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LINEAR OR NONLINEAR DATA  
FILTERING PROCESS?**

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# Is Seasonal Adjustment a Linear or Nonlinear Data Filtering Process?\*

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## Abstract / Résumé

*In this paper, we investigate whether seasonal adjustment procedures are, at least approximately, linear data transformations. This question is important with respect to many issues including estimation of regression models with seasonally adjusted data. We focus on the X-11 program and first review the features of the program that might be potential sources of nonlinearity. We rely on simulation evidence, involving linear unobserved component ARIMA models, to assess the adequacy of the linear approximation. We define a set of properties for the adequacy of a linear approximation to a seasonal adjustment filter. These properties are examined through statistical tests. Next, we study the effect of X-11 seasonal adjustment on regression statistics assessing the statistical significance of the relationship between economic variables in the same spirit as Sims (1974) and Wallis (1974). These findings are complemented with several empirical examples involving economic data.*

Nous examinons si la procédure d'ajustement X-11 est approximativement linéaire. Il y a potentiellement plusieurs sources de non-linéarité dans cette procédure. Le but de l'étude est de savoir si ces sources sont effectivement assez importantes pour affecter, par exemple, des résultats d'estimation dans des modèles de régression linéaire. La seule façon de répondre à cette question est par estimation. Nous proposons plusieurs critères qu'on peut utiliser pour juger si une procédure d'ajustement est approximativement linéaire. Nous examinons également par simulation des propriétés de tests dans le modèle de régression dans le même esprit que Sims (1974) et Wallis (1974).

Key Words: X-11 program, nonlinearity  
Mots-clés : programme X-11, non-linéarité

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## 1. INTRODUCTION

The question whether seasonal adjustment procedures are, at least approximately, linear data transformations is essential for several reasons. First, much of what is known about seasonal adjustment and estimation of regression models rests on the assumption that the process of removing seasonality can be adequately presented as a linear (two-sided and symmetric) filter applied to the raw data. For instance, Sims (1974), Wallis (1974), Ghysels and Perron (1993), Hansen and Sargent (1993), Sims (1993), among others examined the effect of filtering on estimating parameters or hypothesis testing. Naturally, the linearity of the filter is assumed, since any nonlinear filter would make the problem analytically intractable. Second, the theoretical discussions regarding seasonal adjustment revolve around a linear representation. Indeed, for more than three decades, seasonal adjustment has been portrayed in the context of spectral domain representations. See, for instance, Hannan (1963), Granger and Hatanaka (1964), Nerlove (1964), Godfrey and Karreman (1963), among others. The frequency domain analysis led to the formulation of seasonal adjustment as a signal extraction problem in a linear unobserved component ARIMA (henceforth UCARIMA) framework, where the emerging optimal minimum mean-squared error filters are linear.

The theory of signal extraction involving nonstationary processes, which will be the case covered here, was developed by Hannan (1967), Sobel (1967), Cleveland and Tiao (1976), Pierce (1979), Bell (1984), Burridge and Wallis (1988) and Maravall (1988). As a result, the widely used X-11 Census method, and its later developments like X-11 ARIMA, X-12 and REGARIMA, have been examined to determine which UCARIMA model would generate an optimal linear signal extraction filter similar to X-11 and its variants. Moreover, the few attempts that were made to formally model the operations of a statistical agency on the data-generating process of economic time series, as did Sargent (1989) for example, adopted the linear-filtering paradigm. Finally, whenever nonlinearities in time series are discussed, the possibility that such nonlinearities may be (partly) produced by seasonal adjustment is never seriously entertained.

Several authors have examined the linear representation of the X-11 program, notably, Young (1968), Wallis (1974), Bell (1992) and Ghysels and Perron (1993). Young (1968) investigated the question whether the linear filter was an adequate

