Relational Contracts with Private Information:
The Upside of Implicit Downsizing Costs

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Novelty: When deciding on the bonus payment, the principal has private information about the productivity of the agent's effort **in the next period.**

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[$\theta_1 = \theta^h$; $\theta_t = \theta^h$ with probability $q \in (0, 1)$ for all $t = 2, 3, \cdots$ (iid).]

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5. Output $y_t = g(n_t)$ is realized and publicly observed (not contractible!); $g : \mathbb{R}_+ \to \mathbb{R}_+$ is C^2, with $g(0) = 0$, $g' > 0 > g''$, $g'(0) = \infty$, $g'(\infty) = 0$; profit $\theta_t y_t$.

   $\to$ **First-best effort** $n^*(\theta)$ given by $\theta g'(n^*(\theta)) = c$.

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6. Bonus $b_t \geq 0$ is paid by the P to A.

The Players' Payoffs

Principal:

$$d_t\left(\theta_t g(n_t) - w_t\right) + E\left[-b_t + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} d_\tau \left(\theta_\tau g(n_\tau) - w_\tau - b_\tau\right)\right].$$

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Agent:

$$d_t\left(w_t - c\, n_t\right) + E\left[b_t + \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} d_\tau \left(-c\, n_\tau + w_\tau + b_\tau\right)\right].$$

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Look for a best PPE for the principal. This equilibrium also maximizes joint surplus.

Benchmark: Public Info

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2. After receiving w_t, agent must find it optimal to exert the right level of effort:

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\begin{aligned}
-n(\theta^t)c+q\left(b^h(\theta^t)+\delta U^h(\theta^t)\right)+(1-q)\left(b^l(\theta^t)+\delta U^l(\theta^t)\right) \\
\geq -\tilde{n}c+q\left(b^h(\theta^t,\tilde{n})+\delta U^h(\theta^t,\tilde{n})\right) \\
+(1-q)\left(b^l(\theta^t,\tilde{n})+\delta U^l(\theta^t,\tilde{n})\right).
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$$+(1-q)\left(b^l(\theta^t,\tilde{n})+\delta U^l(\theta^t,\tilde{n})\right).$$

3. It must be optimal for the principal to make equilibrium bonus payments

$$-b^h(\theta^t)+\delta\Pi^h(\theta^t)\geq 0 \qquad \text{(DEh)}$$
$$-b^l(\theta^t)+\delta\Pi^l(\theta^t)\geq 0. \qquad \text{(DEl)}$$

Public Info II

(DEh) and (DEl) can be combined into

$$-\left(qb^h(\theta^t) + (1-q)b^l(\theta^t)\right) + \delta\left(q\Pi^h(\theta^t) + (1-q)\Pi^l(\theta^t)\right) \geq 0. \qquad \text{(DE)}$$

Public Info III

Equilibrium effort only depends on the current state: $n(\theta^t) = n(\theta_t)$:

Only observable deviations; no need to destroy surplus on the equilibrium path $\Rightarrow$ Want to be as close to FB-level as possible

Stationary environment (iid): Maximum enforceable effort levels the same for every history θ^t.

Profit-Maximizing Equilibrium with Public Info

Proposition: *Assume the firm's type is publicly observable. Then, there are levels of the discount factor, $\overline{\delta}$ and $\underline{\delta}$, with $0 < \underline{\delta} < \overline{\delta} < 1$, such that*

- n^h *and* n^l *are at their efficient levels for* $\delta \geq \overline{\delta}$.
- $n^h \geq n^l$, *but* n^h *is inefficiently low, and* n^l *is at its efficient level for* $\underline{\delta} \leq \delta < \overline{\delta}$;
- $n^h = n^l$, *and both effort levels are inefficiently low for* $\delta < \underline{\delta}$.

Private Types

Thruthtelling Constraints

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$$-b^h(\theta^t) + \delta\Pi^h(\theta^t) \geq -b^l(\theta^t) + \delta\tilde{\Pi}^l(\theta^t) \quad \text{(TTh)}$$

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where $\tilde{\Pi}^l(\theta^t) = \Pi^l(\theta^t) + \theta^h g(n^l(\theta^t)) - \theta^l g(n^l(\theta^t))$;
$\tilde{\Pi}^h(\theta^t) = \Pi^h(\theta^t) - \theta^h g(n^h(\theta^t)) + \theta^l g(n^h(\theta^t))$.

Uses One-Deviation Principle.

Overview of Constraints

$$U(\theta^t) \geq 0 \quad \text{(IR)}$$

$$-n(\theta^t)c + q\left(b^h(\theta^t) + \delta U^h(\theta^t)\right) + (1-q)\left(b^l(\theta^t) + \delta U^l(\theta^t)\right) \geq 0 \quad \text{(IC)}$$

$$-b^h(\theta^t) + \delta \Pi^h(\theta^t) \geq 0 \quad \text{(DEh)}$$

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The (EC) Constraint

Agency problem with private info boils down to constraint

$$-n(\theta^t)c+\delta q\Pi^h(\theta^t)+\delta(1-q)\Pi^l(\theta^t) \geq \delta q g(n^l(\theta^t))\left(\theta^h - \theta^l\right). \tag{EC}$$

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(LHS) like (DE) constraint

(RHS) New effect: Information Rent of the P, who always has the option of claiming tomorrow's profits are lower (only $\theta^l g(n^l(\theta^t))$) than they actually are ($\theta^h g(n^l(\theta^t))$).

