Competitive Advertising and Pricing

Raphael Boleslavsky Ilwoo Hwang

Kyungmin (Teddy) Kim

University of Miami

November 2018

Raphael Boleslavsky and Ilwoo Hwang

This Paper

- Bertrand competition for differentiated products
  - Perloff and Salop (1985)
  - n firms with horizontally differentiated products
  - $v_i \sim F$: i.i.d. across consumers and products
  - Consumer purchases i if $v_i - p_i > v_j - p_j, \forall j \neq i$
  - Demand for product i:

$$D(p_i, p^*) = Pr\{v_i - p_i > v_j - p^*\} = \int F(v_i - p_i + p^*)^{n-1} dF(v_i).$$

  - Each firm solves $\max_{p_i} D(p_i, p^*) p_i$.

- (Informative) Strategic advertising
  - Each firm decides how much product information to provide.
  - No structural assumption on advertising
  - Seller can choose any mean-preserving contraction G of F
    - No info: $G = \delta_{\mu_F}$, Full info: $G = F$, Cutoff...
  - A way to endogenize F in Perloff and Salop

Research Questions

1. Advertising content under competition
   - Monopoly: pool all values above MC, extract all surplus
   - How competition shapes advertising content?
   - More information as n increases?
2. Effects of strategic advertising on price (welfare)
   - Full information vs. equilibrium information
   - Economic effects of disclosure policies
3. Interaction between pricing and advertising
   - How to adjust advertising strategy as p_i varies?

Most Related Literature

- Classical studies on advertising, product differentiation
- Under structural assumptions
  - Monopoly: Lewis, Sappington (1994), Johnson, Myatt (2006), Anderson and Renault (2006)...
  - Competition: Ivanov (2013)
- Advertising-only game (competitive Bayesian persuasion)
  - Boleslavsky, Cotton (2015, 2018): binary types
  - Au, Kawai (2017): finite types
- Entry game: Boleslavsky, Cotton, Gurnani (2017)
  - New (innovative) firm vs. old (established) firm
  - Binary types, and demonstrations before/after pricing
- Optimal information design with continuous state space
  - Kolotilin (2017), Dworczak, Martini (2018)

The Model

- n sellers with zero MC
- A unit mass of risk-neutral consumers
- Each consumer's (true, underlying) value for i
  - $v_i \sim F[\underline{v}, \overline{v}]$: i.i.d. across consumers and products
  - $\underline{v} = -\infty$, $\overline{v} = \infty$ allowed
  - F has continuous and positive density f
- Each seller chooses G_i (advertising) and p_i
  - G_i: distribution over conditional expectations $E[v|s]$
  - G_i: feasible iff mean-preserving contraction of F
- Each (risk-neutral) consumer purchases i if

$$v_i - p_i > v_j - p_j, \forall j \neq i,$$

where $v_j \sim G_j$ for all j.

Symmetric Pure-Price Equilibrium

- (p^*, G) is a (symmetric pure-price) equilibrium if

$$(p^*, G) \in argmax_{p_i, G_i} D(p_i, G_i, p^*, G) p_i$$

s.t. G_i is a mean-preserving contraction of F, where

$$\begin{aligned} D(p_i, G_i, p^*, G) &= Pr\{v_i - p_i > v_j - p^*, \forall j \neq i\} \\ &= \int G(v_i - p_i + p^*)^{n-1} dG_i(v_i) \end{aligned}$$

Roadmap

1. Characterize equilibrium advertising strategy
   - Given $p_i = p^*$, find G that is best response to G^{n-1}
   - "Advertising-only game"
2. Characterize equilibrium price
   - Given G, find p^* that is a best response to p^*
3. Equilibrium existence
   - Consider all compound deviations (p_i, G_i) from (p^*, G)

Equilibrium Advertising

Theorem

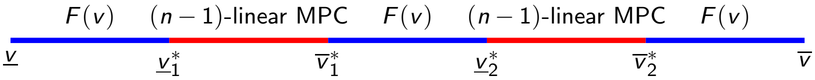
Let G^* be a (unique) MPC of F such that

(i) $(G^*)^{n-1}$ *is convex over its support and*

(ii) *for some partition* $\{\overline{v}_0^* \equiv \underline{v}, \underline{v}_1^*, \overline{v}_1^*, \ldots, \underline{v}_m^*, \overline{v}_m^*, \underline{v}_{m+1}^*\}$,

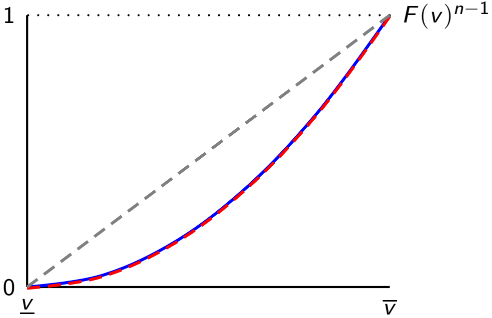
- $G^*[\underline{v}_k, \overline{v}_k]$ *is MPC of* $F[\underline{v}_k, \overline{v}_k]$ *with linear* $(G^*)^{n-1}$ *and*
- $G^*(v) = F(v)$ *if* $v \in (\overline{v}_k, \underline{v}_{k+1})$.

