Network Search: Climbing the Ladder Faster

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Motivation

Networks are important in labor market search

1. Significant fraction of workers search using contacts
   - SCE: $\sim \frac{1}{4}$ found their job by referral from professional-connections (Arbex et al 2018)
   - PSID: $\sim \frac{1}{2}$ found their job through social network (Corcoran, Datcher and Duncan, 1980).
2. Firms use referrals when filling a vacancy.
   - EOPP: 36% of firms filled their last vacancy through a referral (Holzer, 1987).

Networks are “irregular”

People differ in the number of links they have, which:

- implies heterogeneity in finding rate both on and off the job
- implies heterogeneity in the quality of offers drawn.

This paper: Different people climb the ladder differently

What we do

- Put an irregular network into a model of on/off-the-job search
  - Workers find jobs through network
  - Firms' workers become search capital
- Use mean-field approach to tractably describe the network
- Calibrate and compare vis-à-vis common empirical findings
  - New evidence from SCE

Key results

- Use mean-field approach to reduce an ∞-dimensional state to 3
- Analytical results:
  - Network search draws from a “better” (FOSD) distribution than direct contact search
  - Network search reduces firms’ profit

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- Analytical results:
  - Network search draws from a "better" (FOSD) distribution than direct contact search
  - Network search reduces firms' profit
- Calibrate to direct contact & network search. The latter:
  - Have higher wages on acquisition (Marmaros & Sacerdote, 2002)
  - Occur after a shorter unemployment spell (Goel & Lang, 2009)
  - Longer match duration (Dustmann et al 2014)
  - More likely higher on the ladder (Arbex et al. 2018)

Basic environment

On-the-job search (as in Burdett and Mortensen 1998)

- Firms post wages that may be found via direct contact
- Workers are ex ante heterogeneous in their peers
- Employees pass offers to peers for positions just like own

Easily extensible to additional heterogeneity

Before getting into the weeds

The mechanism is:

- Workers with more connections sample jobs more quickly
- They climb the ladder faster
- Referrals are useful for 2 reasons:
    1. Draw from the wage distribution rather than direct offer distribution
    2. Draw from friends who are better connected (*paradox of friendship*)

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Network search is done by better connected workers:

- Jobs through the network are higher paid
- Jobs through the network last longer
- Jobs through the network follow shorter unemployment

Literature

- Network theory: Vega-Redondo (2007), Calvo-Aremengol & Jackson (2007), Calvo-Aremengol & Jackson (2004)
- Empirical finding: Cornelissen, Dustmann & Schoenberg (2015), Hellerstein, Kutzbach, Neumark (2014), Holzer (1988)
- Search and networks: Galenianos (2014), Fontaine (2008), Ioannides & Soetevent (2006), Mortensen & Vishwanath (1995)

Model of search and networks in labor markets

Technology, flows and types

Technologies:

- Workers heterogeneous in number of peers, z
  - Characterized by degree distribution $\Omega(z)$
- Workers homogeneous in non-employment flow value, b
- Firms are homogeneous, with productivity 1

Flows:

- Random search, matched via either *direct* or *network* search
- Jobs break up exogenously at rate δ

Vanilla direct search

- Firms post wages w, distributed as $F(w)$, firm offer distribution
- A worker meets vacancy at rate γ^i
- An unemployed worker exits if $w \geq R(\cdot)$
- An employed worker accepts jobs above her current wage

Networks search

- Employed find & pass along jobs at their firm at rate $\gamma^1\nu$
- Workers sample via their network connections, arrival rate $\rho(\cdot)$
  - Any employed peer equally likely to send a referral
  - Any peer of employed worker equally likely to receive a referral
- Connections pass jobs with the same wage (i.e. same firm)
- Same acceptance rules: reservation wage $R(\cdot)$ or w.

What is a worker type?

