Motivation

- Bergemann and Morris (2016, TE) note that static game of incomplete information can be decomposed into:

  1. *Basic game*: $G = (A, u, p)$ where A are the actions $u : A \times \Omega \longrightarrow R^n$ are payoffs as a function of actions and states and p is a common prior over state spaces. This is a game with symmetric information.

  2. An *information structure* (T, π) where T are the types and $\pi : \Omega \longrightarrow \Delta(T)$.

- Bergemann and Morris (2016) characterize outcomes that can arise as (Bayes) Nash equilibria and obtain a generalization of the equivalence between two representations of correlated equilibria in complete information games.

- The generalized notion of correlated equilibrium is called *Bayes correlated equilibria*. Convenient as equilibrium set characterized by intuitive incentive constraints as opposed to details of the information structure.

- This paper seeks to generalize to *dynamic* games of incomplete information.

- This is a hard. Consistency in sequental equilibrium fixes information structure.

The "Base Game"

- Instead of a decomposing the underlying primitive game into a *basic game* and information structure (where signals don't affect payoffs directly) Makris and Renou calls their primitive a *base game*, denoted Γ.

- The base game defines a (finite) dynamic game of incomplete information (with perfect recall):

  - stages: $t = 1, ..., T$
  - state spaces Ω_t at each stage
  - actions A_t at each stage t
  - information structure (signals) $(S_1, ...S_T, p)$ where at each t p maps histories (sequences of action profiles, past and *current* states and past signals) onto $\Delta(S_t)$

- I miss the decomposition into a symmetric information game and the information structure from Bergemann and Morris (2016).

- Unfortunately, this is a necessary sacrifice because of the generality of the setup. Changing information structure (S, p) allows changing game from what we'd normally consider "simultaneous" moves to a sequential game.

- However, notational issues

  – $H_{i,t} = A_{i,t-1} \times S_{i,t}$ is the "new information"

  – I'm guessing that h_t is supposed to be an element of $\times_{i=1}^{I} H_{i,t}$ and $h^t = (h_1, .., h_t)$

  – But, the notation for the transition probabilities

$$p_{t+1}\left(h_{t+1}, \omega_{t+1} | a_t, h^t, \omega^t\right)$$

  is pretty confusing. The vector h_{t+1} includes *actions*, and one wonders why histories are recorded like transition probabilities? For a while I even thought some equilibrium recommendations were built in.

  – Why not take recording of histories completely out of the processes for states and signals?

- Another notation I don't like is $u_i(h, \omega)$ for the (ex post) utility. The issue is that h contains some components that are pure information that the players should not care about.

Expansions

- An expansion is (if I understand things correctly) just adding more information exactly like in Bergemann and Morris (2016)
- However, "base games" don't just specify the extensive form in case no additional information is added.
- "Base games" with the same associated extensive form have different possibilities for expansions.
- Concretely (I think), you can code a simultaneous move game either as having players literally move at the same time, or as having them move sequentially and making it possible to add information about who did what.
- I think it is tricky to think what the right model is, but maybe there are examples that can be convincing.

A Few Thoughts

- I would drop some generality and let information be just information.

- Example 4 is interesting. This is a setup similar (I think) to Noldeke & Van Damme. If no information is provided, the game is "Cournot". If perfect information is provided game is "Stackelberg". Also illustates that sometimes everything is on the path, so working in strategic form can be useful also with dynamic base games.

- Example generates a nice convex set of Bayes correlated equilibria. But, I think we know that whenever we are between "Cournot" and "Stackelberg" there is a pure strategy equilibrium replicating "Cournot".

- Put, differently, "revelation principles" is about existence of some equilibrium. It seems to me that, in the example in the paper, the most plausible equilibrium for information structures "in the middle" may be pure strategy equilibria where the information structure is irrelevant. So, maybe multiplicity issues can be more of a problem than for typical mechanism design problem?