Information design in multi-stage games

Miltos Makris & Ludovic Renou

The Economics of Strategic Communication and Persuasion:
Application to Evidence-Based Public Policy Workshop

November 16-17 2018

An introductory example: information about past actions

- Two players and two stages.
- Player 1 chooses action $a_1 \in \{T, B\}$ in the first stage. Player 2 is inactive.
- Player 2 chooses $a_2 \in \{L, R\}$ in the second stage. Player 1 is inactive.
- Player 2 has no signals about the action chosen by player 1 before choosing his action.
- The payoffs are:

	L	R
T	2,2	0,1
B	3,0	1,1

Example cont'd

- We are interesting in characterizing the distributions μ over action profiles, which arise as we change the information structure.
- In particular (but not only), this implies varying the observation player 2 has about the action chosen by player 1 before choosing his own action.
- Two obvious distributions:
  - $\mu(B, R) = 1$, which corresponds to the unique Nash equilibrium (no additional signals).
  - $\mu(T, L) = 1$, which corresponds to the Stackelberg outcome (player 2 is perfectly informed of player 1's move).

Example cont'd

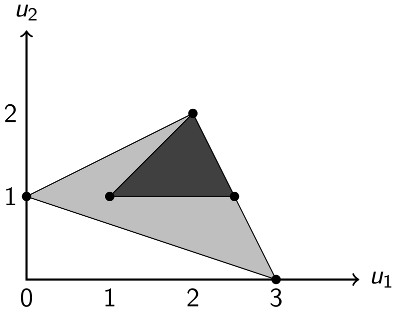
We can get more. For instance, we can get $\mu(T, L) = \mu(B, L) = 1/2$.

- Two equally likely signals t and b at the first stage; player 1 is privately told the first-stage signal.
- Two signals at the second-stage l and r; player 2 is privately told the second-stage signal.
- Player 2 receives l if and only if (T, t) and (B, b) are the first-stage profiles of signals and actions.
- An equilibrium of that augmented game consists in players playing according to their signals. This gives us the desired distribution.

Example cont'd

In fact, the set of possible distributions is

$$\{\mu : \mu(T, L) \geq \mu(B, L), \mu(B, R) \geq \mu(T, R), \mu(T, L) \geq \mu(T, R)\}.$$



The main contribution: we provide revelation principles to characterize the equilibrium distributions of games, as we vary the information structures.

Example cont'd: final remarks

- Bergemann and Morris (2016) introduce the concept of Bayes correlated equilibria for static games of incomplete information.
- At a Bayes correlated equilibrium, players receive recommendations from an *omniscient* mediator and have an incentive to be obedient.
- If we apply the concept of Bergemann and Morris to the strategic-form of the game (i.e., the mediator recommends strategies), the unique equilibrium distribution is $\mu(B, R) = 1$, that is, the distribution corresponding to the unique correlated equilibrium.
- We genuinely need the mediator to make recommendations at each history.
- There is a unique communication equilibrium of the game, which also induces the distribution $\mu(B, R) = 1$. (A unique extensive-form correlated equilibrium too with the same distribution; von Stengel and Forges, 2015)

Multi-stage games Γ

- There are n players and T stages.
- At each stage t, a state $\omega_t \in \Omega_t$ is drawn, player i receives a signal $s_{i,t} \in S_{i,t}$, and chooses an action $a_{i,t} \in A_{i,t}$.
- The joint probability p_t of state ω_t and signal profile s_t at period t depends on all past actions, signals and states.
- The payoff of a player depends on the realized states and chosen actions.
- This defines the base game Γ.

We now consider the additional signals players may receive.

Additional signals/expansions

- At each stage t, player i receives the additional private signal $m_{i,t} \in M_{i,t}$.

- The joint probability π_t of state ω_t and signal profile (s_t, m_t) at period t depends on all past actions, signals (including the additional ones) and states.

We denote Γ_π the expansion of Γ thus obtained.

We restrict attention to *admissible* expansions, to be defined next.

Admissibility: formal definition

Let h^t be the history of actions $(a_1, \ldots, a_{t-1})$ and signals $(s_1, \ldots, s_t)$ at period t. Similarly, m^t is the history of additional signals and ω^t the history of realized states.