Define χ recursively:

- Each worker has z peers
- χ is $z \times 4$. Element c is a triple
  - $i(c)$, the labor status
  - $w(c)$, the wage
  - $k(c)$, the history of wages
  - $\chi(c)$, the position in the network
- $\chi(c)$ is also a $s \times 4$ dimensional object, s.t. s is the number of peers of peer c

To forecast the value of a peer:

- His wage that might be passed
- His *potential* wage next period

The mean-field approach

Goal:

- Remove local information from the state
- Instead of how particular atoms interact, use average atom effect
- Will take the position in network from χ to z

Requires:

1. Incomplete information about peers
2. A locally tree-like structure

We didn't make this up

- Vega-Redondo (2007) uses this approach so that the average state of the network is replicated *locally*: **No neighborhood effects** (Vega-Redondo 2007).
- Good representation of the long-run dynamics of networks (Vega-Redondo 2007, Jackson 2008).
- This or similar idea used in network search papers: Calvo-Armengol & Zenou (2005) or Bramoulle & Saint Paul (2010)

Assumption 1: Tree structure

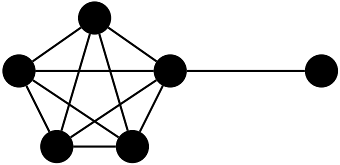
Assumption:

- The network is described *completely* by the degree distribution, Ω
- As nodes $n \to \infty$, probability of a cluster $\to 0$

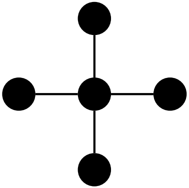
The effect:

- For any χ and χ' if $z = z'$ then $E[s|\chi] = E[s'|\chi']$
- z has no information about local conditions

Networks we rule out



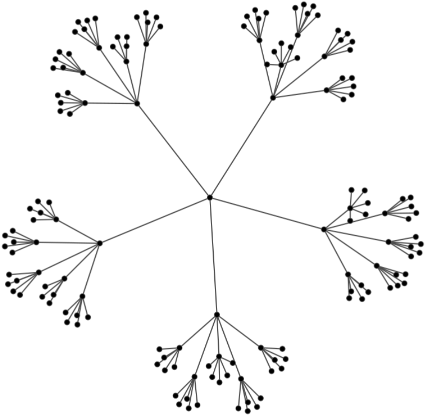
A clustered subgraph where $\langle z \rangle = 4$



A regular subgraph where $\langle z \rangle = 4$

- The clustered network has local structure
- The regular network is uninteresting

Our network structure: A tree



No local “neighborhood,” but number of connections differs

Assumptions 2& 3: Incomplete information/memory

2 Limited observability assumption:
  - Agents do not know the state $(i(c), w(c), k(c), \chi(c))$ of peer c
  - Agents know c exists and can use degree distribution, Ω
  - Use k to form beliefs $(\hat{i}(c), \hat{w}(c), \hat{k}(c), \hat{\chi}(c))\ \forall c$

3 Limited memory assumption
  - Agents know c exists and can use degree distribution, Ω
  - No information on which to form beliefs

Proposition: z is a sufficient statistic

Under each assumption

1. $z = z'$ can differ only in $\{i(c), w(c)\}$
2. Cannot directly observe $\{i(c), w(c)\}$
3. Cannot use k to infer $\{i(c), w(c)\}$

Workers will differ in “connectedness,” but that is unidimensional

Type-distribution of referral passer

- $\Psi(s)$: probability a worker's peer has s peers herself

$$\Psi(s) = \frac{s\Omega(s)}{\langle z \rangle}$$

- $\Psi(s) < \Omega(s)$ is the *paradox of friendship*

Type-distribution of referral passer

- $\Psi(s)$: probability a worker's peer has s peers herself

$$\Psi(s) = \frac{s\Omega(s)}{\langle z \rangle}$$

- $\Psi(s) < \Omega(s)$ is the *paradox of friendship*
- Probability a peer is s and passes referral:

$$\gamma^1 \nu \frac{n(s)}{s} \Psi(s)$$

- Then the distribution is

$$\tilde{\Psi}(s) = \frac{\gamma^1 \nu \frac{n(s)}{s} \Psi(s)}{\int \gamma^1 \nu \frac{n(z)}{z} \Psi(z) dz} = \frac{n(s)\Omega(s)}{\int n(z)\Omega(z) dz}$$

Network search arrival rate

The probability a worker of type z receives an offer via a peer is

$$\rho(z) = \lim_{\Delta \to 0} \left(1 - \left[1 - \int_s \Psi(s) \gamma^1 n(s) \frac{\nu}{s} ds \, \Delta \right]^{z/\Delta} \right)$$
$$= \left(1 - \exp\left(-z\nu\gamma^1 \int \frac{n(s)}{s} \Psi(s) ds \right) \right)$$