approximation and found it to be a reasonable proxy to the operations of the actual program. This result was, to a certain extent, a basic motivation as to why the linear filter representation was extensively used in the literature. The main objective of our paper is to reexamine the question posed by Young. We come to quite the opposite conclusion, namely, that the standard seasonal adjustment procedure is far from being a linear data-filtering process. We reached a different conclusion, primarily because we took advantage of several advances in the analysis of time series, developed over the last two decades, and the leaps in the computational power of computers which enabled us to conduct simulations which could not be easily implemented before. We rely both on artificially simulated data as well as actual series published by the U.S. Census Bureau to address the question of interest. In section 2, we first discuss the attributes of the X-11 program that might be the source of nonlinear features. In section 3, we propose several properties that allow us to assess whether the actual program can be adequately presented by a linear filter. For instance, in the context of a linear UCARIMA, we expect the nonseasonal  $I(1)$  component and its X-11 extraction to be cointegrated and expect the extraction error to be a linear process. Finally, the difference between the unadjusted series filtered with the linear filter and the X-11 adjusted series should not be nonlinearly predictable. Through a combination of simulations and statistical hypotheses, we verify these properties for a large class of model specifications. Finally, we propose to reexamine the effect of X-11 filtering in linear regression models and study whether spurious relationships are produced by the nonlinearities.

In section 4, we report the results from the simulations and for a large class of data published by the U.S. Census Bureau.

## **2. ON POTENTIAL SOURCES OF NONLINEARITY IN THE X-11 PROGRAM**

In this section, we will identify features contained in the X-11 program which may be sources of nonlinearity. Since the program is almost exclusively applied to monthly data, we cover exclusively that case and ignore the quarterly program. In a first subsection, we describe the different versions of the X-11 program. This distinction is important since the operations potentially inducing nonlinearity in the data transformations differ from one version to another. Individual subsections

are devoted to the different features we need to highlight: (1) Multiplicative versus additive, (2) Outlier detection, (3) Moving Average Filter Selection and (4) Aggregation.

## 2.1. The decompositions

One must distinguish between two versions of the X-11 program. One is called the additive version and is based on the following decomposition:

$$X_t \equiv TC_t + S_t + TD_t + H_t + I_t \quad (2.1)$$

where  $X_t$  is the observed process, while  $TC_t$  is the trend-cycle component,  $S_t$  the seasonal,  $TD_t$  and  $H_t$  are respectively the trading-day and holiday components. Finally,  $I_t$  in (2.1) is the irregular component. The second version is called the multiplicative version and is based on the decomposition:

$$X_t \equiv TC_t \times S_t \times TD_t \times H_t \times I_t \quad (2.2)$$

There would be no need to distinguish between the two versions if a logarithmic transformation applied to (2.2) would amount to applying the additive version of the program. Unfortunately, that is not the case as the multiplicative version has features that are distinct from the additive one. These will be discussed shortly. It may parenthetically be noted that one sometimes refers to the log-additive version of X-11 when the additive version is applied to the logarithmic transformation of the data.

The first of several parts in both procedures deals with trading-day and holiday adjustments. Typically, one relies on regression-based methods involving the number of days in a week, etc. as regressors. Since a linear regression model is used, we will not explore this aspect of the program any further. Neither the simulations nor the empirical investigation consider effects related to  $TD_t$  or  $H_t$ . In our empirical analysis, we were careful to select series where no trading-day and holiday effects appear to be significant. For further discussion of trading-day and holiday adjustments, see, for instance, Bell and Hillmer (1984). The extraction of the  $TC_t$ ,  $S_t$  and  $I_t$  components will be more of interest for our purposes. These components are not estimated with regression-based methods, but instead are extracted via a set of moving-average filters. This is the most important part of the X-11 program. While it consists of a series of moving-average filters, it is

important to note that the same set of filters are not necessarily applied to a series through time. Hence, the filter weights may be viewed as time-varying. In addition, both the additive and multiplicative X-11 procedures are designed to identify extreme values, or so-called outliers, and replace them one by one by attenuated replacement values. These two features, namely the scheme determining the selection of moving-average filters and the outlier corrections, make the application of the additive procedure different from the default option linear variant of the program.

A third feature, specific to the multiplicative version, is also a potential source of significant nonlinearity. Indeed, despite the multiplicative structure of the decomposition in (2.2), the program equates the 12-month *sums* of the seasonally adjusted and unadjusted data rather than their products. Since the filters in the X-11 program are two-sided, one must also deal with the fact that, at each end of the sample, the symmetric filters need to be replaced by asymmetric ones due to lack of observations. This feature is also a deviation from the default option linear filter, but it will not be considered in our simulation design, as will be discussed in the next section.