The advertising-only game has a unique symmetric equilibrium in which each firm advertises according to G^*.



Example 1: F^{n-1} convex (increasing density)

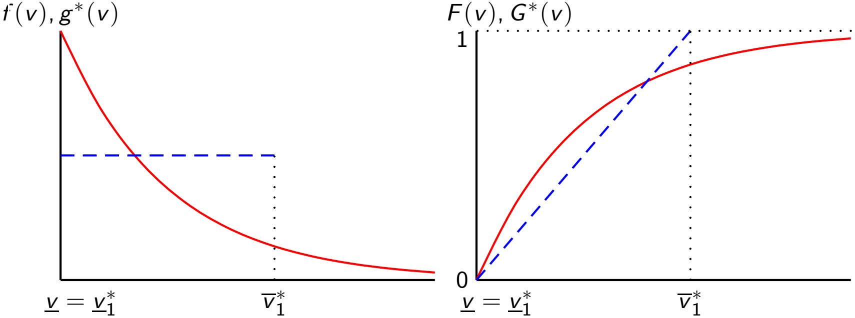
- $G^* = F$: product information fully provided



- Disperse v's as much as possible
- MPC constraint binds.

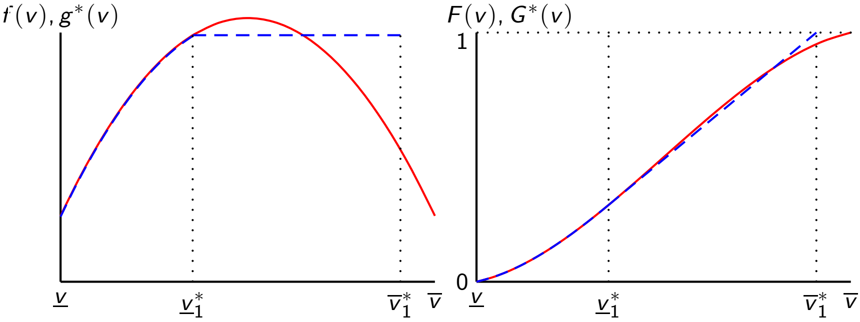
Example 2: F^{n-1} concave (decreasing density)

- Occur only when $n = 2$
- If $\underline{v} = 0$, then $G^* = U[0, 2\mu_F]$

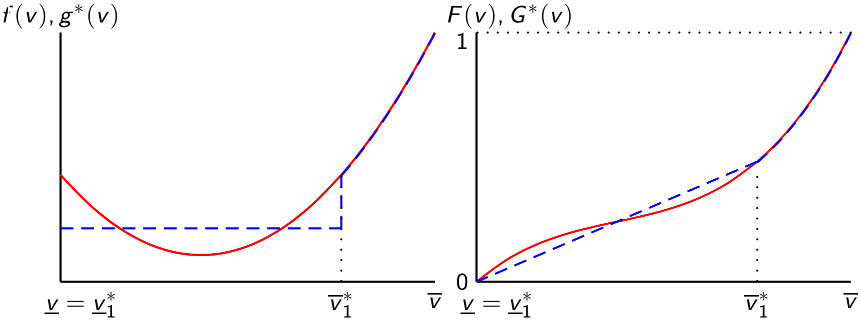


- If $G_j = F$, then $G_i = \delta_{\mu_F}$. But then, $G_j = F$ not optimal...
- G_j linear $\Rightarrow$ neither dispersion nor contraction profitable

Example 3: F^{n-1} convex-concave (single-peaked density)



Example 4: F^{n-1} concave-convex (U-shaped density)



Intuition for Theorem 1

1. $(G^*)^{n-1}$ convex
   - If G_j not convex at $v \in supp(G_j)$, then G_i puts mass on v.
   - Then, $v \notin supp(G_j)$.