- $n(s)\gamma^1$ is the probability this peer is employed and hears of an vacancy
- ν / s is the probability this information is passed along

Network offer distribution/earnings distribution

The earnings distribution among agents of type z

$$G(w,z)$$

Earnings distribution in the population:

$$G(w) = \int_z G(w,z)\Omega(z)dz$$

Network offer distribution:

$$\tilde{G}(w) = \int_s G(w,s)\tilde{\Psi}(s)ds$$

Offers through the network are drawn from $\tilde{G}(w)$

Model of search and networks in labor markets:

Workers' value functions

Unemployed Worker's Value Function

The value function of an unemployed worker of type z is

$$
\begin{aligned}
&rV^0(z) = \\
&b + \underbrace{\gamma^0 \left\{ \int_{R(z)}^{\bar{w}} \left[V^1(z,x) - V^0(z) \right] dF(x) \right\}}_{\text{The value of direct search}} \\
&+ \underbrace{(1-\gamma^0)\rho(z) \int_{R(z)}^{\bar{w}} \left[\left(V^1(z,x) - V^0(z) \right) \right] d\tilde{G}(x)}_{\text{The value of network search}}
\end{aligned}
$$

Employed Worker's Value Function

The value of an employed worker with z connections and wage w is

$$rV^1(z,w) =$$
$$w + \underbrace{\delta\left[V^0(z) - V^1(z,w)\right] + \gamma^1\left\{\int_w^{\bar{w}}\left[V^1(z,x) - V^1(z,w)\right]dF(x)\right\}}_{\text{The value of direct search}}$$
$$+ \underbrace{(1-\gamma^1)\rho(z)\int_{R(z)}^{\bar{w}}\left[\left(V^1(z,x) - V^1(z,w)\right)\right]d\tilde{G}(x)}_{\text{The value of network search}}$$

Reservation Wage

At the reservation wage $R(z)$, we have that $V^1(z, R(z)) = V^0(z)$.

$$R(z) - b = \left(\gamma^0 - \gamma^1\right) \left\{ \int_{R(z)}^{\bar{w}} \left[V^1(z,w) - V^0(z) \right] dF(w) \right\}$$
$$+ \left[\begin{array}{c} (1-\gamma^0)\rho(z) \\ -(1-\gamma^1)\rho(z) \end{array} \right] \left\{ \int_{R(z)}^{\bar{w}} \left[\theta(w) \left(V^1(z,w) - V^0(z) \right) \right] dw \right\}$$

$$= \left(\gamma^0 - \gamma^1\right) \left\{ \int_{R(z)}^{\bar{w}} V_w^1(z,w) \left[1 - F(w) \right] dw \right\}$$
$$+ \left[(\gamma^0 - \gamma^1)\rho(z) \right] \left\{ \int_{R(z)}^{\bar{w}} V_w^1(z,w)(1 - \tilde{G}(w)) dw \right\} \qquad (1)$$

Wage Distribution and Workers per Firm

- $\ell(w, z)$: Labor force of type z per firm at a firm paying wage w
- $L(w)$: Total labor input per firm paying wage w:

$$L(w) = \int_1^{\infty} \ell(w, z) dz \tag{2}$$

- Each employer offers a wage that gives steady state profit:

$$\pi(w) = (1 - w)L(w) \tag{3}$$

Steady state equilibrium and analytic results

Steady State Employment of Workers

Flows in and out of unemployment must balance, give the steady state employment rate:

$$n(z) = \frac{\overbrace{\gamma^0 \left[1 - F\left(R(z)\right)\right]}^{\text{Recruiting from direct search}} + \overbrace{(1-\gamma^0)\rho(z)\left[1 - \tilde{G}\left(R(z)\right)\right]}^{\text{Recruiting from network search}}}{\delta + \gamma^0 \left[1 - F\left(R(z)\right)\right] + (1-\gamma^0)\rho(z)\left[1 - \tilde{G}\left(R(z)\right)\right]}, \tag{4}$$