## **2.2. Multiplicative versus additive**

The bulk of economic time series handled by the U.S. Bureau of the Census and the U.S. Bureau of Labor Statistics are adjusted with the multiplicative version of the program. Only a small portion is treated with the additive version, apparently around one percent of the 3000 series covered by the two aforementioned agencies. The Federal Reserve uses the additive version more frequently, because of the nature of the time series it treats. Roughly 20% of the 400 or so series it deals with are additively adjusted. Young (1968) described the features of the multiplicative version, emphasizing the complications and departures of (log-) linearity due to the equating of the 12-month sums of the seasonally adjusted and unadjusted data. If the equality of sums condition were dropped, then the logarithm of the seasonal factors could be expressed as linear filters of the raw data, just as in the additive version. Young (1968, p. 446) justifies the presence of the feature in the multiplicative X-11 program arguing that “traditionally, economists have not wanted to give up ...(the condition of equating sums)... just to obtain a linear model... the desire to present seasonally adjusted series in which annual totals rather than products are unchanged”.

In the remainder of the paper, we keep in mind the distinguishing features of the additive and multiplicative X-11 programs.

### 2.3. Outlier detections

The treatment of extremes, or outliers, is a key element in seasonal adjustment programs like X-11. Because this feature is similar for the additive and multiplicative versions, we will discuss it using the former as example. The X-11 program produces a first estimate of the seasonal and irregular components  $S_t + I_t$  via a twelve-term MA filter trend-cycle estimator. Seasonal factors are obtained from this preliminary estimate using a weighted five-term moving average. At this point, the program has obtained a first-pass estimate of the irregular component process  $\{I_t\}$ . The scheme to detect outliers is activated at this stage. First, a moving five-year standard deviation of the estimated  $I_t$  process is computed. Hence, extractions of  $I_t$  will be evaluated against a standard-error estimate only involving the past five years, i.e., sixty observations in a monthly setting. We shall denote the standard error applicable to  $I_t$  as  $\sigma_t^{(1)}$ , where the superscript indicates that one has obtained a first estimate. The standard error is reestimated after removing any observations on  $I_t$  such that  $|I_t| > 2.5 \sigma_t^{(1)}$ , yielding a second estimate  $\sigma_t^{(2)}$ , where the number of observations entering the second estimate is random. The second-round estimated standard error  $\sigma_t^{(2)}$  is used to clear the  $S_t + I_t$  process from outlier or influential observations. The rules followed to purge the process can be described as:

- (1) A weighting function  $w_t$  is defined as:

$$w_t = \begin{cases} 1 & \text{if } 0 \leq I_t \leq 1.5 \sigma_t^{(2)} \\ 2.5 - I_t/\sigma_t^{(2)} & \text{if } 1.5 \sigma_t^{(2)} < I_t \leq 2.5 \sigma_t^{(2)} \\ 0 & \text{if } I_t > 2.5 \sigma_t^{(2)} \end{cases} \quad (2.3)$$

- (2)  $S_t + I_t$  is replaced by an average of two annual leads and lags plus the contemporary observation weighted by  $w_t$  if

$$w_t < 1. \quad (2.4)$$

The formula in (2.4) replaces any perceived outlier by the smoothed nearest neighbor estimate. The 1.5 and 2.5 values in (2.3), setting the benchmarks of the

weighting function play, of course, a key role besides the two-step standard-error estimate  $\sigma_t^{(2)}$  described earlier. The (2.3) - (2.4) schemes are, however, entirely based on rules of thumb and not so easy to rationalize. The value of  $1.5 \sigma_t^{(2)}$  in (2.3) which sets off the correction scheme, since it determines whether  $w_t < 1$ , is quite tight.

