

2004s-11

Ratchet vs Blasé Investors and Asset Markets

Pascal St-Amour

**Série Scientifique
Scientific Series**

**Montréal
Mars 2004**

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Ratchet vs Blasé Investors and Asset Markets

Pascal St-Amour *

Résumé / Abstract

Cet article propose une nouvelle fonction d'utilité contingente à la richesse pour le problème de la consommation et de portefeuille inter-temporels, où le niveau de subsistance (félicité) de la consommation dépend de la richesse. Des effets de rochet sont obtenus lorsque la richesse accroît le niveau de félicité, des effets blasés lorsqu'elle le réduit. Nous présentons trois contributions : (i) nous identifions des formes réduites pour la consommation et les portefeuilles optimaux; (ii) nous utilisons ces règles pour estimer le modèle à partir de la consommation et des positions financières agrégées et (iii) nous dérivons et discutons des implications de valorisation des actifs. Nos résultats indiquent la présence d'effet blasé et d'aversion contra-cyclique.

Mots clés : choix de portefeuille, préférences contingentes à la richesse, préférence pour le statut social, valorisation des actifs, prime de risque, taux sans risque, prévisibilité.

This paper proposes a new wealth-dependent utility function for the inter-temporal consumption and portfolio problem, in which the subsistence (bliss) consumption level is a function of wealth. Ratchet effects obtain when higher wealth increases the subsistence consumption level; blasé behavior occurs when higher wealth reduces it. We have three contributions: (i) we identify closed-form solutions for optimal consumption and portfolio rules; (ii) we use the optimal rules to estimate the model using aggregate portfolio data, and (iii) we derive and discuss the pricing implications of our results. Our estimates are consistent with blasé behavior and counter-cyclical risk aversion.

Keywords: portfolio choice, wealth-dependent preferences, preference for status, asset pricing, equity premium, risk-free rate, predictability.

Codes JEL : G11, G12

* Department of Finance, HEC Montreal, CIRPEE and CIRANO, email: Pascal.St-Amour@hec.ca.

1 Introduction

1.1 Motivation and outline

Recent research on preference-based explanations of asset market dynamics focuses on the minimum admissible (also known as subsistence, reference or bliss) consumption level. Standard representations of preferences such as the widely-used Constant Relative Risk Aversion (CRRA) utility, restrict the minimum consumption level to zero. Hyperbolic Absolute Risk Aversion (HARA) utility is more general in allowing for a constant non-zero bliss level. Conversely, the more recent habit literature allows for bliss to be determined by conditional state variables, such as lagged idiosyncratic or aggregate consumption.

Focusing on minimum admissible consumption instead of curvature or impatience indices is sensible. Indeed, bliss and attitudes toward risk are closely related. The closer consumption is to bliss, the steeper is the marginal utility schedule. This implies that small movements in consumption cause larger fluctuations in marginal utility. To the extent that the agent selects his portfolio so as to hedge away these risks, in HARA utility, the distance between consumption and bliss directly determines risk aversion and portfolio. Moreover, the CRRA restriction of zero bliss is not innocuous. Under the more general HARA utility, non-zero bliss implies that both consumption and asset holding schedules have a state-independent non-zero intercept. This is important to the extent that it allows consumption and portfolio *shares* of wealth to be time-varying, a feature that is also found in the data (see Figure 3 in Appendix E).

Furthermore, allowing for time-varying instead of constant bliss is intuitively appealing. In both CRRA and HARA, this level is taken as a deep parameter. This restriction appears excessive. Subsistence may be interpreted as a subjective, as well as physiological, measure. It seems more realistic to allow for our basic needs to evolve with age, habits or wealth levels. What is considered basic minimum when young or poor need not be the same at an older age or when richer. If this is the case, bliss consumption should be allowed to evolve with time and/or economic conditions.

Unfortunately, the empirical gains of habit preferences has been mitigated. On one hand, the higher savings rate required to maintain habits helps to understand the low observed returns on risk-less assets. However, ultimately, the only source of inter-temporal marginal rates of substitution (IMRS) risk remains consumption. This risk is the only determinant of excess returns in preference-based models of asset returns. Since aggregate consumption is a despairingly smooth series, it is weakly correlated with returns. The quantity of consumption risk is consequently too low to justify the high observed premia on risky assets.

These elements suggest that bliss consumption level should be (i) nonzero, and (ii) state-dependent. Since consumption was found to be inadequate, a natural alternative as a determinant of bliss is wealth. Wealth, as primary state variable, is a good proxy for economic conditions. Furthermore, direct preference for wealth can be explained through a preference-for-status, ‘*capitalistic spirit*’, argument. We therefore introduce a wealth-dependent utility (WDU) function in which the Bernoulli concave transform is applied to an affine function of consumption and wealth. This utility simplifies to a HARA class under wealth independence, otherwise bliss is a linear function of wealth. Our application focuses on the inter-temporal consumption–portfolio problem. We have three contributions: (i) we identify closed-form solutions for optimal consumption and portfolio rules; (ii) we use the optimal rules to estimate the model using aggregate portfolio data, and (iii) we derive and discuss the pricing implications of our results.

When wealth enters negatively in the utility, increases in wealth cause clockwise rotations in the marginal utility (MU) schedule. Higher wealth causes a *ratchet* effect whereby the bliss level increases. Since bliss is the minimum admissible consumption level, this also causes an increase in consumption risk aversion, i.e. risk aversion is pro-cyclical. When wealth enters positively in the utility, increases in wealth cause a counter-clockwise rotation in the MU schedule. A higher level of wealth causes a *blasé* effect whereby the same level of consumption is valued less by the wealthier investor. Since the slope of the MU schedule falls, this implies that consumption risk aversion is counter-cyclical.¹

Contrary to habit models, bliss is only indirectly related to past consumption. Movements in the reference point can be caused by factors that are independent of the agent's decisions, such as the realizations of individual returns that compose the total wealth portfolio. Therefore, the agent has only partial control over movements in bliss through his savings and portfolio decisions. We show that this will have important consequences for the MU risk, and therefore the optimal rules and pricing implications.

Our first main contribution is that we derive closed-form expressions for optimal consumption and portfolio rules. We find that the value function remains iso-morphic to the instantaneous utility. Consequently, both consumption, and the value invested in assets are affine functions of wealth. This implies that neither the average propensity to consume, nor the portfolio shares are constant, but move depending on the wealth level. This result is useful to the extent that it predicts cyclical movements in the value invested in assets relative to one another. Moreover, time variation in the consumption-wealth ratio accords with the findings of Lettau and Ludvigson (2001a,b) that this variable is counter-cyclical, and has predictive power over returns.

Secondly, estimation of the model focuses on the closed-form consumption and portfolio. More precisely, we estimate a multivariate system composed of instantaneous changes in consumption, asset holdings, wealth, and asset returns. The first three elements incorporate the full theoretical restrictions, both on the conditional first, and conditional second moments. Asset returns are unrestricted and included to correct inference for the uncertainty regarding the distributional parameters used in the closed-form expressions. We innovate from standard approaches which typically treat consumption growth as exogenous and estimate the model applying the theoretical restrictions on returns instead.

The resulting empirical model is a multivariate Brownian motion that presents estimation challenges as both drifts and diffusions are affine functions of the state variable. These functions do not admit closed-form expressions for the transition density. We therefore resort to a homoscedasticity-inducing transformation which *also* stationarizes the drift term. The transformed model can consequently be estimated using a discrete time differencing approach without inducing any time discretization biases.

We estimate the model by maximum likelihood for three utility functions: CRRA, HARA, and WDU. Our first step is to test the theoretical restrictions that guarantee monotonicity. We next discuss the estimated parameters, inference, and derived variables of interest, followed by formal specifications tests. Under a suitable identification strategy, we find that our estimates are (i) theoretically acceptable, (ii) intuitively realistic. In particular, both the curvature parameter and risk aversion index are within reasonable bounds when the subjective discount rate is calibrated to a realistic value. Our results are also indicative of a *blasé* behavior in portfolio choices. Moreover, risk aversion is found to be counter-cyclical, increasing in downturns and falling during recoveries. This

¹A positive effect of wealth can also be related to durability. Since total wealth includes durable goods, positive cross effects on nondurables consumption utility would be captured by our model.

result is consistent with findings in returns space that allow for time-varying risk aversion (Gordon and St-Amour, 2000; Melino and Yi, in press; Gordon and St-Amour, in press). Finally we find that the null of CRRA preferences is strongly rejected when tested against HARA or WDU utility. However, we do not reject the null of HARA when the alternative is taken to be WDU.

Our third contribution is a derivation of the corresponding expressions for the assets' returns. Our model generates a linear multi-factor premia in which both consumption, and total wealth (i.e. market) risks are theoretically valued by investors. Moreover, the price of consumption risk remains the Arrow-Pratt coefficient of relative consumption risk aversion, whereas the price of market risk has the intuitive interpretation of being the Arrow-Pratt coefficient, evaluated over relative *wealth* risk aversion. Hence, our pricing kernel can be interpreted as a weighted average between a standard, wealth-independent C-CAPM, and a static CAPM, where the weights are given by the relative importance of consumption, versus wealth risk aversion. Two-factor pricing kernels can also be obtained using non-expected utility. In contrast, our approach is derived under Von Neumann–Morgenstern (VNM) preferences; a test that the market risk is valued simplifies to a test of wealth dependence, rather than a joint test of state- and time- non-separability.

Our framework has the potential to explain the three main empirical anomalies of the C-CAPM. First, the additional risk contributed by covariances of individual returns with total wealth is likely to help explain the high observed premia on risky assets. Secondly, since both consumption and wealth risk aversion follow cyclical movements, this model can address the predictability found in excess returns. Finally, we show that our model may explain the low rate of return on a risk-free asset through the effect on the mean and variance of the inter-temporal marginal rate of substitution.

The rest of this paper is organized as follows. After discussing the relevant literature in Section 1.2, we outline the model in Section 2 and the closed-form solutions. Next, we introduce the empirical methods in Section 3, and present the estimation results in Section 4. We discuss the pricing implications of these results in Section 5, before concluding in Section 6. All proofs and most figures are regrouped in the Appendix.

1.2 Relevant literature

Preferences Our modelling approach for preferences can be related to the literature on state-dependent preferences. These preferences assume that the agent's within-period utility $U_t = U(C_t, Z_t)$ is a function of consumption C_t , as well as one (or many) state variable(s) Z_t . These variables are usually restricted to be *conditional* states, i.e. they do not belong to the control set at the time of the decision. The models essentially differ in (i) their choice of the state Z_t , and (ii) the functional form for $U(\cdot, \cdot)$. Table 1 describes a sample of state-dependent models.

Sundaresan (1989) as well as Ferson and Constantinides (1991) are both early examples of habit models. Specifically, the time-varying bliss factor $-\eta_t$ is a function of past consumption profiles which represent the state. If $\eta'_t < 0$, then high current consumption imply larger future bliss levels. Ferson and Constantinides (1991) find that habit preferences are able to explain the low risk-free rate, but not the high equity premium. Similarly, Campbell and Cochrane (1999) allow for bliss to be related to consumption. However, they restrict consumption to be *aggregate* consumption profile \bar{C}_t , i.e. both a conditional and an unconditional state variable. Similar to Ferson and Constantinides (1991), they find that this slow-moving habit model is unable to explain the equity premium puzzle but can successfully address the low risk-free rate.

Table 1: State-Dependent Preferences

Authors	$U_t = U(C_t, Z_t)$
Sundaresan (1989), Ferson and Constantinides (1991)	$[C_t + \eta(C_{t-1})]^{1-\gamma}$
Campbell and Cochrane (1999)	$[C_t + \eta(\bar{C}_t)]^{1-\gamma}$
Bakshi and Chen (1996), Gong and Zou (2002)	$C_t^{1-\gamma} g(W_t, \bar{W}_t)$
Falato (2003)	$[h(C_t, W_t)]^{1-\gamma}$
Barberis et al. (2001)	$C_t^{1-\gamma} [1 + v(X_{t+1}, S_t, v_t)/C_t]$
Gordon and St-Amour (2000, in press), Danthine et al. (2002)	$C_t^{1-\gamma(Z_t)}$
St-Amour (2004)	$U_t = [C_t + \eta(W_t)]^{1-\gamma}.$

The intuition for these mitigated results is straightforward. A habit investor needs to save more in order to maintain his future habits. Consequently, he is willing to pay a higher price for the risk-less asset. However, the only source of MU risk for habit preferences remains consumption. Since this series is quite smooth, its covariance with returns is too low to justify the high premia, unless risk aversion is excessive (see also Barberis et al., 2001, for a discussion).

Our model is similar in terms of functional form, but differs in the choice of state variable. We also focus on the bliss point but instead relate the state to the investor's total wealth. Because consumption risk is minimal, we resort to total wealth risk. This is achieved by introducing wealth as the preference state variable. If this second source of risk is large, and if this risk is positively valued by the market, then its presence should help resolving the high equity premium puzzle. By keeping the habit perspective, we hope to be able to preserve the model's favorable results with respect to the predicted risk-free rate.