The economy's employment rate is given by

$$n = \int_1^\infty n(z)\Omega(z)dz \tag{5}$$

Steady State Earnings Distribution

$$G(w,z) =$$

$$\frac{[1-n(z)]\left\{\overbrace{\gamma^0\left[F(w)-F(R(z))\right]}^{\text{Direct search effect}}+\overbrace{(1-\gamma^0)\rho(z)\left\{\tilde{G}(w)-\tilde{G}(R(z))\right\}}^{\text{Network search effect}}\right\}}{n(z)\left[\delta+\gamma^1\left[1-F(w)\right]+(1-\gamma^1)\rho(z)(1-\tilde{G}(w)\right]}$$

Because $\frac{F(w)-F(R)}{(1-F(w))} \geq \frac{\tilde{G}(w)-\tilde{G}(R)}{(1-\tilde{G}(w))}$, averaging in dominating distribution

Steady State Firm Size

Separating of z-type workers equal the z-type workers:

$$\ell(w,z)\beta(w,z) = h(w,z) \tag{6}$$

where

- $\beta(w,z) =$
  $$\delta + \underbrace{\gamma^1(1-F(w))}_{\text{Loss to poaching via direct search}} + \underbrace{(1-\gamma^1)\rho(z)\left[1-\tilde{G}(w)\right]}_{\text{Loss to poaching via network search}}$$
- $h(w,z) =$

$$\underbrace{\frac{\Omega(z)}{M}\overbrace{\left\{[1-n(z)]\,\gamma^0 \mathbb{I}_{R(z)\leq w} + n(z)\gamma^1 G(w,z)\right\}}^{\text{Hired via direct search}} + \gamma^1 \int \ell(w,t) t\Psi(z)\left\{[1-n(z)]\,\nu \mathbb{I}_{R(z)\leq w} + n(z)\nu G(w,z)\right\} dt}_{\text{Hired via network search}}$$

The steady state equilibrium

Definition

A *Sufficient Recursive Equilibrium*: V^0, V^1, R, π and $F(w)$, $G(w,z)$, $n(w)$, such that:

- V^0, V^1, R solve household problem
- G, n consistent with worker flows
- F implies $\pi(w) = \pi\ \forall w$

Ordering offer distributions, $\tilde{G} \leq F$

Proposition

$\tilde{G}$ First Order Stochastically Dominates F

- As in Burdett Mortensen, $G \leq F$ because $\gamma^1 > 0$
- $\tilde{G}$ weights G by $n(\cdot)$: $\int \frac{n(s)G(w,s)\Omega(s)}{\int n(z)\Omega(z)} ds$
- $n' > 0$, which is guaranteed by
  - $\rho' > 0$ by definition
  - $R' < 0$ because $V^1_{wz}(z, R(z)) > 0$

The equilibrium effect of network search

Proposition

Beginning from $\nu = 0$*, for sufficiently high* γ[1]

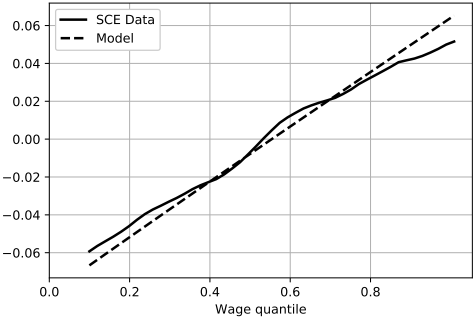
- $\frac{\partial \underline{w}}{\partial \nu} \leq 0$ *and* $\frac{\partial \bar{w}}{\partial \nu} \geq 0$
- $\frac{\partial L(\underline{w})}{\partial \nu} \leq 0$ *and* $\frac{\partial L(\bar{w})}{\partial \nu} \geq 0$
- $\frac{\partial \pi}{\partial \nu} \leq 0$

- $\frac{\partial \underline{w}}{\partial \nu} \leq 0$ because $\frac{\partial R(z)}{\partial \nu} \leq 0$
- $\frac{\partial L(\underline{w})}{\partial \nu} \leq 0$ because poaching is faster
- $\frac{\partial \pi}{\partial \nu}$ depends on $\frac{\partial L(\underline{w})}{\partial \nu}$ (Envelope condition takes care of $\frac{\partial \underline{w}}{\partial \nu}$)
- $\frac{\partial L(\bar{w})}{\partial \nu} \geq 0$ because own workers increase hiring

Results from the calibrated economy

SCE: Higher wage workers use networks more

- Model prediction: higher-wage workers find jobs through networks
- Survey of Consumer Expectations (SCE) asks workers their current job's finding method