#### 2.4. Moving average filter selection

We will continue with the additive version of the program again for the sake of discussion. The seasonal plus irregular components modified through (2.3) - (2.4) will be denoted  $(S_t + I_t)$ . The series is used to compute a new set of seasonal factors which are applied to the original raw series, yielding a first estimate of the seasonally adjusted series, which we shall denote  $X_{1t}^{SA}$ . Obviously, if the outlier correction procedure were turned off, then  $S_t + I_t$  would be used to compute the seasonal factors and, as a result, different estimates of seasonally adjusted series would already be obtained at this stage. The X-11 procedure continues with a second and final iteration of seasonal adjustment. As a first step in this second stage, one extracts again the trend-cycle component by applying a thirteen-term Henderson moving-average filter to the seasonally adjusted  $X_{1t}^{SA}$  series [the design of Henderson MA filters is described in the papers covering the linear X-11 approximation, formulae for the Henderson filter weights also appear in Macauley (1931) or Gouriéroux and Monfort (1990)]. The trend-cycle component estimate obtained at this point will be denoted  $TC_t^{(1)}$ . The moving-average filter selection scheme now comes into play. To describe the scheme, let us define two annual average percentage changes:  $\mu_{1t}$  is the average change of  $(X_{1t}^{SA} - TC_t^{(1)})$ , and  $\mu_{2t}$  the average change of  $TC_t^{(1)}$ . The averages are updated as new raw data are added to the sample and are therefore made time-varying. The filter selection scheme can then be formulated as follows:

- (1) apply nine-term Henderson MA if

$$\mu_{1t} < 0.99 \mu_{2t}; \tag{2.5}$$

- (2) apply thirteen-term Henderson MA if

$$0.99 \mu_{2t} \leq \mu_{1t} < 3.5 \mu_{2t}; \tag{2.6}$$

(3) apply twenty-three-term Henderson MA if

$$3.5 \mu_{2t} \leq \mu_{1t}. \quad (2.7)$$

The Henderson MA filter thus selected is reapplied to  $X_{1t}^{SA}$  to yield a second estimate  $TC_t^{(2)}$ . This yields a new estimate of the seasonal and irregular component. The program then repeats the process of estimating a standard error  $\sigma_t^{(i)}$ ;  $i = 1, 2$  and proceeds with a second application of the outlier correction process described in (2.3) and (2.4).

## 2.5. Aggregation

So far, we have highlighted the two distinct features that represent the possible causes of nonlinearity and/or time variation in the actual X-11 filtering process. However, another source of nonlinearity also needs to be highlighted. It is not related to the intrinsic operational rules of the program but rather to the *modus operandi* of its application to several series. Indeed, seasonal adjustment procedures are quite often applied to disaggregated series, like narrowly defined industrial sectors or components of monetary aggregates, and the output is then aggregated to produce a seasonally adjusted aggregate. Obviously, the separate decomposition (2.1) for two series, say  $X_t$  and  $Y_t$ , is not the same as the decomposition for a  $Z_t$  process defined as  $Z_t \equiv X_t + Y_t$ . The question whether seasonal adjustment should precede or follow aggregation is discussed in Geweke (1978) and was recently reexamined by Ghysels (1993). When the seasonal-adjustment process is linear and uniform, then aggregation and seasonal adjustments are interchangeable. Another potential source of nonlinearity is introduced, however, when seasonal adjustment and aggregation are not interchangeable, and one applies the procedure to disaggregated series with only the aggregated series available to the public. In practice, this setup is quite common. We therefore included in our simulation design a setup similar to the effect of aggregation combined with seasonal adjustment. This issue was, of course, studied separately. We first investigated the potential sources of nonlinearity produced by the internal design of X-11.

### 3. A SIMULATION STUDY

The effect of filtering on the statistical properties of time series and properties of estimators in linear regression models and cointegration tests are reasonably well understood when the adjustments are performed with a linear filter. The seminal papers by Sims (1974) and Wallis (1974) justified the routine use of seasonally adjusted series in linear regression models. Their result, namely that linear regressions with filtered series yielded consistent estimators, together with the more recent developments by Hansen and Sargent (1993), Ghysels and Perron (1993), Sims (1993), Ericsson, Hendry and Tran (1994) and Ghysels and Lieberman (1994) all rely on the key assumption that the filter is linear and uniformly applied to all series (and also in certain cases that it is two-sided and symmetric like the linear X-11 filter). In dealing with the question of potential nonlinearities in the actual X-11 procedure, we have to give up the elegance of econometric theory as there is no longer an explicit and easy characterization of the operations of the filter. The key question then is whether the features described in the previous section intervene to a degree that the linear filter can no longer be viewed as an adequate representation of the adjustment procedure in practice. A subsidiary question is to find out what effects are produced by the actual procedure if in fact the linear approximation is inadequate. The only way to address these questions is through simulations.