Recent research also explores the implications of wealth-dependent utility for asset prices. Bakshi and Chen (1996) incorporate direct preference for wealth to the standard consumption utility. They rationalize wealth dependency through a preference-for-status argument put forward by Robson (1992). Agents care about their social position, relative to their reference group. As a result, the pricing kernel incorporates both consumption and total wealth risk. They show that this increment in MU risk can successfully address the main pricing anomalies. This is confirmed by Gong and Zou (2002) who extend the analysis to a stochastic growth setting. They find that preference for wealth results in a higher premia, more investment in risky assets, a lower consumption-wealth ratio and consequently a larger growth rate. Similarly, Falato (2003) introduces direct preference for wealth in the utility function. Imposing a pro-cyclical effect of wealth ('happiness maintenance'), he derives the pricing implications, and shows that mild pro-cyclicality generates a larger volatility in the price-dividends ratio and consequently, a higher premia.

Our approach is qualitatively similar. In our discussion of results in Section 5, we also derive pricing implications and closed-form solutions. However, our specification for within-period utility bears closer resemblance to the habit literature. Our focus is on the role of wealth in determining the bliss factor, rather than using a multiplicative specification. As already mentioned, an advantage of our approach is on the intuitive interpretation of the price of total wealth risk as the Arrow-Pratt level of relative risk aversion. Moreover, contrary to Bakshi and Chen (1996) and Gong and Zou (2002), our structure explicitly generates time-varying consumption risk aversion. Movements in bliss cause rotations, rather than shifts, in the MU schedule and corresponding movements in

risk aversion. Finally, contrary to Falato (2003), we do not impose any *ex-ante* pro- or counter-cyclical on attitudes toward risk, but let this issue be determined by the data.

Barberis et al. (2001) consider a model where utility is subject to (i) loss aversion, and (ii) wealth dependence. Their framework uses the prospect theory of Kahneman and Tversky (1979) to allow for an asymmetric utility effect from losses and gains, whether current (X_{t+1}), or past (z_t) in addition of consumption utility. Moreover, the level of *risky financial* wealth S_t affects consumption utility. They find an additional financial risk in the pricing equation which reduces the emphasis placed upon consumption risk, and allows them to solve the main pricing anomalies.

Their framework raises a number of questions. First, it seems debatable that only financial wealth should affect utility. As is well known, financial wealth, despite recent increases, remains a relatively modest share of net worth (Canner et al., 1997; Heaton and Lucas, 2000a). Recent research by Heaton and Lucas (2000a,b), as well as by Jagannathan and Wang (1996) shows that other assets, such as residential assets, proprietary entrepreneurial assets, and human capital are important in terms of shares of total wealth, and asset pricing impacts. Second, their model requires a series of auxiliary assumptions to be operational. In particular, four additional parameters for which theory offers little guidance need to be calibrated. These parameters govern the relative importance of the loss aversion versus wealth dependency, as well as the derivation of the benchmark used to evaluate performance. Finally, and related to the last point, their results show that loss aversion alone cannot solve the pricing puzzles; they do not consider the possibility for financial wealth in solving the anomalies by itself.

Contrary to Barberis et al. (2001), our theoretical model does not restrict preference for status to specific components of total wealth.² Moreover, our framework does not require a differential treatment of losses and gains, i.e. we abstract completely from loss aversion. Our approach is therefore considerably simpler to implement; we require identification of a single additional parameter. Importantly, this parameter is estimated, rather than calibrated. Its reasonableness can be established by (i) formally testing the theoretical restrictions and (ii) deriving the asset pricing implications of the model evaluated at its estimated parameters to compare it with known facts characterizing returns.

Finally, Gordon and St-Amour (2000, in press), as well as Danthine et al. (2002) remain agnostic about the conditioning preference state Z_t , while treating the Arrow-Pratt index γ_t as state-dependent. Gordon and St-Amour (2000) find that a two-state Markov process can rationalize stock and bond prices, whereas Gordon and St-Amour (in press) extend this analysis to a continuum of states. Both models generate considerable counter-cyclical in risk aversion indices which helps addressing the predictability puzzle. Danthine et al. (2002) use a state-dependent utility approach in a Mehra-Prescott economy where the state is correlated with consumption growth regimes. They show that whether risk aversion is pro- or counter-cyclical has little impact on the theoretical mean premia and risk-free rate, but do not discuss its implications for the predictability of the premia.

Our approach differs considerably. First our functional form imposes a constant curvature index; fluctuations in risk aversion are caused by movements in and around bliss, rather than by movements in γ . Second we select total wealth as the state variable. Finally, our focus is on predictions in goods rather than price space. Nonetheless, as will be seen later, counter-cyclical risk aversion remains a salient feature of our empirical findings.

²Although, our *empirical* implementation relies on elements of financial wealth for practical reasons which are discussed below.

Other literature From a different perspective, observing that returns display considerable predictability, Lettau and Ludvigson (2001a,b) analyze the role of the consumption-wealth ratio as a conditioning variable in assessing the performance of the conditional CAPM and C-CAPM. The intuition is that changes in the consumption share reflects changes in expected future returns on total wealth induced by changes in individual returns. As such, it should have predictive power in explaining *ex-post* returns. Since human wealth is unobservable, they assume that labor income is the annuity value accruing to the holder of human capital. This procedure allows them to identify wealth based only on observable non-human net worth, and labor income. They find that the consumption share improves the performance of the CAPM and C-CAPM to a level comparable to the Fama and French (1992, 1993) three-factor model.

We differ from Lettau and Ludvigson (2001a,b) in a number of important ways. First, we do not focus on conditional version of the C-CAPM with time-varying loadings, but rather explicitly derive pricing restrictions on the kernel. Importantly, we derive and focus on closed-form solutions for optimal consumption and portfolio. As anticipated, all our shares are explicitly time-varying, whereas Lettau and Ludvigson (2001a) need to assume *constant* portfolio shares to derive the *time-varying* consumption-wealth ratio. This restriction is at odds with the evidence presented in Figure 3 in which both consumption and portfolio are time-varying.

2 Model

2.1 Economic environment and preferences

Uncertainty We assume that the stochastic environment is described by a standard Brownian motion $\mathbf{Z}_t \in \mathbb{R}^d$ on a complete probability space (Ω, \mathcal{F}, Q) , with filtration $\mathbb{F} = \{\mathcal{F}_t : t \geq 0\}$ and infinite time horizon.

Securities The investment set consists of $n + 1$ securities, in positive net supply, with prices $\mathbf{P}_t \in \mathbb{R}^{n+1}$, which are assumed to be adapted Itô processes:

$$\frac{d\mathbf{P}_t}{\mathbf{P}_t} = \boldsymbol{\mu}_{p,t} dt + \boldsymbol{\sigma}_{p,t} d\mathbf{Z}_t, \quad (1)$$

$$\frac{dP_{0,t}}{P_{0,t}} = r_t dt, \quad (2)$$

where $\boldsymbol{\mu}_{p,t} \in \mathbb{R}^n$ and $\boldsymbol{\sigma}_{p,t} \in \mathbb{R}^{n \times d}$ are respectively a drift and diffusion term, and r_t is the rate of return on a risk-free asset indexed 0. All processes are \mathcal{F}_t -measurable. The prices \mathbf{P}_t include all associated payments, such as dividends.

Budget constraint A representative agent's budget constraint is given by:

$$dW_t = \{[\mathbf{v}'_t(\boldsymbol{\mu}_{p,t} - r_t) + r_t]W_t - C_t\}dt + W_t \mathbf{v}'_t \boldsymbol{\sigma}_{p,t} d\mathbf{Z}_t, \quad (3)$$

where W_t is current-period wealth, $\mathbf{v}_t \in \mathbb{R}^n$ is a vector of portfolio weights and C_t is consumption.

Preferences The representative agent's preferences are characterized by wealth-dependent utility:

$$E_0 \int_0^\infty \exp(-\rho t) U(C_t, W_t) dt, \quad (4)$$

where E_0 is a conditional expectations operator, $\rho > 0$ is a subjective discount rate, and $U(\cdot, W_t)$ is a monotone increasing and concave instantaneous utility function that will be discussed shortly.

Objectives The agent's objective will be to maximize expected utility (4) subject to the budget constraint (3). Denote by $J_t = J(W_t)$ the value function, and by $J_{i,t}$, and $J_{ij,t}$ its first and cross derivatives with respect to argument i and ij . For this program, the Bellman equation can be written as:

$$0 = \max_{\{C_t, \mathbf{v}_t\}} U(C_t, W_t) - \rho J_t + J_{w,t} \{[(\boldsymbol{\mu}_{p,t} - r_t)' \mathbf{v}_t + r_t] W_t - C_t\} + 0.5 W_t^2 J_{ww,t} \mathbf{v}_t' \boldsymbol{\Sigma}_{pp,t} \mathbf{v}_t \quad (5)$$

where $\boldsymbol{\Sigma}_{ij,t} \equiv E_t[\boldsymbol{\sigma}_{i,t} d\mathbf{Z}_t d\mathbf{Z}'_{j,t} \boldsymbol{\sigma}'_{j,t}]$ is positive semi-definite matrix of instantaneous covariances for processes i, j .

Instantaneous utility The agent's within-period utility is given by:

$$U(C, W) = \begin{cases} \frac{(\eta_c C + \eta_0 + \eta_w W)^{1-\gamma}}{1-\gamma}, & \text{if } \gamma \neq 1; \\ \log(\eta_c C + \eta_0 + \eta_w W), & \text{if } \gamma = 1 \end{cases} \quad (6)$$

Utility (6) belongs to the HARA class advocated by Rubinstein (1974), modified to allow for wealth dependence. Following Merton (1990, p. 137), the necessary HARA restrictions are:

$$\eta_c > 0, \quad \eta_c C + \eta_0 + \eta_w W > 0, \quad \gamma \geq 0. \quad (7)$$

These restrictions are required to guarantee monotonicity and concavity. To these, we add a further theoretical restriction that bounds below and above the term η_w :

$$-1 < \eta_w / \eta_c < \rho. \quad (8)$$

This restriction allows for negative or positive values of the loading of wealth in the utility function, η_w , but limits the size of the effect.³

³The theoretical restriction (8) is suggested from the financial problem we are analyzing. Consider the discrete-time analog of maximizing (4) subject to (3). First-order and Envelope conditions yield the following:

$$U_{c,t} = \exp(-\rho) E_t \{ [U_{c,t+1} + U_{w,t+1}] R_{i,t+1} \},$$

or,

$$1 = \exp(-\rho) (1 + \eta_w / \eta_c) E_t \left\{ \left(\frac{\eta_c C_{t+1} + \eta_0 + \eta_w W_{t+1}}{\eta_c C_t + \eta_0 + \eta_w W_t} \right)^{-\gamma} R_{i,t+1} \right\}.$$

This Euler equation has a familiar representation, with the exception that the subjective discount factor is now modified to allow for wealth dependence. It is reasonable to expect that the *effective* discount factor, $\exp(-\rho)(1 + \eta_w / \eta_c) \in (0, 1)$. Restriction (8) follows immediately.

Subject to restrictions (7) and (8), the utility function (6) has interesting properties. The expression $C_{\text{bliss},t} \equiv -(\eta_0 + \eta_w W_t)/\eta_c$ has the interpretation of a reference (bliss) level of consumption. More precisely, as consumption falls toward bliss, marginal utility goes to infinity, such that $C_{\text{bliss},t}$ is the minimum admissible consumption level. In addition, under WDU, the bliss level changes because of changes in wealth. In contrast, slow-moving habit or durability models let the reference level be a function of past consumption, whether idiosyncratic, or aggregate. Finally, CRRA utility and HARA utility fix the bliss consumption to 0 and $-\eta_0/\eta_c$ respectively, both state-independent levels.

The marginal utility, cross effect and the Arrow-Pratt coefficient of absolute risk aversion (calculated with respect to consumption and wealth) are respectively:

$$U_x = \frac{\eta_x}{(\eta_c C + \eta_0 + \eta_w W)^\gamma}, \quad (9)$$

$$U_{xy} = \frac{-\gamma \eta_x \eta_y}{(\eta_c C + \eta_0 + \eta_w W)^{\gamma+1}}, \quad (10)$$

$$R^{ax} \equiv \frac{-U_{xx}}{U_x} = \frac{\gamma \eta_x}{(\eta_c C + \eta_0 + \eta_w W)}, \quad (11)$$

where U_x denotes the partial derivative with respect to argument $x, y \in \{c, w\}$, R^{ax} is the Arrow-Pratt absolute x -risk aversion index, and R_y^{aw} is its derivative with respect to argument y . We summarize the sign of these as well as other variables of interest in the following Table 2.

Table 2: Sign of variables of interest

	$\eta_w < 0$ (ratchet)	$\eta_w > 0$ (blasé)
U_c	positive	positive
U_w	negative	positive
U_{cw}	positive	negative
R^{ac}	positive	positive
R^{aw}	negative	positive
R_w^{ac}	positive	negative
R_c^{ac}	negative	negative
R_w^{aw}	negative	negative
R_c^{aw}	positive	negative

A first observation is that wealth-dependent utility is monotone increasing in consumption, and monotone increasing or decreasing in wealth depending on the sign of η_w . Second, utility is always concave in consumption and in wealth, regardless of the sign of η_w . It follows that the sign of η_w also determines the sign of the Arrow-Pratt absolute risk aversion index; an agent can simultaneously be risk averse over consumption and risk lover over wealth. Note also that the ratio of marginal utilities of consumption and of wealth is constant and equal to the ratio of the consumption and wealth risk aversion:

$$\frac{U_w}{U_c} = \frac{R^{aw}}{R^{ac}} = \frac{\eta_w}{\eta_c}. \quad (12)$$

This last result will have important implications for both returns and optimal rules, as will be discussed later.