Dynamics of Equilibrium Employment

Lemma: There exists an optimal equilibrium with the property that, for every two histories θ^t and $\tilde{\theta}^{\tilde{t}}$, $n^h(\theta^t) = n^h(\tilde{\theta}^{\tilde{t}})$. Furthermore, for every history θ^t, $n^l(\theta^t) = n_i^l$, where $i \in \{0, 1, 2, ...\}$ denotes the number of previous consecutive periods τ with $\theta_\tau = \theta^l$.

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n^h only enters the (LHS) of the (EC) constraint; reduction of $n^h(\theta^t)$ does not increase P's commitment. $\Rightarrow$ Have the n^h that is the closest possible to the FB after any history θ^t.

Environment stationary $\Rightarrow$ Closest n^h to the FB possible is the same after any history θ^t.

By contrast, reduction of n^l enhances P's commitment.

Result: High δ

Proposition: There exists a $\overline{\delta} \in (0, 1)$ such that optimal equilibrium profits are equal to first-best surplus *for all* $\delta > \overline{\delta}$. In this case, for every history θ^t, first-best effort levels $n^*(\theta_t)$ can be implemented.

Intermediate δ

Proposition: There exist discount factors $\underline{\delta}$ and $\overline{\delta}$, with $0 < \underline{\delta} < \overline{\delta} < 1$, such that, in an optimal equilibrium, for $\delta \in (\underline{\delta}, \overline{\delta})$, n^h and n_0^l are inefficiently low; for all $i \geq 1$, $n_i^l = n^*(\theta^l)$.

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(ECh) binds; need to reduce n^h.

n_0^l is also reduced! $\Rightarrow$ Cost of not telling the truth in high state goes up; "transferring effort from low to high state"

n_i^l at FB-levels! Discount factor is still high enough for $n^*(\theta^l)$ to be enforceable.

Optimal effort in low periods immediately following a high period is **not sequentially optimal**.

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$\Rightarrow$ *Implicit Downsizing Costs*

Extensions

Persistent Shocks

- Assume that principal starts with a high type; type remains high for another period with probability q. With probability $1-q$, type becomes low and remains low forever.

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- Assume that principal starts with a high type; type remains high for another period with probability q. With probability $1-q$, type becomes low and remains low forever.

- After claiming type is low, have to stick to that forever after, yielding constraints

$$-n^h c + \delta q \Pi^h + \delta \left(1-q\right) \Pi^l_0 \geq \left(\theta^h - \theta^l\right) \sum_{i=0}^{\infty} \left(\delta q\right)^{i+1} g(n^l_i) \tag{ECh}$$

$$-n^l_i c + \delta \Pi^l_{i+1} \geq 0 \tag{DEli}$$

Expression for information rent is now $\left(\theta^h - \theta^l\right) \sum_{i=0}^{\infty} \left(\delta q\right)^{i+1} g(n^l_i)$, while it was $\delta q \left(\theta^h - \theta^l\right) g(n^l_0)$ before.

Results – Persistent Downsizing

- Suppose $q < \frac{\theta^l g(n^l_{FB})\left(n^h_{FB} - n^l_{FB}\right)}{\theta^h n^l_{FB}\left(g(n^h_{FB}) - g(n^l_{FB})\right)}$.
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- Intermediate discount factor where (DEli) holds for FB effort levels, but (ECh) does not:

$$n^l_i < n^l_{i+1} < n^l_{FB}, \text{ with } \lim_{i\to\infty} n^l_i = n^l_{FB}$$

Persistent Shock–Overshooting

- Overshooting still observed, but recovery gradual and never complete:
  - Falsely claiming that type is low forces principal to stick to claiming low state forever thereafter
  - In expectation, costs of distortion to off-path principal remain higher than those to on-path principal throughout
  - But: Increasing likelihood that the state will indeed have switched to low $\Rightarrow$ Decrease in the cost difference over time (faster the lower q)

$\Rightarrow$ Optimal to distort the less the further past the announcement of the switch to the low state is.

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  - θ_t is learnt at the start of period t
    - Private info is not costly; sequential optimality!
  - θ_{t+1} is learnt at the start of period t
    - Effort depends only on *current* type
    - Reason: Need to shut down P's incentives to **over-**report

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  - Reduction of labour input not sequentially optimal!
  - On-path destruction of surplus (even though private info is one-sided)

Literature

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- Bull (1987); MacLeod & Malcolmson (1989)
- Levin (2003)
- Halac (2012): P has private info about his (persistent) outside option.
- Li & Matouschek (2013): P has private information about cost of compensating the agent.
- Malcomson (2015): P has private info about the value of A's effort in the **current** period; A has private info about costs
- Malcomson (2016): A's persistent cost type is private information; full separation not possible when continuation payoffs are on the Pareto frontier