Unfortunately, the question of the simulation design is not simply one of a judicious choice of data generating processes. It is first and foremost a question about what we characterize as departures from a linear filter and how these are measured. We settled for a design centered around two broad topics which follow certain established traditions in the literature. First, we define a set of desirable properties which any filtering procedure should have to ensure that the linear approximation is adequate. This part of the design follows a tradition in the time series statistics literature concerned with defining properties that seasonal adjustment procedures ought to have [see, for instance, Bell and Hillmer (1984) for discussion and references]. Second, we also focus on questions which have a tradition rooted in the econometrics literature, particularly as established since Sims (1974) and Wallis (1974). Here we are not so much concerned with univariate filtering but rather with the measurement of relationships among economic time series through linear regression analysis. It is perhaps worth noting that since Young (1968) did not examine nonlinearities through simulated data we cannot

really make any comparison with his study. He took three test series, U.S. imports from 1948 to 1965, Unemployed Men from 1950 to 1964 and Carbon Steel production from 1947 until 1964, and reported a very detailed study of the seasonal factors produced by the X-11 method and its linear version. We take advantage of advances on two fronts: (1) an incredible leap in the computational power of computers, and (2) progress in the theory of time series analysis. Like Young, we will also study real data except that our analysis of actual series will only be complementary to the simulation results to verify the similarities between the two.

Examining (statistical) properties of adjustment procedures and studying regression output will require, in both cases, generating data which subsequently are filtered with the linear filter and the X-11 adjustment program. We will therefore devote a first subsection to the description of the data generating processes. A second subsection deals with the properties of linear approximation while a third subsection covers seasonal adjustment and regression analysis. A final and fourth subsection deals with technical notes regarding the simulations.

### 3.1. The data generating processes

We generated data from a set of linear UCARIMA models, with Gaussian innovations. Each process consisted of two components, including one exhibiting seasonal characteristics. Let the  $X_t$  process consist of two components:

$$X_t = X_t^{NS} + X_t^S \quad (3.1)$$

where  $X_t^{NS}$  represents a nonseasonal process and  $X_t^S$  displays seasonal characteristics. Obviously, equation (3.1) is adapted to the additive decomposition (2.1). The multiplicative one will be discussed later. The first component in (3.1) has the following structure:

$$(1 - L)X_t^{NS} = (1 + \alpha_{NS}L)\varepsilon_t^{NS} \quad (3.2)$$

with  $\varepsilon_t^{NS}$  i.i.d.  $N(0, \sigma_{NS}^2)$  and where  $\alpha_{NS}$  is the moving-average parameter. The process is chosen to be I(1) and invertible, determined only by two parameters, namely,  $\alpha_{NS}$  and  $\sigma_{NS}^2$ . The (monthly) seasonal component has the following structure:

$$(1 + L + \dots + L^{11})X_t^S = (1 - \alpha_S L^{12})\varepsilon_t^S \quad (3.3)$$

with  $\varepsilon_t^S$  again *i.i.d.*  $N(0, \sigma_S^2)$ . Here also two parameters determine the process. Obviously, the data generated have neither trading-day or holiday effects, nor is there an explicit distinction made between the  $TC_t$  and  $I_t$  components appearing in (2.1). This simplification was done purposely. Indeed, it is well known that the decomposition of a time series into a trend cycle, a seasonal and irregular components is not unique. Hence, it is not clear at the outset that if we were to define a structure for  $X_t^{NS}$  as the sum of two components,  $TC_t$  and  $I_t$ , the X-11 program would select exactly that same decomposition. For similar reasons, it is not clear that the X-11 procedure will identify  $S_t$  as exactly equal to  $X_t^S$ . Consequently, we must view our design as one where four parameters are selected to form an  $X_t$  time series with the stochastic structure

$$(1 - L^{12})X_t = \psi_x(L)\varepsilon_t \tag{3.4}$$

where  $\varepsilon_t$  is *i.i.d.*  $N(0, \sigma_x^2)$  and

$$\begin{aligned} \sigma_x^2 \psi_x(z) \psi_x(z^{-1}) &\equiv \sigma_S^2 [(1 + z + \dots + z^{11})(1 + z^{-1} + \dots + z^{-11}) \times \\ &\quad (1 - \alpha_S z^{12})(1 - \alpha_S z^{-12})] + \\ &\quad \sigma_{NS}^2 [(1 - z)(1 - z^{-1})(1 - \alpha_{NS} z)(1 - \alpha_{NS} z^{-1})]. \end{aligned}$$

The additive version of the X-11 program will operate on the time series  $X_t$  and choose a decomposition  $TC_t + S_t + I_t$ . Theoretically, this decomposition is defined by taking the maximal variance of the irregular component [see for instance Bell and Hillmer (1984) or Hotta (1989) for further discussion].