Third, the sign of η_w can be associated to the presence of ‘visceral factors’ identified by Loewenstein (2000). These are taken to represent emotions that affect an individual’s behavior. For example, a negative visceral factor such as hunger increases the marginal utility of food, but reduces the overall utility level if left unaddressed. A similar effect is obtained when $\eta_w < 0$ (i.e. $U_{cw} > 0, U_w < 0$), whereas positive visceral factors are derived for $\eta_w > 0$. Loewenstein (2000) argues that visceral factors have important implications for decision making. In particular, he maintains that effective time discounting and the divergence between cognitive and objective evaluations of risk are altered. We will show later that this is effectively the case in our setting.

Fourth, note that, as for other quasi-homothetic preferences, relative risk aversion is not constant. However, contrary to standard HARA utility, such movements are caused by movements in wealth, in addition to changes in consumption. To see this, consider the effect on marginal utility of consumption following an increase in wealth. As shown

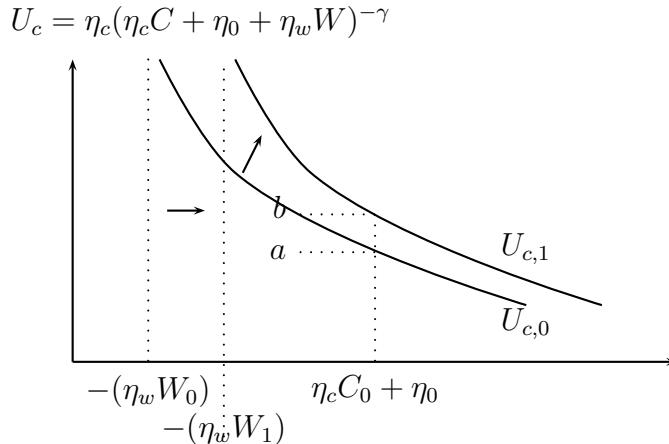


Figure 1: Effects of increase in wealth on marginal utility, Ratchet Investors ($\eta_w < 0$)

in Figure 1, when $\eta_w < 0$, as wealth increases, so does the minimum level of consumption required for marginal utility to be well defined, from $-\eta_w W_0$, to $-\eta_w W_1$. Hence, a negative wealth dependence involves a *ratchet* effect whereby the bliss consumption level increases in wealth. This leads to a clockwise rotation in the marginal utility schedule from $U_{c,0}$ to $U_{c,1}$, and increases marginal utility from a to b . Put differently, as wealth increases, the agent approaches his reference consumption, and becomes more averse toward consumption risk.

Next, these movements in marginal utility and risk aversion are reversed for $\eta_w > 0$, as shown in Figure 2. An increase in wealth now reduces minimum admissible consumption from $-\eta_w W_0$, to $-\eta_w W_1$. This causes a counter-clockwise rotation in the marginal utility schedule from $U_{c,0}$ to $U_{c,1}$, and a reduction in the marginal utility of consuming the same level of nondurable good falls from a to b . This effect could be related to *blasé* behavior; as the investor becomes richer, for a given level of consumption, both marginal utility and consumption risk aversion fall.

Finally, Table 2 establishes that the decreasing absolute risk aversion (DARA) property is maintained in all cases for *blasé* utility. Ratchet preferences on the other hand maintains DARA only in the case of the own effect, i.e. absolute consumption (wealth) risk aversion is decreasing in consumption (wealth), but not for cross effects. DARA is widely considered as desirable in the specification of preferences (e.g. Gollier, 2002; Dionne

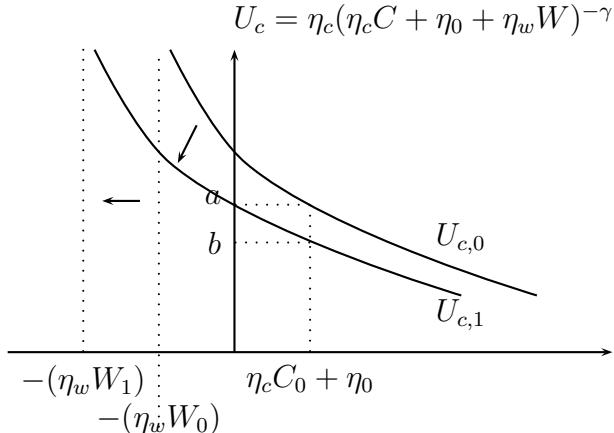


Figure 2: Effects of increase in wealth on marginal utility, Blasé Investors ($\eta_w > 0$)

and Ingabire, 2001). However it is usually understood in the sense of own, rather than cross effects.

Optimum Returning to the agent's problem, the $n + 1$ first-order conditions for an interior optimum are:

$$U_{c,t} - J_{w,t} = 0, \quad (13)$$

$$J_{w,t}(\boldsymbol{\mu}_{p,t} - r_t)W_t + J_{ww,t}\boldsymbol{\Sigma}_{pp,t}\boldsymbol{v}_tW_t^2 = \mathbf{0}. \quad (14)$$

Equations (13) and (14) can be solved either (i) from a goods space perspective, i.e. solving for optimal consumption and portfolio, given an exogenous forcing process for excess returns, or (ii) from a price space perspective, i.e. solving for the risk premium $\boldsymbol{\mu}_{p,t} - r_t$, given an exogenous process for consumption and portfolio shares. In what follows, we focus on the goods space analysis and postpone our discussion of asset returns until Section 5.

2.2 Optimal Consumption and Portfolio Rules

Up to now, our economic environment is general in the sense that it allows for a time-varying investment set. In order to derive closed-form solutions however, it is more convenient to start with a constant set assumption. We relax this restriction later when we discuss the asset pricing implications of our results. As usual, a constant set requires constant drift and diffusion terms.

Definition 1 (CIOS) *A constant investment opportunity set (CIOS) obtains when:*

$$\boldsymbol{\mu}_{p,t} - r_t = \boldsymbol{\mu}_p - r; \quad r_t = r; \quad \boldsymbol{\sigma}_{i,t} = \boldsymbol{\sigma}_i, \quad \forall i, t. \quad (15)$$

The following intermediate result will be useful for the derivation of the optimal rules.

Lemma 1 *Under CIOS, the indirect utility function is:*

$$J(W_t) = \frac{(G + FW_t)^{1-\gamma}}{1-\gamma}, \quad (16)$$

where G and F are constants that depend only on the model's primitives:

$$F = \eta_c \left\{ \left(\frac{\gamma - 1}{\gamma} \right) \left(r + \frac{\eta_w}{\eta_c} + \frac{\rho}{\gamma - 1} + 0.5Q/\gamma \right) \right\}^{\gamma/(\gamma-1)}, \quad (17)$$

$$G = \frac{\eta_0}{\eta_c} \left\{ \left(\frac{\gamma}{\gamma - 1} \right) \frac{1}{\eta_c} \left(\frac{F}{\eta_c} \right)^{-1/\gamma} - \frac{\rho}{F(\gamma - 1)} - \frac{0.5Q}{\gamma F} \right\}^{-1}, \quad (18)$$

Proof. See Appendix A. ■

Lemma 1 reveals interesting characteristics of the value function. First, we find that $J(W_t)$ is iso-morphic to the instantaneous utility function $U(C_t, W_t)$ in (6). The particular form of wealth dependence that we are considering supposes that the Bernoulli transform is applied to an affine function of wealth. This functional has the property that the value function is also in the HARA class. Note in particular that $\eta_0 = 0$ implies $G = 0$, such that the value function becomes iso-elastic despite the wealth dependence.

Second, following our previous analysis, we can analyze risk aversion using the marginal utility of wealth schedule, $J_{w,t} = J'(W_t)$, and the distance of an arbitrary wealth level W_t from minimum admissible bliss level. In particular, straightforward manipulations reveal that:

$$W_{bliss} \equiv \frac{-G}{F} = \frac{-\eta_0}{\eta_c r + \eta_w}, \quad (19)$$

$$\frac{-W_t J_{ww,t}}{J_{w,t}} = \frac{\gamma W_t}{W_t - W_{bliss}}. \quad (20)$$

The bliss level of wealth (19) can take on negative or positive values depending on the parameters $\eta_i, i = 0, c, w$ and on the interest rate r . In particular, since $\eta_c, r > 0, \eta_w < 0$ pushes the bliss level away from zero, $\eta_w > 0$ pushes it toward zero. For $W_{bliss} < 0$, a positive η_w (blasé) moves the bliss asymptote to the right. Given any wealth level, the agent is closer to bliss, and therefore characterized by a higher degree of absolute risk aversion. A negative η_w (ratchet) decreases absolute risk aversion for the opposite reason. When $W_{bliss} > 0$, movements in bliss are reversed, and we find that a blasé investor has lower absolute risk aversion than a ratchet investor. With respect to relative risk aversion (20), a (negative) positive bliss implies that risk aversion is (pro-) counter-cyclical. These elements will have important implications for the optimal consumption and portfolio rules, an issue to which we now turn.

Proposition 1 Under CIOS, the optimal consumption and value invested in assets are:

$$\begin{aligned} C_t &= \frac{\eta_0}{\eta_c} \left\{ \left(\frac{\gamma - 1}{\gamma} \right) \left(\frac{\rho/(\gamma - 1) + 0.5Q/\gamma}{r + \eta_w/\eta_c} \right) - \frac{1}{\gamma} \right\} \\ &\quad + \left\{ \left(\frac{\gamma - 1}{\gamma} \right) (r + \rho/(\gamma - 1) + 0.5Q/\gamma) - \frac{\eta_w}{\eta_c \gamma} \right\} W_t, \end{aligned} \quad (21)$$

$$V_t = \left(\frac{\eta_0/\eta_c}{r + \eta_w/\eta_c} \right) \frac{\Sigma_{pp}^{-1}(\mu_p - r)}{\gamma} + \frac{\Sigma_{pp}^{-1}(\mu_p - r)}{\gamma} W_t, \quad (22)$$

where $Q \equiv (\mu_p - r)' \Sigma_{pp}^{-1} (\mu_p - r) \geq 0$, and where $\mathbf{V}_t \equiv \mathbf{v}_t W_t$.

Proof. See Appendix B. ■

Proposition 1 shows that the optimal rules are affine in wealth. As for the standard HARA utility, imposing $\eta_0 = 0$ results in the iso-elastic case of both rules being proportional to net worth. Otherwise, wealth dependence affects both the intercept (C_t, \mathbf{V}_t) and the slope (C_t) of the closed-form solutions. To isolate these effects, it is useful to resort to our previous analysis of the value function. Note that we can indeed rewrite the optimal rules as:

$$\begin{aligned} C_t &= - \left\{ \left(\frac{\gamma - 1}{\gamma} \right) W_{bliss} [\rho/(\gamma - 1) + 0.5Q/\gamma] + \frac{\eta_0}{\gamma\eta_c} \right\} \\ &\quad + \left\{ \left(\frac{\gamma - 1}{\gamma} \right) [r + \rho/(\gamma - 1) + 0.5Q/\gamma] - \frac{\eta_w}{\gamma\eta_c} \right\} W_t, \end{aligned} \quad (23)$$

$$(\boldsymbol{\mu}_p - r)' \mathbf{V}_t = \frac{Q}{\gamma} \{-W_{bliss} + W_t\}, \quad (24)$$

where W_{bliss} is given by (19). For the rest of this section's analysis, assume that the investor is at least moderately risk averse, i.e $\gamma > 1$.

First, turning to consumption, we obtain the intuitive result that for positive W_{bliss} , minimum consumption, i.e the intercept in (23), is negative (or less positive), and positive (or less negative) otherwise. *Ceteris paribus*, a positive bliss implies a steeper marginal utility of wealth at the optimum, and consequently, greater IMRS risk. The risk-averse investor reacts to this by increasing wealth away from bliss. This is achieved by decreasing consumption and increasing savings. Secondly, regardless of bliss, it is straightforward to show that a blasé investor always has a higher minimum consumption, and a lower marginal propensity to consume out of wealth than a ratchet investor. This result is again intuitively appealing. Because higher wealth reduces his marginal utility of consumption more rapidly, a blasé investor always consumes less at the margin. At very low wealth levels however, from the cross effects in Table 2, marginal utility of consumption is higher and the blasé agent consumes more.

Third, (24) expresses the expected excess return (in \\$ terms) on the optimal total wealth portfolio. As usual, higher curvature γ results in more conservative positions. Again, bliss levels of wealth influence the intercept terms. A positive W_{bliss} implies more MU risk at any wealth levels. The risk-averse agent hedges away these risks by selecting more conservative portfolios. Negative bliss values however reduce risk aversion and increase the asset values held in risky assets. Fourth, for reasons discussed earlier, when bliss is negative, $\eta_w < 0$ shifts bliss to the left and decreases risk aversion; the ratchet investor therefore selects a more risky portfolio, the blasé a more conservative one. These positions are reversed for $W_{bliss} > 0$; the blasé investor's portfolio is more risky compared to the ratchet's.