Parameter values

Parameter	Value	Moment	Model	Data
γ^0	0.24	Finding rate UE	0.24	0.25
γ^1	0.10	Finding rate EE	0.02	0.02
ν	0.04	Hires through the network	0.13	0.23
α	2.34	Network finding slope	0.26	0.25
δ	0.013	Average EU		

Average offer distribution by type

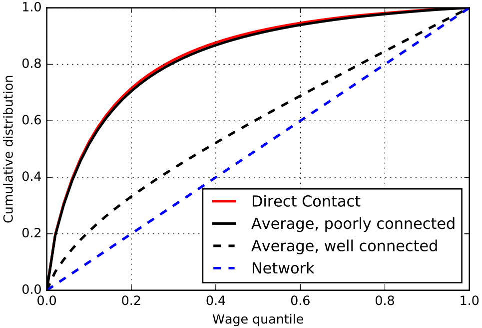


Figure: Average distribution of wage offers by contact method conditional on number of peers.

Average hiring method by wage

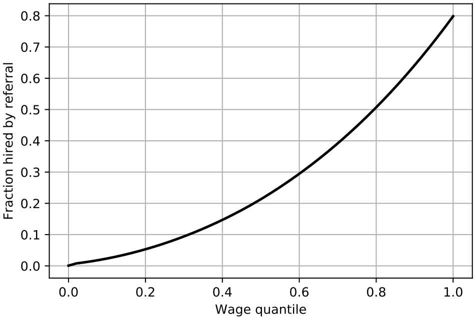


Figure: At higher wage levels, most hiring occurs through referral.

The half-life by connections

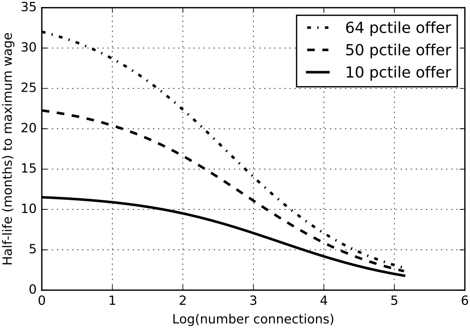


Figure: Half-life of wage growth paths to maximum wage: different starting wages and different network connections z.

The effect is not just heterogeneous search

We let arrival rates differ by z, but not the offer distribution

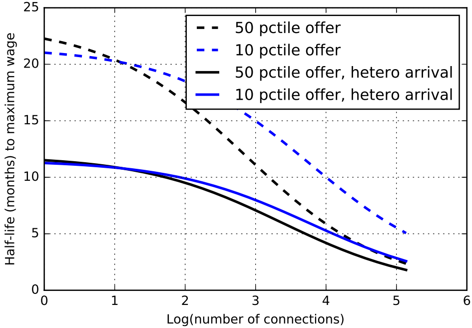


Figure: Half-life of wage growth comparing heterogeneous search rates and network search model

Results from the calibrated economy:

Relationship to empirical findings

The different distributions of workers

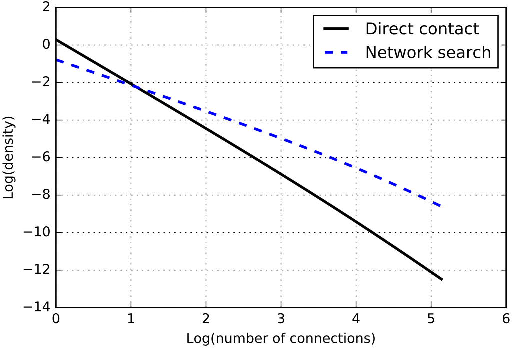


Figure: Distribution of number of peers: Direct search and network search.

Summarizing the effects

	Network Search	Direct Search
Average z relative to unemployed	4.14	0.93
Expected wage qtile; UE	0.251	0.075
Search time relative to avg	0.951	1.001
Average z relative to employed	2.67	0.80
Expected wage qtile; job-to-job	0.444	0.224
Expected duration of job match	4.87 years	2.70 years

Table: Expected differences between workers finding jobs through network or directed search. Above the line describe finding from unemployment, below adds features of job-to-job transitions.

Conclusion

Conclusions

- We presented a model of network search
- The mean-field approach allows for tractable, irregular networks
- Highly extensible to other search frameworks
- Empirical findings on search consistent with type heterogeneity and job ladders