In section 3.4, we will provide further technical details regarding parameter values and sample sizes. Before leaving the subject, however, we would like to conclude with a few words regarding the multiplicative decomposition. The same steps as described in (3.1) through (3.4) were followed except that the generated series were viewed as the logarithmic transformation of the series of interest. Hence,  $\exp(X_t) = \exp(X_t^{NS}) \exp(X_t^S)$  was computed before applying the multiplicative X-11 program.

### 3.2. Properties of linear approximations

The design of seasonal adjustment filters is typically motivated on the basis of a set of desirable properties which the procedure ideally should exhibit. Most often,

these theoretical discussions revolve around a linear representation. In reality however, as we noted in section 2, there are many potential sources of nonlinearity. This raises the question which properties one would like to advance so that the linear filter approximation is reasonably adequate. The purpose of this section is to exploit certain properties of the *linear X-11* filter which will allow us to predict what will happen if the actual procedure were approximately linear. Let us denote the seasonally adjusted series, using the linear X-11 filter, as:

$$X_t^{LSA} \equiv \Theta_{X-11}^L(L)X_t \quad (3.5)$$

where the linear polynomial lag operator in (3.5) represents the X-11 filter. It has been shown that the linear filter includes the  $(1 + L + \dots + L^{11})$  operator [see, e.g., Bell (1992) for further discussion]. Moreover, the filter has the properties that  $\Theta_{X-11}^L(1) = 1$  [see Ghysels and Perron (1993)] implying that it will leave the zero frequency unit root in the  $X_t$  process unaffected when the process follows the specification described in section 3.1.

The purpose now is to identify a set of properties that would hold if X-11 were linear and to associate with those properties statistical tests which can be conducted either with simulated data, with real data or both.

We will first consider a class of relatively weak conditions applicable to simulated data, in particular we know that:

*Property 1L: The  $X_t^{NS}$  and  $X_t^{LSA}$  processes are cointegrated.*

Obviously, we would also like the actual X-11 procedure to yield an estimate of the nonseasonal component which is cointegrated with  $X_t^{NS}$ . Suppose that we denote  $X_t^{SA}$  as the seasonally adjusted series using the actual X-11 procedure. Then the following property should also hold:

*Property 1X: The  $X_t^{NS}$  and  $X_t^{SA}$  processes are cointegrated.*

Failure of property 1X to hold is an indication of inconsistencies when the actual X-11 program is applied to the data. Some caution is necessary, however, with the use of cointegration arguments. In principle, one should not expect cointegration properties possessed by the linear approximation to X-11 to translate *exactly* to the X-11 program itself. Indeed, cointegration is defined as two series

being exactly I(1) and for which there is an exact (though not necessarily unique) linear relationship canceling the zero frequency unit roots. In our context, it is perhaps more appropriate to interpret cointegration as a property we expect to hold *approximately* for the X-11 adjusted data when the filter approaches its linear version.

A second property is much stronger as it is borrowed directly from the theoretical linear signal extraction framework where we know that the extraction error defined as:

$$\delta_t^{LSA} \equiv X_t^{NS} - X_t^{LSA} \equiv [1 - \Theta_{X-11}^L(L)]X_t^{NS} - \Theta_{X-11}^L(L)X_t^S \quad (3.6)$$

will also be a linear process. Moreover, as  $\Theta_{X-11}^L(1)$  and  $X_t^S$  do not have a zero-frequency unit root, it follows that  $\delta_t^{LSA}$  is stationary. This yields a second property of interest, namely:

*Property 2L: The extraction-error process  $\delta_t^{LSA}$  is linear and stationary.*