Clearly linearity for the optimal rules (21), and (22) implies that the change in consumption and portfolio are $dC_t = c_w dW_t$, and $d\mathbf{V}_t = \mathbf{v}_w dW_t$, where c_w, \mathbf{v}_w are constants defined by Proposition 1. We can also substitute the solutions in the budget constraint (3) to obtain the closed-form expression for instantaneous changes in wealth. Consequently the following results are obtained:

Corollary 1 *The instantaneous changes in consumption, the value invested in assets, and wealth are:*

$$dC_t = \left\{ \left(\frac{\gamma - 1}{\gamma} \right) (r + \rho/(\gamma - 1) + 0.5Q/\gamma) - \frac{1}{\gamma} \frac{\eta_w}{\eta_c} \right\} dW_t, \quad (25)$$

$$d\mathbf{V}_t = \frac{\Sigma_{pp}^{-1}(\boldsymbol{\mu}_p - r)}{\gamma} dW_t, \quad (26)$$

$$\begin{aligned} dW_t &= \left[\frac{\eta_0/\eta_c + (r + \eta_w/\eta_c)W_t}{\gamma(r + \eta_w/\eta_c)} \right] \left\{ \left[\left(\frac{\gamma + 1}{\gamma} \right) 0.5Q + r + \eta_w/\eta_c - \rho \right] dt \right. \\ &\quad \left. + (\boldsymbol{\mu}_p - r)' \Sigma_{pp}^{-1} \boldsymbol{\sigma}_p d\mathbf{Z}_t \right\}. \end{aligned} \quad (27)$$

3 Estimation

3.1 Econometric Model

Estimation focuses on the multivariate Brownian motion given by Corollary 1, which can be written as:

$$dC_t = c_w dW_t, \quad (28)$$

$$d\mathbf{V}_t = \mathbf{v}_w dW_t, \quad (29)$$

$$dW_t = [\mu_0 + \mu_w W_t]dt + [\sigma_0 + \sigma_w W_t]dZ_t, \quad (30)$$

where $c_w, \mathbf{v}_w, \mu_0, \mu_w, \sigma_0, \sigma_w$ are constant loadings that depend only on the deep parameters.

In principle, estimation of the model could be undertaken in price space or in quantity space (i.e. imposing the restrictions implied by Proposition 1). We select the second approach for a number of reasons. First, the quantity space results impose considerably more theoretical restrictions on the moments of interest. These restrictions are summarized in Table 3.

Table 3: Comparisons of Moments Restrictions

	Price space analysis	Quantity space analysis	Proposition 1
Conditional means			
– quantities	unrestricted		restricted
– returns	restricted		unrestricted
Conditional variances			
– quantities	unrestricted		restricted
– returns	unrestricted		unrestricted
Conditional covariances			
– quantities–quantities	unrestricted		restricted
– quantities–returns	unrestricted		restricted
– returns–returns	unrestricted		unrestricted

Standard analyses in price space treat the equilibrium quantities in the pricing kernels as exogenous; the theoretical restrictions are imposed on the prices of risk exclusively, with conditional second moments left unrestricted. In comparison, the quantity space analysis produces theoretical restrictions on both first and second moments of changes

in consumption, asset holdings and wealth, while returns are treated as exogenous. In the absence of prior information on η_w in particular, these additional restrictions will be useful in identifying the preference parameters of interest.

Second, empirical studies of aggregate optimal consumption and asset holdings are much less frequent than asset pricing studies. We believe that focusing on quantities rather than on returns thus provides another perspective that complements existing results. We also elect to study instantaneous changes in quantities rather than levels. The motivation behind this choice is that aggregate consumption, asset holdings and wealth are likely to be non-stationary. To avoid well-known inference problems associated with integrated data, we prefer to estimate the model using time-differenced data.

Transformation One major problem in estimating (28)–(30) is that there exists no closed-form transition density for multi-variate Brownian motions with affine drifts and diffusions. Indeed, analytical expressions for the likelihood function exist only for a limited class of Itô processes (Melino, 1996). Unfortunately, our multi-variate process does not belong to this class. Alternative solutions include discrete (Euler) approximations, and simulating the continuous-time paths between the discretely-sampled data, either through classical (Durham and Gallant, 2002) or through Bayesian (Eraker, 2001) approaches.

Our solution to this problem is different and considerably simpler to implement. It is based on a homoscedasticity-inducing transformation for general Brownian motions. It will be shown that this approach also stationarizes the drift term. Consequently, a standard discretized approximation is appropriate, efficient, and unbiased. In particular, a straightforward application of Itô's lemma reveals the following.

Lemma 2 *Let $X_t \in \{C_t, V_t\}$ be defined as follows:*

$$X_t = x_0 + x_w W_t, \quad (31)$$

$$dW_t = (\mu_0 + \mu_w W_t) dt + (\sigma_0 + \sigma_w W_t) dZ_t, \quad (32)$$

where $\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\sigma}$ are constants defined in Proposition 1, and in Corollary 1, and consider the following transformation:

$$\tilde{X}_t = \frac{\log[x_w \sigma_0 + \sigma_w (X_t - x_0)]}{\sigma_w}, \quad (33)$$

Then, \tilde{X}_t has constant drift and diffusion given by:

$$d\tilde{X}_t = \left[\frac{\mu_w}{\sigma_w} - 0.5\sigma_w \right] dt + dZ_t. \quad (34)$$

Proof. See Appendix C. ■

The transformation (33) requires that its first derivative with respect to the Itô process X_t is the inverse of the diffusion. It is usually introduced in order to stationarize the diffusion (Shoji and Ozaki, 1998; Ait-Sahalia, 2002; Durham and Gallant, 2002). In our case, both drift and diffusion are affine and have intercept and slope coefficients that are closely inter-related. Consequently the theoretical restrictions implied by the model result imply that the transformation *also* stationarizes the drift term. This is fortunate since the resulting transformed model is easily estimated by maximum likelihood. In particular, the discretization of (34):

$$\Delta \tilde{X}_t = \left[\frac{\mu_w}{\sigma_w} - 0.5\sigma_w \right] + \epsilon_t \quad (35)$$

where ϵ_t is a standard Gaussian term, can be consistently and efficiently estimated by MLE (e.g. Gourieroux and Jasiak, 2001, pp. 287–288).

Likelihood function The optimal rules in Proposition 1 take the moments of the returns' distribution $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$, as well as the risk-free rate r as given. These moments could be estimated in an external round, using a two-step procedure, and substituted back into the optimal rules to obtain the predicted rules. Instead, we perform a single-step procedure and incorporate the risky returns into the calculation of the likelihood function.⁴ This approach has the advantage of incorporating the parametric uncertainty concerning $\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_{pp}$ into the calculation of the standard errors of the deep parameters. Specifically, denote by $\tilde{\mathbf{X}}_t \equiv [\tilde{C}_t, \tilde{\mathbf{V}}_t, \tilde{W}_t]'$ the $n + 2$ vector of transformed variables, the model to be estimated is the following:

$$\begin{pmatrix} \Delta \tilde{\mathbf{X}}_t \\ \Delta \mathbf{P}_t / \mathbf{P}_{t-1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\mu}_x \\ \boldsymbol{\mu}_p \end{pmatrix} + \begin{pmatrix} \boldsymbol{\epsilon}_x \\ \boldsymbol{\epsilon}_p \end{pmatrix}, \quad \begin{pmatrix} \boldsymbol{\epsilon}_x \\ \boldsymbol{\epsilon}_p \end{pmatrix} \sim \text{N.I.D.} \left(\begin{pmatrix} \mathbf{0}_x \\ \mathbf{0}_p \end{pmatrix}, \begin{pmatrix} \mathbf{I}_x & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Sigma}_{pp} \end{pmatrix} \right), \quad (36)$$

where $\boldsymbol{\mu}_x$ is given by (35), and \mathbf{I}_x is an $n + 2$ identity matrix.

First, in accordance with the maintained assumption of the model, all the innovations are Gaussian. Second, as mentioned earlier, the transformation in Lemma 2 implies that the quantities innovations are standardized white noise. Third, consistent with the model, the covariance matrix is block diagonal, i.e. we impose the absence of cross-correlations between innovations in quantities and returns. Any potential covariance between the two is fully taken into account in the closed-form solutions; allowing for additional correlations is not justified.

With these elements in mind, the contributions to the likelihood function (with constant term omitted) are given by:

$$f_t = -0.5 \log[\det(\boldsymbol{\Sigma})] + \log[\det(\mathbf{K}_t)] - 0.5 \boldsymbol{\epsilon}'_t \boldsymbol{\Sigma}^{-1} \boldsymbol{\epsilon}_t \quad (37)$$

where $\boldsymbol{\Sigma}$ is defined implicitly in (36), while $\mathbf{K}_t \equiv \text{Diag}([K_{c,t}, \mathbf{K}_{v,t}, K_{w,t}, 1, \dots, 1])$ and $K_{x,t} = 1/[x_w \sigma_0 + \sigma_w(X_t - x_0)]$ is a Jacobian correction term associated with the transformation (35). The parameter vector is then $\boldsymbol{\theta} \equiv \{\gamma, \rho, \eta_0, \eta_c, \eta_w, \boldsymbol{\mu}_p, \mathbf{Q}_{pp}\}$, where $\mathbf{Q}_{pp} \equiv \text{Chol}(\boldsymbol{\Sigma}_{pp})$ is the n -dimensional triangular Cholesky root of the returns covariance matrix.

Hypothesis tests It will be recalled that theoretical restrictions for HARA and WDU utility are necessary to guarantee that marginal utility is non-negative. In particular, for both models, restriction (7) is required for monotone preferences, whereas for WDU utility, (8) verifies that the agent has a positive effective discount rate.

We also consider two benchmarks in assessing the performance of the WDU model. As mentioned earlier, CRRA utility is obtained by imposing that $\eta_0, \eta_w = 0$, whereas HARA utility imposes $\eta_w = 0$. To the extent that it has been studied extensively in asset pricing models, CRRA utility constitutes a natural benchmark. HARA utility, although less popular, has the advantage of optimal rules which are not proportional to wealth (see the previous discussion). Both the theoretical restrictions and the model selection tests will be performed and discussed below.

3.2 Data

Our data set consists of post-war U.S. quarterly observations on aggregate consumption, asset holdings and corresponding returns indices. The time period covered ranges from 1952:II to 2000:IV, for a total of 195 observations. All quantities are expressed in real, per-capita terms, where the aggregate price index is taken to be the implicit GDP deflator. Similarly, all returns are converted in real terms by subtracting the inflation index.

⁴Following standard practices, the risk-free rate r is calibrated to its mean value.

Consumption The consumption series is the aggregate expenditure on Non-Durables and Services. The source of the data is the Bureau of Economic Analysis NIPA series. This series has been used in most asset pricing studies.

Assets The aggregate portfolio holdings are defined as follows:

$$\begin{aligned}\mathbf{V}_t &= [V_{0,t}, V_{1,t}, V_{2,t}] \\ &= [\text{Deposits, Bonds, Stocks}].\end{aligned}$$

Each asset holdings are obtained from the Flow of Funds Accounts made available by the Board of Governors of the Federal Reserve (Table L.100). They correspond to the level values of asset holdings by households and non-profit organizations (see also Lettau and Ludvigson, 2003). More precisely the individual assets (mnemonic) are:

- Deposits (FL1540005): Includes foreign, checkable, time, savings deposits and money market fund shares.
- Bonds (FL153061005): U.S. government securities (Treasury and Agency).
- Stocks (FL153064105) Corporate equities directly held by households.

Deposits will thus be taken to represent the risk-free asset, whereas both long-term government bonds and corporate equity are proxies for the risky assets.

The choice of specific portfolio holdings was dictated by a number of practical elements. First, these assets correspond to some of the largest asset holdings for U.S. households, and their returns have been studied extensively in the asset pricing literature, thus providing useful benchmarks for our analysis. In particular, we are interested in verifying whether the pricing anomalies are also observable in the quantity space. Second and related, these assets have corresponding returns series. Those returns are required to evaluate the distributional parameters μ_p , Σ_{pp} that are used to compute the theoretical asset holdings. Other assets such as home equity or pension and life insurance reserves are also important in relative size. However, no clear returns indices are available for these assets.⁵

Wealth Our empirical implementation defines wealth as the sum of the individual asset holdings previously defined:

$$W_t = V_{0,t} + V_{1,t} + V_{2,t}.$$

This definition has been used in theoretical models of portfolio choice (e.g. Campbell et al., 2003). Its main advantage is that wealth is thus observable and the definition provides more structure on the econometric model since one of the theoretical asset holding is the defined residually.⁶ However, the definition is narrow in the sense that it abstracts from

⁵For example, pension reserves are typically invested differently by fund managers whether they are defined benefit or defined contribution. Finding a unique pricing index for this series in the absence of detailed information on the funds' composition is impractical.

⁶In particular, (22) reveals that, for the risk-free asset:

$$V_{0,t} = v_{00} + v_{w,0} W_t \tag{38}$$

where,

$$v_{00} = -v_{10} - v_{20}, \quad v_{w0} = 1 - v_{1w} - v_{2w}. \tag{39}$$

tangible (real) and human wealth. Unfortunately, real returns indices on housing and durable goods are difficult to evaluate, and these assets were omitted from our selected holdings series \mathbf{V}_t . Moreover, human wealth is not observable, whether in levels or in rates of returns and thus also eliminated.

Table 4 reports the sample moments for the consumption and asset holdings in percentage of wealth. Figure 3 in Appendix E plots these shares. A first observation is that consumption, deposits and stocks are roughly of the same order of magnitude, and similarly volatile. Bonds on the other hand represent a much lower share of wealth and are smoother.

Table 4: Sample moments: Shares of wealth

	mean	std	correlation			
Consumption	0.525	0.096	1.000	0.907	-0.233	-0.947
Deposits	0.473	0.123		1.000	-0.519	-0.970
Bonds	0.088	0.031			1.000	0.295
Stocks	0.440	0.110				1.000

Secondly, shares experience considerable time variation. In particular, cash and consumption display a strong positive correlation, particularly between the 1970's and 90's. Apparently, households were keeping cash balances closely-aligned with consumption needs during a high interest rate period. Conversely, stocks exhibit a strong negative correlation with cash and consumption. Equity holdings appear particularly pro-cyclical: decreasing during downturns, and picking up during recoveries. Finally, we find that bond holdings are declining until the 80's, and pretty much stagnant afterwards.