2. Either $(G^*)^{n-1}$ linear or $G^* = F$
   - Since G^* is a MPC of F and $(G^*)^{n-1}$ convex,

$$\int (G^*)^{n-1} dG^* \leq \int (G^*)^{n-1} dF.$$

   - This must hold with equality: o/w $F \succ G^*$
   - Either $G^* = F$ or $(G^*)^{n-1}$ linear (risk neutral)

($*$) The second needs modification if $supp(G^*) \neq supp(F)$.

Competition Intensity on Advertising Content

Proposition

As $n \to \infty$, G^* converges to F.

Proof.

- F^{n-1} becomes more convex as n increases:

$$(F^{n-1})'' = (n-1)((n-2)F^{n-3}f^2 + F^{n-1}f').$$

□

- As $n \to \infty$, making a few loyal consumers becomes more important.
- Ivanov (2013)
  - Identical economic result based on *rotation order* by Johnson and Myatt (2006)

Equilibrium Price

- Optimal pricing: Since $\pi_i = D_i p_i$,

$$\text{(F.O.C)}\ D_i + \frac{\partial D_i}{\partial p_i} p_i = 0 \Rightarrow p_i = \frac{D_i}{-\partial D_i / \partial p_i}.$$

- In symmetric equilibrium, $D_i = 1/n$, and thus

$$p^* = \frac{1}{n(n-1) \int (G^*)^{n-2} g^* dG^*}.$$

- Under full information (i.e., $G_i = F$),

$$p^F = \frac{1}{n(n-1) \int F^{n-2} f dF}.$$

Strategic Advertising vs. Full Information

- Intuitively, $p^* \leq p^F$
  - $p > 0$ because of preference diversity (product differentiation)
  - G^* is a MPC of (so less dispersed than) F
- How to measure preference diversity?
  - Perloff and Salop (1985): MPS (SOSD) not work in general
  - Zhou (2017), Choi, Dai, Kim (2018): *dispersive order* works!
- G^* and F not ranked in dispersive order
  - Zhou and CDK not apply
- G^* is a particular type of MPC of F
  - PS not apply either

Strategic Advertising vs. Full Information

1. Exponential: $F(v) = 1 - e^{-\lambda v}$
   - Well-known that $p^F = 1/\lambda$, independent of n
   - $G^*(v) = F(v)$ until $v^*(> 0)$, then $(G^*)^{n-1}$ linear, but...

$$p^* = \frac{1}{\lambda}, \forall n \geq 2.$$

2. Duopoly: $n = 2$, $\mu_F = 1$, $G^* = U[0,2] \Rightarrow p^* = 1$
   - Dec. linear density: $f(v) = b - av \Rightarrow p^F > 1$
   - Half-normal, truncated exponential $\Rightarrow p^F > 1$
   - U-shaped density symmetric around $\mu_F = 1 \Rightarrow p^F < 1$

Strategic Advertising vs. Full Information

- If $n = 2$, then

$$p^* = \frac{1}{2\int g^* dG^*} = \frac{1}{2\int (g^*)^2 dv}.$$

- Under full information (i.e., $G_i = F$),

$$p^F = \frac{1}{2\int f^2 dv}.$$

- When $n = 2$,

$$p^F \geq p^* \Leftrightarrow \int f^2 dv \leq \int (g^*)^2 dv.$$

Two Effects of Mean-Preserving Contraction

1. Support effect
   - Combine $f(v_1)$ and $f(v_2)$ into one

$$(f(v_1) + f(v_2))^2 > f(v_1)^2 + f(v_2)^2.$$

   - Always $\uparrow$

2. Marginal effect
   - Let $v_1 < v_3 < v_4 < v_2$, and $f_i = f(v_i), \forall i$
   - $f_1 - d$, $f_3 + d$, $f_4 + d$, $f_2 - d$

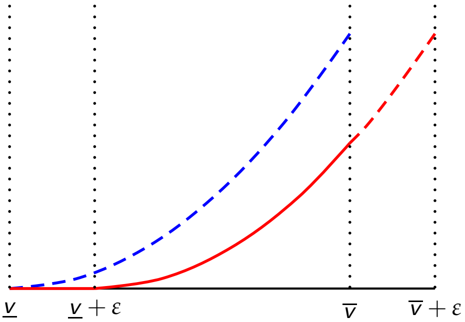
$$\begin{aligned} & (f_1 - d)^2 + (f_3 + d)^2 + (f_4 + d)^2 + (f_2 - d)^2 - \sum f_i^2 \\ = \; & 2d\left[(f_3 + f_4) - (f_1 + f_2)\right] > 0 \text{ iff } f_3 + f_4 > f_1 + f_2. \end{aligned}$$

