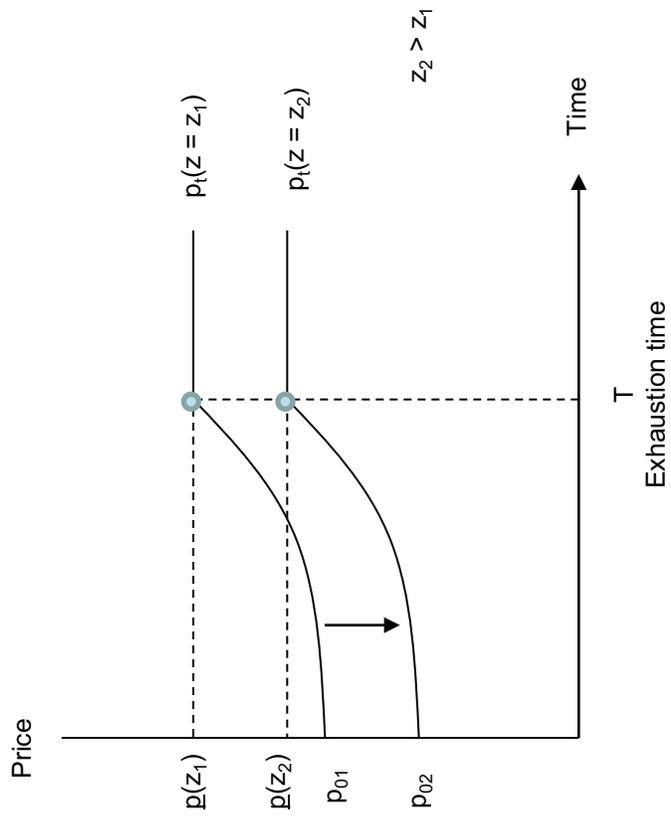


Figure 3 – Green paradox: higher biofuel subsidy causes faster exhaustion