In summary, consumption and portfolio shares are clearly time varying, and subject to cyclical movements. This suggests that preference specifications that yield constant shares, such as CRRA utility, are clearly at odds with the data. Both HARA and WDU on the other hand generate time variation in shares as long as $\eta_0 \neq 0$.

Returns We follow Campbell et al. (2003) in constructing the returns series that correspond to our assets definition. The return on cash is taken to be the real return on 3-months Treasury Bills. The return on bonds is proxied by the real return on 5-years T-Bills. Finally, stock returns are evaluated as the value-weighted returns on the NYSE, NASDAQ and AMEX markets. Bond and stock returns were obtained from the CRSP data file. Again, the inflation series is computed from the GDP deflator.

Table 5 presents sample moments of the real returns. These series have been widely discussed in the asset pricing literature, so we only briefly outline their main features. First, we observe that both bonds and stocks warrant a positive premia. Equity returns however are clearly larger, and much more volatile. Next, we find that both cash and bonds as well as bonds and stocks are positively, and similarly correlated. Cash and stocks on the other hand display no covariance.

Table 5: Sample moments: Real Returns

	mean	std	correlation			
Deposits	0.017	0.022	1.000	0.236	0.005	
Bonds	0.036	0.130		1.000	0.212	
Stocks	0.137	0.342				1.000

4 Results

4.1 Estimation details

Identifiability The theoretical model in Corollary 1 presents important challenges for identification. Indeed, the parameters are often expressed as ratios of one another which usually results in poor identifiability. These problems affect both the HARA and WDU models, but not the CRRA model. As is well known, utility is defined only up to an affine transformation such that the parameter η_c plays no role in the optimal rules when $\eta_0, \eta_w = 0$. In preliminary estimation rounds, we experimented with numerous identification strategies which we briefly discuss.⁷

A first approach was to fix the subjective discount rate ρ to a realistic value, and to let η_c be flexible. We found that both HARA and WDU models were then poorly identifiable; results were highly dependent on starting values, and convergence problems were noticed. Second, we let ρ be flexible, and fixed η_c . Whereas HARA utility was well identified and yielded realistic ρ estimates, the WDU model was not. In particular, we found that we could not identify ρ and η_w separately. Nonetheless, the effective discount rate $\rho - \eta_w/\eta_c$ was uniquely identifiable, and realistic. Finally, we fixed both ρ, η_c , and found this approach to be the most satisfactory. Both models were then clearly identified, with robustness to starting values and rapid convergence. We found that the curvature parameter γ was completely independent from the choice of calibration for η_c . Moreover, changing η_c resulted in changing the estimated η_0, η_w in the same proportions, such that the T-statistics were always unaffected by η_c . This again suggests that although the ratios $\eta_0/\eta_c, \eta_w/\eta_c$ are well identified, the separate parameters are not. We therefore fix $\rho = (1 + 0.035)^{1/4} - 1$, a realistic value, and arbitrarily impose $\eta_c = 1$ to estimate γ, η_0, η_w . The vector of free parameters is then $\{\gamma, \eta_0, \eta_w, \boldsymbol{\mu}_p, \mathbf{Q}_{pp}\}$.

4.2 Parameter estimates

Table 6 presents the estimated parameters for model (36). Panel A imposes the CRRA restrictions that $\eta_0 = \eta_w = 0$; Panel B imposes the HARA restriction that $\eta_w = 0$. Panel C relaxes these restrictions altogether for the WDU model.

Theoretical restrictions First, regarding the monotonicity restriction, CRRA utility trivially respects non-negative marginal utility. In the case of HARA and WDU, this condition needs to be verified. We test that monotonicity is always maintained by evaluating (7) at the minimum consumption and wealth levels:

$$\eta_c \min(C_t) + \eta_0 + \eta_w \min(W_t) > 0$$

Since $\eta_c \equiv 1$ and η_w is estimated positive, this approach is sufficient to guarantee monotonicity throughout our sample. For HARA utility, the statistic (standard error) is 6,474 (0.35); for WDU, it is 6,6157 (87.95). We thus conclude that monotonicity condition (7) is verified for both HARA and WDU.

Second, we verify that the effective discount rate for WDU preferences is positive as in (8). Since $\eta_c \equiv 1$, this is obtained by testing

$$H_0 : \rho - \eta_w > 0.$$

⁷The full results can be obtained upon request.

Table 6: Parameter Estimates

Param.	Estim.	T-stat.	Estim.	T-stat.	Estim.	T-stat.
	A. CRRA		B. HARA		C. WDU	
γ	6.187	4.822	5.799	2.747	5.340	4.059
η_0	0.000	0.000	-17.319	-49.462	-93.809	-1.876
η_w	0.000	0.000	0.000	0.000	0.019	1.530
μ_1	0.007	3.329	0.007	2.395	0.008	3.471
μ_2	0.026	7.602	0.028	6.129	0.023	5.575
$Q(1, 1)$	0.031	19.369	0.031	19.685	0.030	19.519
$Q(1, 2)$	0.017	2.573	0.017	3.008	0.016	2.504
$Q(2, 2)$	0.080	19.073	0.079	19.914	0.080	19.303
LLF	6490.541		6382.977		6382.266	

Note: Estimated model (36). Fixed parameters $\rho = (1 + 0.035)^{1/4} - 1$, and $\eta_c = 1$. $\boldsymbol{\mu}_p$ are the drift parameters, $\mathbf{Q}_{pp} \equiv \text{Chol}(\boldsymbol{\Sigma}_{pp})$ is the Cholesky root of the covariance matrix of the returns process.

Evaluated at our parameter estimates in Panel C, the effective discount rate is -0.0099 (0.0121), a negative but low value that is not statistically significant. A test of the null hypothesis yields a p-value of 0.282, such that the null is not rejected. We therefore conclude that all three models satisfy the theoretical restrictions and proceed with the analysis of the point estimates.

Individual estimates The estimates for the curvature parameter γ in Table 6 are positive, significant and realistic for all three preference specifications. Indeed, it is widely accepted that this parameter should be positive, but less than 10 for iso-elastic utility (e.g. Mehra and Prescott, 1985). Moreover, the point estimates are lower for WDU. Whether or not this translates into a lower level of risk aversion for these functionals will be addressed below.

Next, we find that the bliss parameter η_0 is negative and very significant for HARA utility, and even more negative, but less significant under WDU. This suggests that the reference consumption level is positive under HARA preferences. Under WDU, the minimum admissible consumption level ranges between -100 and -800 and remains negative throughout. Third, the wealth dependence parameter η_w is positive, although weakly significant. This would be consistent with blasé investors.

The other parameters are the drift and diffusion parameters of the returns process. These elements are only instrumental to our analysis and need not be discussed. Note that, as expected, they are almost completely unaffected by our preference specifications.

Hypothesis tests We next perform Likelihood Ratio (LR) tests of the parametric restrictions associated with the three preference specifications. Table 7 reports the test statistics with the associated P-values.

When the alternative is taken to be either HARA or WDU, the null of CRRA utility is strongly rejected. This suggests that the standard practice of focusing on iso-elastic preferences in empirical pricing studies could fruitfully be revised. Consistent with our earlier results, we see however that the null of HARA utility is rejected only at the 25% level when tested against the alternative of WDU.

Table 7: LR Tests

Null	Alternative	Statistic	P-Value
CRRA	HARA	215.13	0
CRRA	WDU	216.55	0
HARA	WDU	1.42	0.23

4.3 Risk Aversion Estimates

Figure 4 in Appendix E plots R_t^{rc} for the three utility functions. We find that CRRA utility generates the highest, and WDU the lowest level of consumption risk aversion. Moreover, consumption risk aversion under HARA is almost flat compared to that obtained under WDU, i.e. HARA generates no perceptible cyclical variation in attitudes towards consumption risk. Figure 5 plots the wealth risk aversion R_t^{rw} . Clearly, this index is zero for both CRRA and HARA. The level for WDU is positive, generally lower, and less volatile compared to consumption risk aversion.

A more definitive interpretation of the representative agent's risk aversion can be obtained from the indirect utility function J_t , and its corresponding relative risk aversion index $-W_t J_{ww,t} / J_{w,t}$ in (20). This variable is plotted in Figure 6. Because the indirect utility is iso-morphic to the instantaneous utility, the CRRA function has a constant index equal to γ . For most of our sample, this level is lower than that obtained under HARA and WDU. Note finally that the risk aversion under WDU is lower than that obtained under HARA, with parallel time paths.

We can explore the issue of cyclical movements of attitudes toward risk by comparing risk aversion series with indices of the state of the economy. For that purpose, we use the University of Michigan Consumer Confidence Index, a subjective measure, which we plot against the various measures of risk aversion obtained under WDU. Figure 7 plots the consumption risk aversion against the confidence index. Clear counter-cyclical patterns emerge. Risk aversion is initially decreasing until the late 60's, when the confidence index is stable. Then, risk aversion increases as the index falls in the early and mid 70's. The gradual recovery in consumer sentiment is associated with a smooth decline in consumption risk aversion.

The correspondence between attitudes toward risk and consumer confidence is even more striking for wealth risk aversion in Figure 8. Pro-cyclical movements in wealth risk aversion mimic almost exactly those in confidence, particularly up until the mid 70's. After that period, the gradual increase in confidence is associated with a smooth increase in wealth risk aversion.

We therefore find strong counter-cyclical movements in consumption risk aversion, and strong pro-cyclical movements in wealth risk aversion. To verify which one of those two conflicting influences dominates, we plot the indirect risk aversion (20) against the confidence index in Figure 9. Again, a counter-cyclical movement clearly emerges. To understand this result, our estimates reveal that the bliss level of wealth (19) is $W_{bliss} = 4126.1$, a positive value, whereas wealth in our sample ranges between 11 thousand and 48 thousand \$. As wealth increases above bliss, movements in marginal utility are reduced and risk aversion falls. This accords with our previous discussion of the value function in Lemma 1 that for positive bliss, a blasé investor has lower and counter-cyclical risk aversion.

5 Discussion: Implications for Asset Returns

Our empirical results obtained from the estimation of optimal consumption and portfolio rules can be summarized as follows:

1. The intercept parameter η_0 is negative.
2. The wealth-dependence term η_w is positive.
3. The curvature parameter γ is positive, realistic, and lower under WDU.
4. Risk aversion is counter-cyclical.

Because there is a relative paucity of empirical results in goods space for these models, it is difficult to establish whether or not our findings make sense on a comparative basis. Nonetheless, we can use the price space results to obtain further perspective.

We consequently consider the implications of our model and of our results for asset returns. For that purpose, we take the more standard approach used in the empirical asset pricing literature of solving the first-order conditions for mean excess and risk-free returns. In a pure exchange economy, equilibrium is straightforward to characterize: identical agents hold their asset endowments and consume the associated dividends flow. Note that the constant investment set assumption (15) is no longer necessary so that the following results are derived under the more general time-varying investment opportunity set.⁸

Proposition 2 *The vector of risk premia on risky assets is given by:*

$$\boldsymbol{\mu}_{p,t} - r_t = R_t^{rc} \text{Cov}_t \left(\frac{dC_t}{C_t}, \frac{d\mathbf{P}_t}{\mathbf{P}_t} \right) + R_t^{rw} \text{Cov}_t \left(\frac{dW_t}{W_t}, \frac{d\mathbf{P}_t}{\mathbf{P}_t} \right). \quad (40)$$

The risk-free rate is given by:

$$r_t = \rho - \eta_w/\eta_c + R_t^{rc} \text{E}_t \left(\frac{dC_t}{C_t} \right) + R_t^{rw} \text{E}_t \left(\frac{dW_t}{W_t} \right) - 0.5 \left(\frac{\gamma+1}{\gamma} \right) \times \left[(R_t^{rc})^2 \text{Var}_t \left(\frac{dC_t}{C_t} \right) + 2R_t^{rc} R_t^{rw} \text{Cov}_t \left(\frac{dC_t}{C_t}, \frac{dW_t}{W_t} \right) + (R_t^{rw})^2 \text{Var}_t \left(\frac{dW_t}{W_t} \right) \right] \quad (41)$$

where:

$$R_t^{rx} \equiv \frac{-X_t U_{xx,t}}{U_{x,t}} = \left(\frac{\gamma \eta_x X_t}{\eta_c C_t + \eta_0 + \eta_w W_t} \right)$$

Proof. See Appendix D. ■

⁸Note that the rules in Proposition 1, which were obtained under a CIOS assumption, do not contradict the premia and risk-free rate in Proposition 2. In particular, the optimal rules (21), (22) as well as Corollary 1 can be used to compute closed-form expressions for risk aversion, conditional means, variances, and covariances. Substituting the resulting expressions in (40) and (41) readily verifies that $\boldsymbol{\mu}_{p,t} - r_t = \boldsymbol{\mu}_p - r$, and $r_t = r$ as initially postulated in Proposition 1.