   - $\uparrow$ if f $\cap$-shaped, while $\downarrow$ if f $\cup$-shaped

Compound Deviations

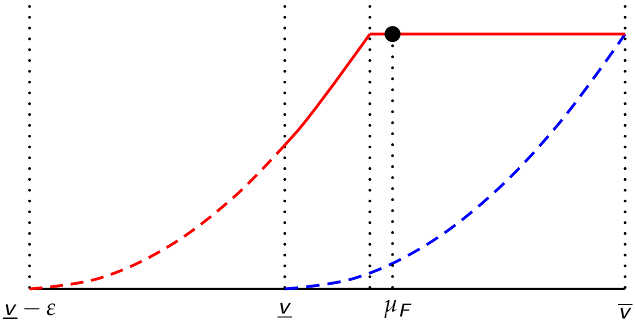
- So far, only simple deviations
  - G^* only by considering $G_i \neq G^*$, while fixing $p_i = p^*$.
  - p^* only by considering $p_i \neq p^*$, while fixing $G_i = G^*$.
- Compound deviations: $p_i \neq p^*$ *and* $G_i \neq G^*$
  - How a firm's advertising and pricing decisions interact each other.
  - (p^*, G^*) is an equilibrium if and only if no (p_i, G_i) is profitable.
- Our strategy: for each p_i, identify optimal G_i^*.
  - (p^*, G^*) is an equilibrium iff no (p_i, G_i^*) is profitable.
  - Today, only the case where $G^* = F$

When p_i is larger than p^*



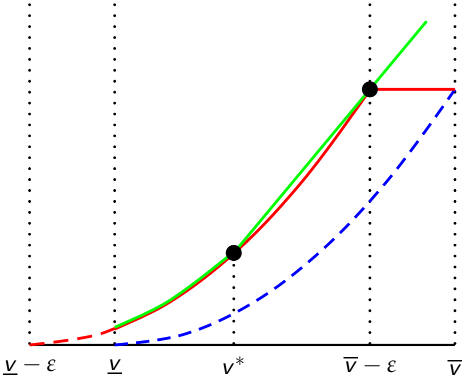
- $\varepsilon \equiv p_i - p^*$
- $G^*(v - p_i + p^*)^{n-1}$ is convex over $[\underline{v}, \overline{v}]$.
- Therefore, $G_i^* = F$.

When p_i is sufficiently smaller than p^*



- $\varepsilon \equiv p^* - p_i$
- No information is optimal: $G_i^* = \delta_{\mu_F}$

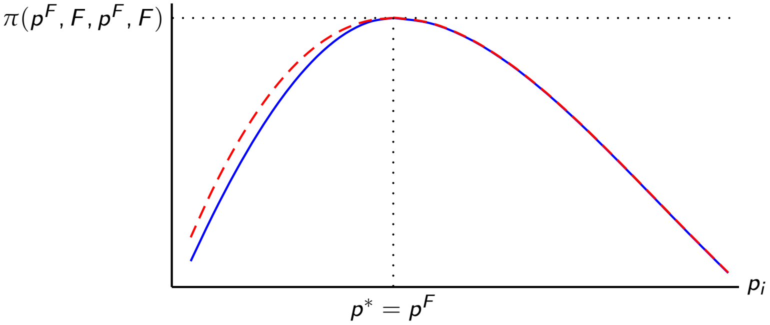
When p_i is slightly smaller than p^*



- $\varepsilon \equiv p^* - p_i$
- $G_i^* = F$ if $v \leq v^*$ and then put all remaining mass on $\overline{v} - \varepsilon$.

Existence of Full Information Equilibrium

- $G^* = G_i^* = F$ if $p_i \geq p^* = p^F$.
- $G_i^* \neq G^* = F$ if $p_i < p^*$.
- Relative to the full info benchmark where $G_i = F$ always,
  - upward deviation $(p_i > p^F)$ is equally profitable, while
  - downward deviation $(p_i < p^F)$ is more profitable.
- Need stronger condition for equilibrium existence than in Perloff and Salop.



Conclusion

1. Bertrand competition with strategic advertising
   - Endogenous F in the Perloff-Salop model
2. Competitive advertising (information disclosure)
   - With continuous underlying distributions (F)
   - Look for G^*!
     - $(G^*)^{n-1}$ is convex and linear unless $G^*(v) = F(V)$
   - More competition $\Rightarrow$ more informative advertising
3. Effects of advertising on price
   - p^* may or may not be smaller than p^F.
   - Stricter disclosure requirements may not help.
4. Effects of pricing on advertising
   - Optimal advertising strategy depends on p_i