An expansion is *admissible* if there exist kernels $(\xi_t)_t$ such that

$$\pi_1(h_1, m_1, \omega_1) = \xi_1(m_1 | h_1, \omega_1) p_1(h_1, \omega_1),$$

for all (h_1, m_1, ω_1), and

$$\pi_{t+1}(h_{t+1}, m_{t+1}, \omega_{t+1} | \overbrace{a_t, h^t, m^t, \omega^t}^{\text{past}}) =$$

$$\xi_{t+1}(m_{t+1} | h^{t+1}, m^t, \omega^{t+1}) \underbrace{p_{t+1}(h_{t+1}, \omega_{t+1} | a_t, h^t, \omega^t)}_{\text{no causal effects}},$$

for all $(a_t, h^t, m^t, \omega^t, h_{t+1}, m_{t+1}, \omega_{t+1})$, for all t.

Example of a non-admissible expansion

- Let $\Omega_1 = \Omega_2 = \{0, 1\}$, independently and uniformly distributed.
- Let $M_1 = \{0, 1\}$ and assume that $\omega_2 = (\omega_1 + m_1) \bmod_2$, with ω_1 and m_1 independent and uniformly distributed.
- In words, state tomorrow = state today + shocks, and the player is informed of both the state today and the shock in the expansion.
- We have that $\pi_1(m_1, \omega_1) = 1/4$ for all (m_1, ω_1) and $\pi_2(\omega_2 | m_1, \omega_1) = 1$ if and only if $\omega_2 = (\omega_1 + m_1) \bmod_2$. Admissibility is violated.
- A possible fix: Assume that states and signals are realized at the first stage. This fix does not work in general when the players control the transitions.

Conditional probability perfect Bayesian equilibria

A conditional probability PBE is a profile of strategies σ and a conditional probability system β such that players are sequentially rational given the belief system induced by β.

Let $\mathcal{CPPBE}(\Gamma_\pi)$ be the set of distributions over states and actions induced by all the conditional probability perfect Bayesian equilibria of Γ_π.

Today's objective is to characterize the set:

$$\bigcup_{\Gamma_\pi \text{ an admissible expansion of } \Gamma} \mathcal{CPPBE}(\Gamma_\pi) = ?$$

without any explicit reference to expansions.

Sequential Bayes correlated equilibria

Consider the following mediated extension of Γ (not Γ_π; it is not a typo):

- At each period t, player i:
  - observes the private signal $s_{i,t}$,
  - receives a private recommendation $\hat{a}_{i,t} \in R_{i,t}(h_i^t, \hat{a}_i^{t-1}) \neq \emptyset$ from the mediator,
  - and chooses an action $a_{i,t}$.

A sequential Bayes correlated equilibrium of Γ is a collection of kernels $(\overline{\mu}_t)_t$ such that the players have an incentive to be obedient at all histories, consistent with the mediation ranges $(R_{i,t})$.

Let $\mathcal{SBCE}(\Gamma)$ be the set of distributions over states and actions induced by the sequential Bayes correlated equilibria of Γ.

Equivalence

Theorem

We have the following equivalence:

$$\bigcup_{\Gamma_\pi \text{ an admissible expansion of } \Gamma} \mathcal{CPPBE}(\Gamma_\pi) \quad = \quad \mathcal{SBCE}(\Gamma).$$

Idea of the proof: From $\mathcal{CPPBE}$ to $\mathcal{BCE}$

- Fix an expansion Γ_π of Γ and a CPPBE (σ, β) of Γ_π.
- Consider the "fictitious" mediated game, where at each stage,
  - player i receives the signal $s_{i,t}$; the mediator is told of (s_t, a_{t-1}, ω_t);
  - the mediator reports $m_{i,t}$ to player i, with $\xi_t(m_t|h^t, m^{t-1}, \omega^t)$ the probability of m_t conditional on (h^t, m^{t-1}, ω^t);
  - player i chooses an action $a_{i,t}$.
- By construction, (σ, β) is also an equilibrium of this fictitious mediated game.
- Now invoke the revelation principle due to Sugaya and Wolitzky (2017) to go from the fictitious mediated game to the fictitious canonical mediated game.

Intuition: From $\mathcal{SBCE}$ to $\mathcal{CPPBE}$

- The idea is to construct an expansion Γ_π of Γ, which has $M_{i,t} = A_{i,t}$ as the set of additional signals, i.e., additional signals are the recommendations (as in a canonical communication equilibrium).

- An equilibrium of Γ_π then consists in playing $a_{i,t}$ when the additional signal is $m_{i,t} = a_{i,t}$ (i.e., to be obedient).

Concluding remarks

- Today's talk was a taster: we consider many more solution concepts in the paper and an application to bilateral bargaining problems.
- Open issues:
  - restrictions on information structures beyond admissibility,
  - dispensing with admissibility,
  - other solution concepts, particularly non-equilibrium ones,
  - full implementation, etc.