Risk premia The premia (40) is a two-factor pricing model, with the C-CAPM consumption beta supplemented by the CAPM total wealth return beta. In particular, (11) reveals that the price of consumption risk is the Arrow-Pratt risk aversion level, measured with respect to *consumption*, and that the price of the market risk is the corresponding Arrow-Pratt risk aversion, measured with respect to *wealth*. The model can be interpreted as a weighted average of a static CAPM ($\eta_c = 0$), and a standard C-CAPM ($\eta_w = 0$), where the weights depend on the relative contributions of consumption and wealth to the agent's utility. Epstein and Zin (1991) also obtain a two-factor model, although their model is derived under non-expected utility, rather than VNM preferences. In addition, the relative weights depend on the distance between risk aversion, and the inverse of the elasticity of intertemporal substitution. In contrast, our measure assumes that the agent is an expected-utility maximizer. Moreover, the relative weights under WDU reflect the importance of consumption versus wealth risk aversion.

Our estimates indicate that optimal consumption is not proportional to wealth (finding 1). This has important consequences for the pricing equations. To see this consider the case where $\eta_0 = 0$ in (21). Then, the consumption/wealth ratio is constant, and the growth rates on consumption and wealth are equal: $dC_t/C_t = dW_t/W_t$. Consequently, so are the covariance terms. Substitute in the premium (40) to obtain that:

$$\begin{aligned}\mu_{p,t} - r_t|_{\eta_0=0} &= \left(\frac{\gamma\eta_c C_t}{\eta_c C_t + \eta_w W_t} + \frac{\gamma\eta_w W_t}{\eta_c C_t + \eta_w W_t} \right) \text{Cov}_t \left(\frac{dC_t}{C_t}, \frac{d\mathbf{P}_t}{\mathbf{P}_t} \right), \\ &= \gamma \text{Cov}_t \left(\frac{dC_t}{C_t}, \frac{d\mathbf{P}_t}{\mathbf{P}_t} \right),\end{aligned}$$

which is simply the standard C-CAPM with CRRA preferences, in which wealth dependence plays no role. Hence, our finding 1 that $\eta_0 \neq 0$ is important to allow for wealth dependence to impact asset returns. Furthermore, our unequivocal rejection of the CRRA model can be interpreted as the dual in goods space of its empirical anomalies in price space.

Second, the presence of a second source of IMRS risk is a welcomed addition in finding a solution for the equity risk premium puzzle. Finding 2 establishes that $\eta_w > 0$ such that the price of the market risk, R_t^{rw} , is positive. This result is consistent with the multi-factor empirical literature which finds that market risk is positively valued by the market (Chen et al., 1986; Ferson and Harvey, 1991). If the quantity of market risk is also positive, then a high equity premia need not be explained by consumption risk alone. This is also confirmed in our data set. Table 8 establishes that the total wealth risk of corporate stocks is much larger (by a ratio of 91:1) than consumption risk. A consequence

Table 8: Sample moments: Consumption, wealth growth and stock returns

	mean	std	covariances		
Consumption growth	0.02356	0.02079	0.00043	0.00051	0.00051
Wealth growth	0.03658	0.16291		0.02654	0.04701
Stock returns	0.13811	0.34218			0.11709

of estimating $\eta_w > 0$ is that this larger market risk can justify the high observed premia at a lower level of risk aversion. This is consistent with our finding 3 that the curvature parameter γ , and the risk aversion estimates in general, are lower under WDU.

Third, note that the prices of both risks will in general be time-varying. Our results indicate that relative risk aversion with respect to consumption (wealth) was counter- (pro-)

cyclical, with the overall indirect utility risk aversion being counter-cyclical (finding 4). This result would be consistent with the predictability puzzle whereby the conditional premia are observed to fall during booms, and pick up during recessions (Cochrane, 1997; Guvenen, 2003). In the absence of strong conditional heteroscedasticity effects in the quantities of consumption or market risks, predictability would be explained in our model by cyclical movements in risk aversion.

Risk-free rate As is well known, the risk-free rate puzzle is a by-product of the equity premium puzzle (Weil, 1989; Kocherlakota, 1996). A high risk aversion implies a low elasticity of inter-temporal substitution, and a high risk-free rate to induce savings. We have already mentioned that wealth dependence result in lower curvature indices (finding 3), thereby potentially addressing the risk-free rate puzzle.

Nonetheless, it is interesting to study the impact of WDU for the predicted risk-free rate. As in the standard case, the risk-free rate (41) captures a first-order and a second-order effect reflecting the mean and the variance of the IMRS. In our case, however, marginal utility depends on movements in both consumption and in wealth.

Our empirical findings would be consistent with a low risk-free for a number of reasons. First, the effective discount rate in (41) is now $\rho - \eta_w/\eta_c$; a positive η_w consistent with blasé behavior (finding 2) reduces it and consequently helps in reproducing the low observed r_t . Second, a low r_t is achieved if the second-order effect on IMRS is stronger than the first-order one. More precisely, allowing for wealth dependence affects both the mean (through the conditional mean terms for consumption and wealth growth) and the variance of the IMRS (through their conditional variance and covariance terms).

In particular, regardless of the sign of η_w , the variance of innovations in wealth enters negatively and reduces the risk-free rate. Table 8 shows that the volatility of wealth growth is much larger than that of consumption growth (by a ratio of 61:1; Lettau and Ludvigson, 2003, p. 2 also find that measured wealth growth is much more volatile than consumption growth over short horizons). This effect should therefore be important towards reducing the predicted rates. Moreover, the sample moments indicate that consumption growth is positively correlated with wealth growth. Since η_w was estimated to be positive this covariance in (41) tends to reduce further the predicted rate. Note however that $\eta_w > 0$ implies that the mean growth rate of wealth affects positively the predicted risk-free rate. Since the empirical moments in Table 8 indicate that mean consumption and wealth growth rates are roughly equal, this first-order effect could be important in increasing the predicted rate.

To conclude, our wealth-dependent framework has the theoretical potential to successfully address the three main pricing anomalies of the C-CAPM. Our empirical findings in goods space are consistent with a WDU explanation of empirical asset returns puzzles. Whether or not similar estimation results in price space are obtained will require further analysis which we leave on the research agenda.

6 Conclusion

Summary This paper proposes a new instantaneous utility function in which the bliss level of consumption is an affine function of wealth. The specification is related to preference for status, as well as wealth-dependent habit formation and durability.

The empirical implementation focuses on the closed-form expressions in the quantity space. The structural econometric model imposes the full theoretical restrictions on the

conditional drifts and diffusions of a multi-variate Brownian motion composed of changes in consumption, asset holdings and wealth. We estimate the model using aggregate data for these variables. We fully control for any discretization bias in evaluating the transition density. Our main results show the presence of a blasé effect whereby higher wealth reduces the marginal utility of consumption. Moreover, we find that the implied risk aversion is realistic, and counter-cyclical. This is consistent with heuristic arguments and other findings in price space that also relax constant risk aversion. Importantly, we strongly reject the null of CRRA preferences. Our discussion of results focuses on their asset pricing implications. We show that wealth-dependent utility generates a larger IMRS risk. This risk justifies a larger premium on risky assets and a lower risk-free rate.

Future research Future research should study the pricing kernels of WDU preferences. In particular, it would be useful to test whether our findings are confirmed when the empirical implementation is taken in price, rather than quantity space. Moreover, our estimation relies on a narrow definition of wealth (cash + bonds + stocks). Future research should allow for a more comprehensive measure of total wealth for estimation purposes.

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A Lemma 1

Proof. Solve for C_t in (13), and \mathbf{v}_t in (14), and substitute both in the Bellman equation (5). The indirect utility must satisfy:

$$0 = \frac{\gamma}{(1-\gamma)\eta_c} \left(\frac{J_{w,t}}{\eta_c} \right)^{-1/\gamma} - \frac{\rho J_t}{J_{w,t}} + \left(rW_t + \frac{\eta_0}{\eta_c} + \frac{\eta_w}{\eta_c} W_t \right) - 0.5 \frac{J_{w,t}}{J_{ww,t}} Q, \quad (42)$$

where $Q \equiv (\boldsymbol{\mu}_p - r)' \boldsymbol{\Sigma}_{pp}^{-1} (\boldsymbol{\mu}_p - r) > 0$. Consider the candidate solution for indirect utility as:

$$J(W_t) = \frac{(G + FW_t)^{1-\gamma}}{1-\gamma}.$$

Substitute in (42) to obtain that:

$$\begin{aligned} 0 &= \left\{ \frac{\gamma}{(1-\gamma)\eta_c} \left(\frac{F}{\eta_c} \right)^{-1/\gamma} G - \frac{\rho G}{(1-\gamma)F} + \frac{\eta_0}{\eta_c} + 0.5Q \frac{G}{\gamma F} \right\} \\ &\quad + \left\{ \frac{\gamma}{1-\gamma} \left(\frac{F}{\eta_c} \right)^{(\gamma-1)/\gamma} - \frac{\rho}{1-\gamma} + r + \frac{\eta_w}{\eta_c} + \frac{0.5Q}{\gamma} \right\} W_t \end{aligned} \quad (43)$$

The unique solution in which F, G are at most functions of time is when both elements in curly brackets are zero. Starting with the second term uniquely identifies F as in (17). Substitute in the first term to solve for G as in (18). ■

B Proposition 1

Proof. Using (13), and (14) reveals that the optimal rules are:

$$C_t = \frac{1}{\eta_c} \left(\frac{J_{w,t}}{\eta_c} \right)^{-1/\gamma} - \frac{\eta_0}{\eta_c} - \frac{\eta_w}{\eta_c} W_t, \quad (44)$$

$$\mathbf{v}_t W_t = \frac{-J_{w,t}}{J_{ww,t}} \boldsymbol{\Sigma}_{pp}^{-1} (\boldsymbol{\mu}_p - r). \quad (45)$$

Use the indirect function (16) in Lemma 1 to obtain that the optimal rules are:

$$C_t = \left[\frac{1}{\eta_c} \left(\frac{F}{\eta_c} \right)^{-1/\gamma} G - \frac{\eta_0}{\eta_c} \right] + \left[\left(\frac{F}{\eta_c} \right)^{(\gamma-1)/\gamma} - \frac{\eta_w}{\eta_c} \right] W_t, \quad (46)$$

$$\mathbf{v}_t W_t = \frac{G \boldsymbol{\Sigma}_{pp}^{-1} (\boldsymbol{\mu}_p - r)}{\gamma F} + \frac{\boldsymbol{\Sigma}_{pp}^{-1} (\boldsymbol{\mu}_p - r)}{\gamma} W_t. \quad (47)$$

Use (17) and (18) to substitute for F and G . ■

C Lemma 2

Proof. First, (31) and (32) reveal that:

$$dX_t = [x_w \mu_0 + \mu_w (X_t - x_0)] dt + [x_w \sigma_0 + \sigma_w (X_t - x_0)] dZ_t \quad (48)$$

$$= \mu(X_t) dt + \sigma(X_t) dZ_t. \quad (49)$$

Next, by Itô's lemma, we have for $\tilde{X}_t = \tilde{X}(X_t)$:

$$d\tilde{X}_{j,t} = \left[\mu(X_t) \tilde{X}'(X_t) + 0.5\sigma(X_t)^2 \tilde{X}''(X_t) \right] dt + \sigma(X_t) \tilde{X}'(X_t) dZ_t \quad (50)$$

Observe that $\mu_0/\mu_w = \sigma_0/\sigma_w$ to substitute in (50) to obtain (34). ■

D Proposition 2

Proof.

Risk premia Using (13) and (14) reveals that:

$$\begin{aligned}\boldsymbol{\mu}_{p,t} - r_t &= \frac{-J_{ww,t}}{J_{w,t}} W_t \boldsymbol{\Sigma}_{pp,t} \mathbf{v}_t, \\ &= -\frac{U_{cc,t}}{U_{c,t}} C_{w,t} W_t \boldsymbol{\Sigma}_{pp,t} \mathbf{v}_t - \frac{U_{cw,t}}{U_{c,t}} W_t \boldsymbol{\Sigma}_{pp,t} \mathbf{v}_t\end{aligned}\quad (51)$$

Following Breeden (1979), it is straightforward to show that:

$$\begin{aligned}\text{Cov}_t \left(dW_t, \frac{d\mathbf{P}_t}{\mathbf{P}_t} \right) &= W_t \boldsymbol{\Sigma}_{pp,t} \mathbf{v}_t \\ \text{Cov}_t \left(dC_t, \frac{d\mathbf{P}_t}{\mathbf{P}_t} \right) &= C_{w,t} \text{Cov}_t \left(dW_t, \frac{d\mathbf{P}_t}{\mathbf{P}_t} \right).\end{aligned}$$

Substitute in (51) to obtain:

$$\boldsymbol{\mu}_{p,t} - r_t = \left(\frac{-U_{cc,t}}{U_{c,t}} \right) \text{Cov}_t \left(dC_t, \frac{d\mathbf{P}_t}{\mathbf{P}_t} \right) + \left(\frac{-U_{cw,t}}{U_{c,t}} \right) \text{Cov}_t \left(dW_t, \frac{d\mathbf{P}_t}{\mathbf{P}_t} \right), \quad (52)$$

and use the utility function (6) to obtain (40).

Risk-free rate We follow Cox et al. (1985) by first establishing the relation between the interest rate and the expected rate of growth of the marginal utility of wealth. First, pre-multiply first-order condition (14) by \mathbf{v}'_t and divide by W_t to obtain that:

$$J_{w,t} \mathbf{v}'_t (\boldsymbol{\mu}_{p,t} - r_t) + J_{ww,t} \mathbf{v}'_t \boldsymbol{\Sigma}_{pp,t} \mathbf{v}_t W_t = 0. \quad (53)$$

Next, by Itô's lemma, we have for $J_{w,t} = J_w(W_t)$:

$$dJ_{w,t} = J_{ww,t} dW_t + 0.5 J_{www,t} dW_t^2, \quad (54)$$

such that,

$$\mathbb{E}_t(dJ_{w,t}) = J_{ww,t} \mathbb{E}_t(dW_t) + 0.5 J_{www,t} \text{Var}_t(dW_t), \quad (55)$$

Returning to the Bellman equation (5), use Envelope theorem and take derivatives with respect to W to obtain:

$$\begin{aligned}0 &= U_{w,t} - \rho J_{w,t} + J_{ww,t} \{ [\mathbf{v}'_t (\boldsymbol{\mu}_{p,t} - r_t) + r_t] W_t - C_t \} + J_{w,t} [\mathbf{v}'_t (\boldsymbol{\mu}_{p,t} - r_t) + r_t] \\ &\quad + J_{ww,t} W_t \mathbf{v}'_t \boldsymbol{\Sigma}_{pp,t} \mathbf{v}_t + 0.5 J_{www,t} W_t^2 \mathbf{v}'_t \boldsymbol{\Sigma}_{pp,t} \mathbf{v}_t.\end{aligned}\quad (56)$$

Use (53) and (55) to simplify this expression to:

$$0 = U_{w,t} - \rho J_{w,t} + r_t J_{w,t} + \mathbb{E}_t(dJ_{w,t}) \quad (57)$$

But, note from (12) and first-order condition (13) that:

$$U_{w,t} = U_{c,t} (\eta_w / \eta_c) = J_{w,t} (\eta_w / \eta_c).$$

Substitute into (57) to obtain that:

$$r_t = \rho - \eta_w/\eta_c - E_t \left(\frac{dJ_{w,t}}{J_{w,t}} \right). \quad (58)$$

Next, use again Itô's lemma in first-order condition (13) to obtain that:

$$\begin{aligned} E_t \left(\frac{-dJ_{w,t}}{J_{w,t}} \right) &= \frac{-U_{cc,t}}{U_{c,t}} E_t(dC_t) + \frac{-U_{cw,t}}{U_{c,t}} E_t(dW_t) \\ &- 0.5 \left\{ \frac{U_{ccc,t}}{U_{ct}} \text{Var}_t(dC_t) + 2 \frac{U_{ccw,t}}{U_{c,t}} \text{Cov}_t(dC_t, dW_t) + \frac{U_{cww,t}}{U_{c,t}} \text{Var}_t(dW_t) \right\}. \end{aligned} \quad (59)$$

Use utility (6) to substitute the expressions for the first, second, third and cross derivatives in order to obtain (41).

■

E Additional Figures

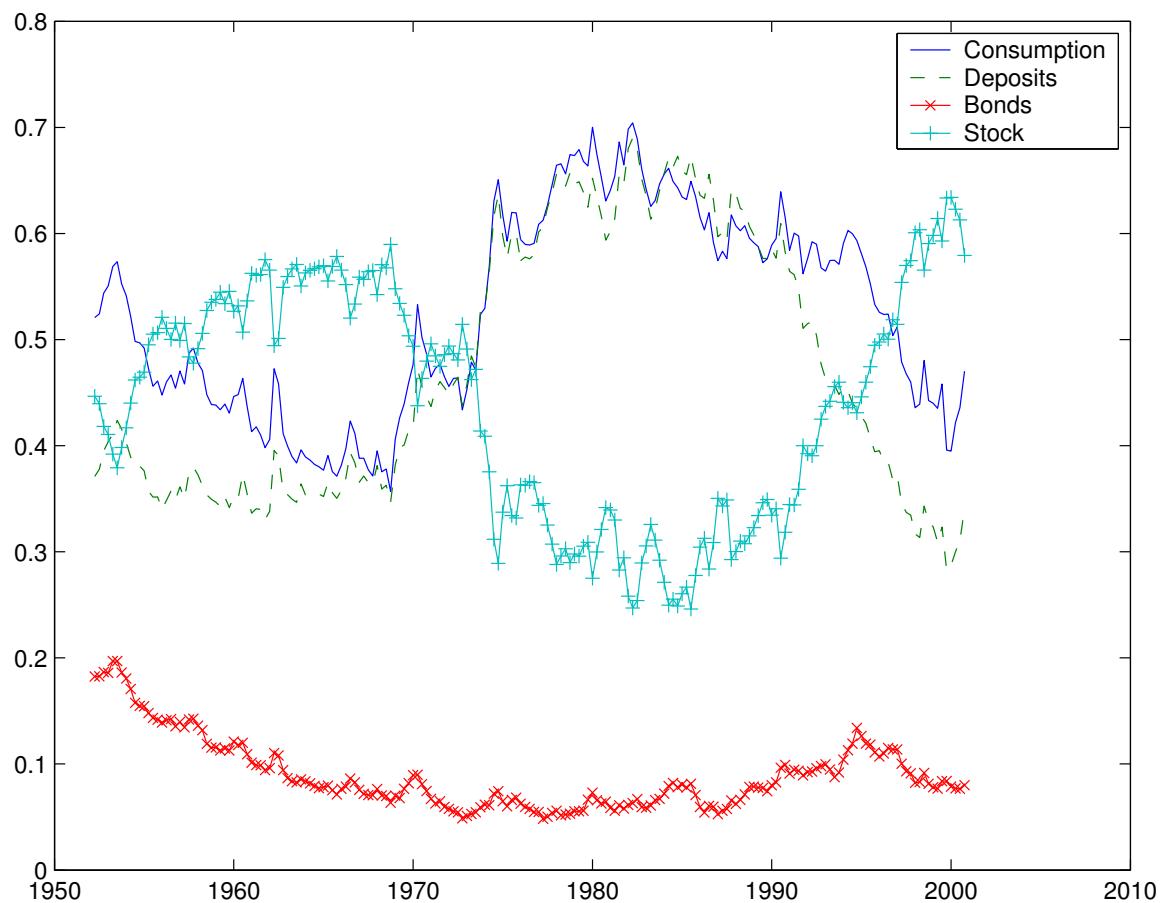


Figure 3: Shares of Wealth, C_t/W_t , V_t/W_t

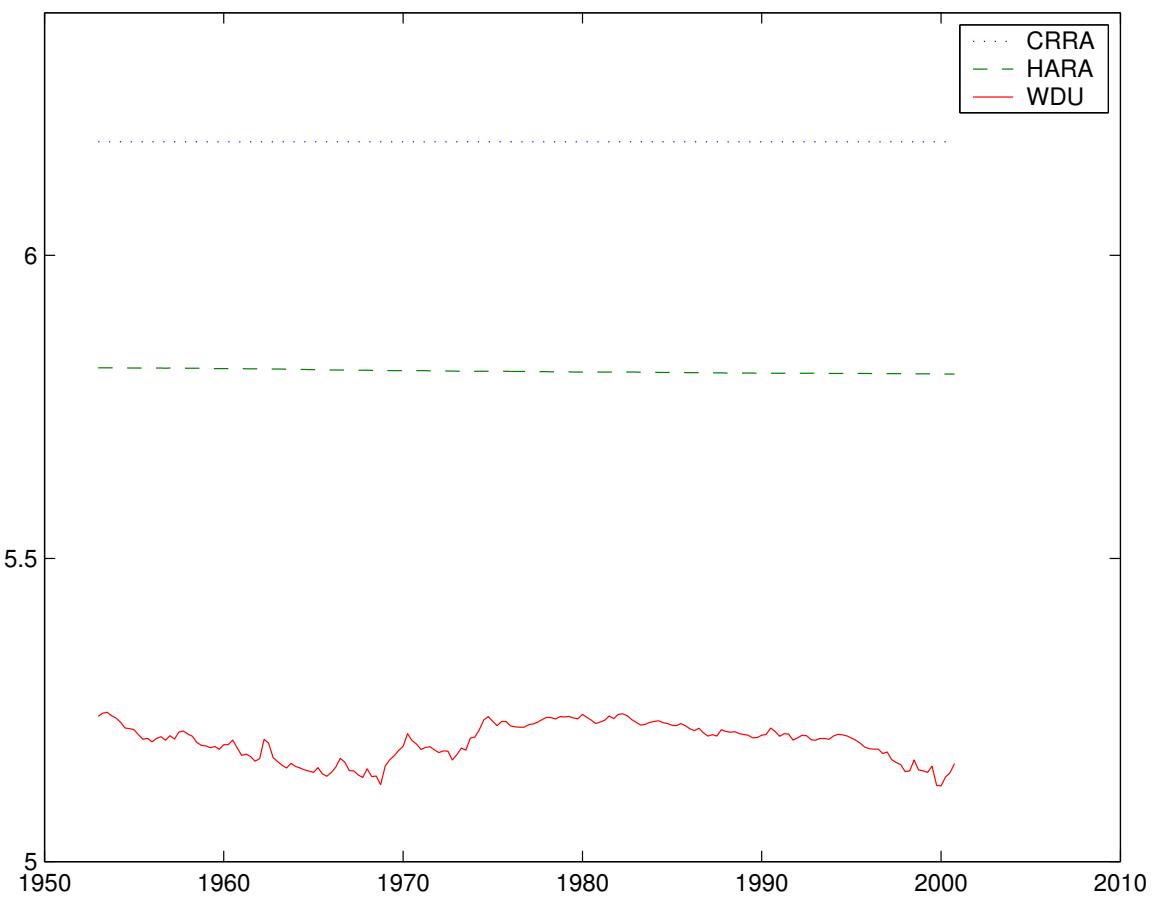


Figure 4: Consumption Risk Aversion, $-C_t U_{cc,t}/U_{c,t}$

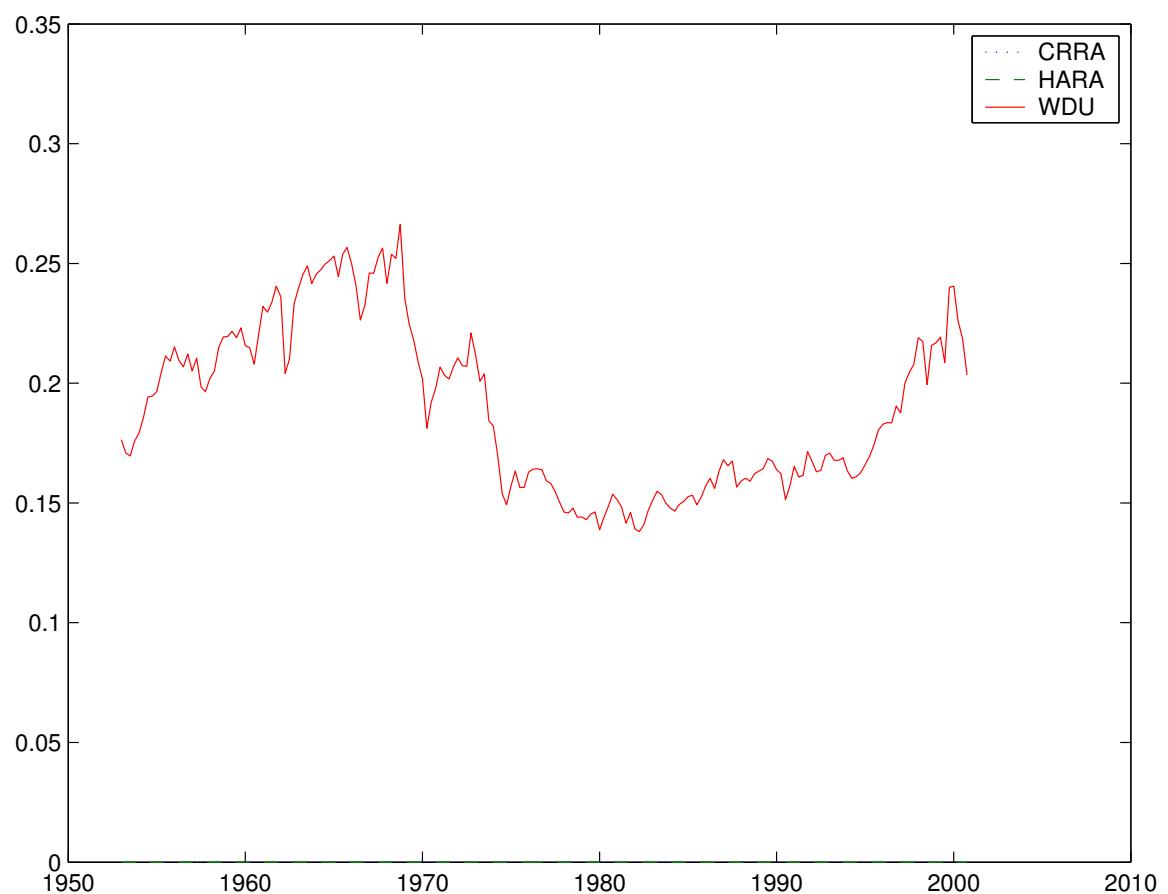


Figure 5: Wealth Risk Aversion, $-W_t U_{ww,t} / U_{w,t}$

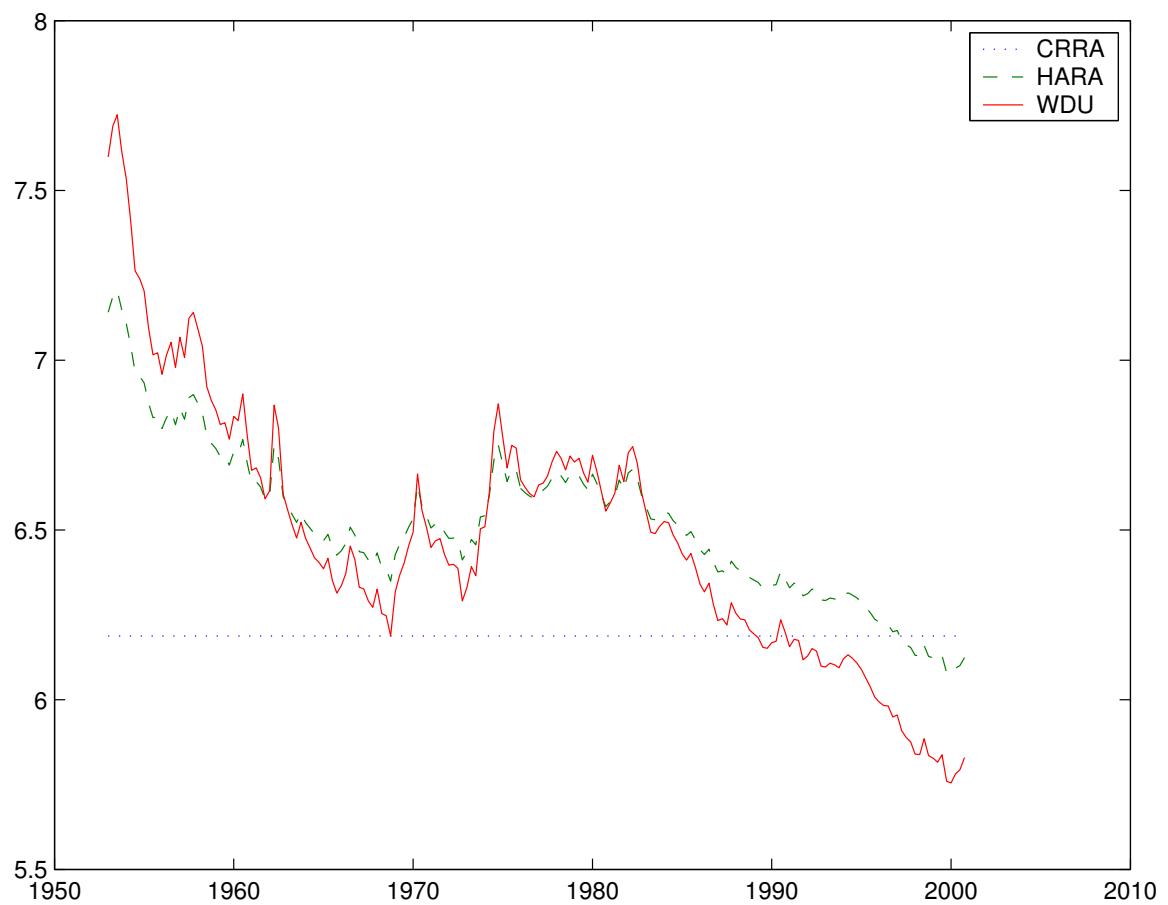


Figure 6: Risk Aversion, $-W_t J_{ww,t} / J_{w,t}$

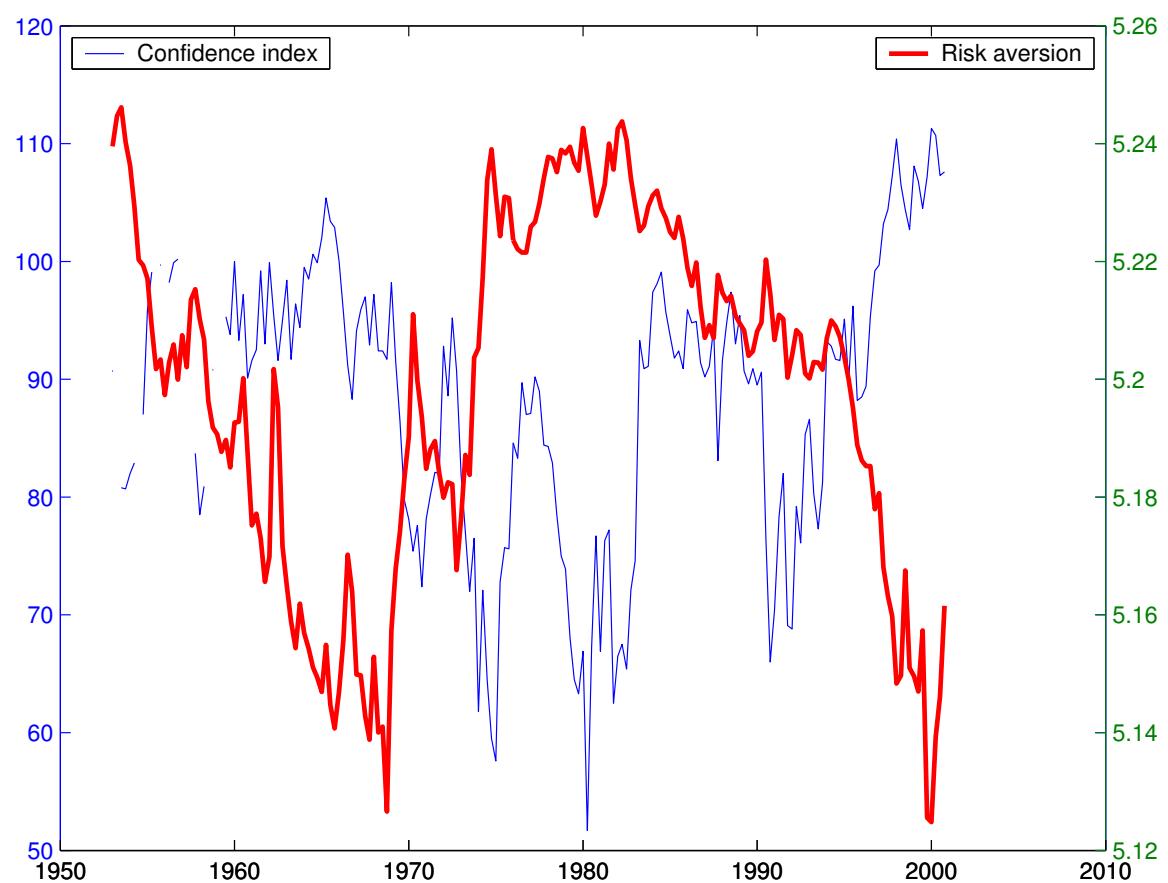


Figure 7: Consumption Risk Aversion, $-C_t U_{cc,t}/U_{c,t}$, and Consumer Confidence Index

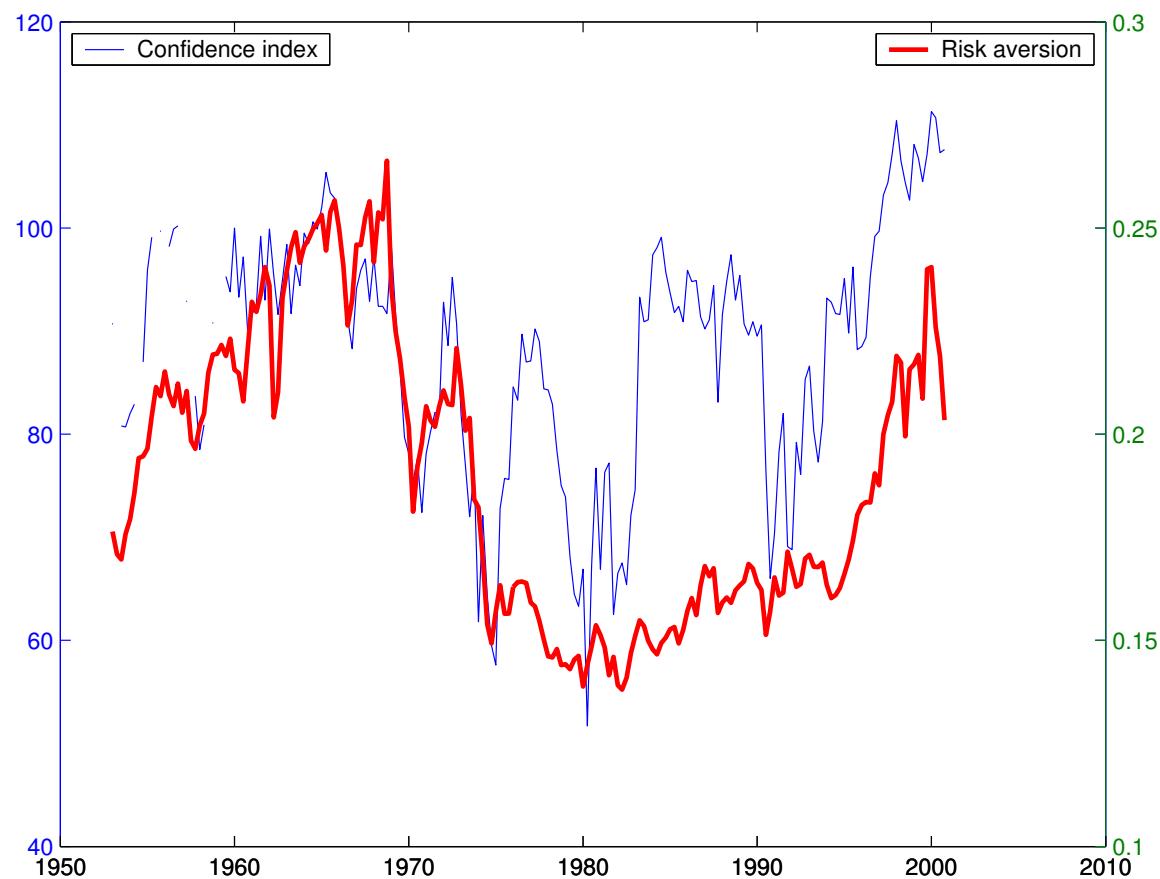


Figure 8: Wealth Risk Aversion, $-W_t U_{ww,t} / U_{w,t}$, and Consumer Confidence Index

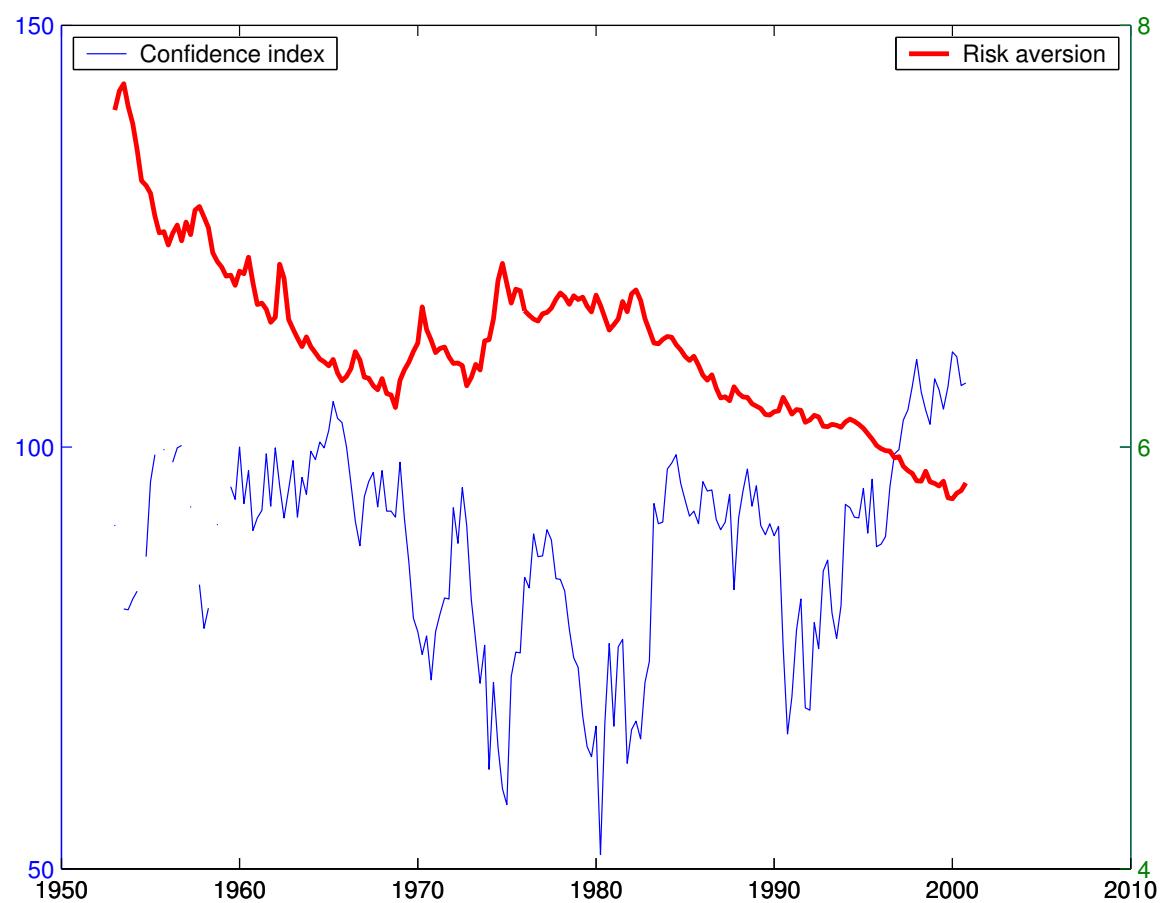


Figure 9: Risk Aversion, $-W_t J_{ww,t} / J_{w,t}$, and Consumer Confidence Index