It will be interesting, once again, to investigate whether a similar property holds for the X-11 program. Let  $\delta_t^{SA}$  be the extraction-error process defined as in (3.6) yet involving  $X_t^{SA}$  instead of  $X_t^{LSA}$ . We are then interested in:

*Property 2X: The extraction-error process  $\delta_t^{SA}$  is linear and stationary.*

Again, if this property fails to hold this is an indication that there are significant departures from linearity. So far, we examined properties which are only applicable to simulated series since they involve the unobserved component series. Clearly, instead of comparing  $X_t^{NS}$  with  $X_t^{LSA}$  and  $X_t^{SA}$ , respectively, it is also useful to analyze  $X_t^{LSA}$  and  $X_t^{SA}$  in terms of cointegration and linearity. This yields two additional properties, namely,

*Property 3: The  $X_t^{LSA}$  and  $X_t^{SA}$  processes are cointegrated.*

*Property 4: The  $(X_t^{LSA} - X_t^{SA})$  process is linear and stationary.*

The latter is simply a combination of Properties 2L and 2X, since  $X_t^{LSA} - X_t^{SA} \equiv \delta_t^{LSA} - \delta_t^{SA}$ . Likewise, the former is a consequence of Properties 1X and 1L. Properties 3 and 4 are relatively straightforward to implement both with actual and simulated series.

The properties discussed so far pertain to the possible sources of nonlinearity associated with the internal operations of the program discussed in the previous section. At the end of section 2, it was noted that the combination of seasonal adjustment and aggregation can also be a source of nonlinear features. To investigate this aspect of the problem, we included in the simulation design a second process, called  $Y_t$ , with the same stochastic properties as the  $X_t$  process. It should be noted though that while  $Y_t$  is a replica of  $X_t$  in terms of laws of motion, its path generated by an independent realization of the innovation processes for the unobserved components, which will be denoted by analogy,  $Y_t^{NS}$  and  $Y_t^S$ . We also define the  $Y_t^{LSA}$  and  $Y_t^{SA}$  processes to describe extractions. The process of ultimate interest for our purposes will be the  $Z_t$  process defined as  $Z_t \equiv X_t + Y_t$ . Given the nature of aggregation, we restrict our attention to the additive version of the X-11 program. Hence,  $Z_t$  consists of two components, namely,  $Z_t^S \equiv X_t^S + Y_t^S$ . For the linear X-11 filter, one can unambiguously define the  $Z_t^{LSA}$  process since summation and linear filtering are interchangeable. For the X-11 procedure, however, one must distinguish between two potentially different outcomes. If seasonal adjustment is performed on the  $Z_t$  process using the X-11 program, then the outcome will be denoted  $Z_t^{SAA}$ . The superscript A indicates that the aggregated series was adjusted. Conversely, if  $X_t$  and  $Y_t$  are adjusted separately, then  $Z_t^{SAD} \equiv X_t^{SA} + Y_t^{SA}$ . We could investigate Properties 1 through 4, again, using the  $Z_t$  process and its extractions. This, to a certain extent, would be repetitive, except for the fact that the stochastic properties of the  $Z_t$  process would differ from those of  $X_t$  in each case. Instead of repeating such analysis, we will instead focus exclusively on the aggregation effects. In particular, we will be interested in:

*Property 5: The  $Z_t^{SAA}$  and  $Z_t^{SAD}$  processes are cointegrated.*

*Property 6: The  $(Z_t^{SAA} - Z_t^{SAD})$  process is linear and stationary.*

Both properties follow naturally from arguments similar to those used to formulate Properties 3 and 4.

### 3.3. Linear regression and filtering

Ultimately, economists are interested in understanding the comovements between economic time series. Until the work of Sims (1974) and Wallis (1974) discussions

regarding seasonal adjustment were mostly centered on a single economic series. We now have some strong results regarding (linear) filtering and seasonality in (linear) regression models. To date there has been no attempt to assess how fragile this finding is when faced with the practical and routine application of the X-11 procedure. In this section, we describe how our simulation design attempts to shed light on this relatively simple and fundamental question.

We propose to look at the linear regression model

$$y_t^i = \beta_0 + \beta_1 x_t^i + \varepsilon_t^i \quad (3.7)$$

for  $i = NS, LSA$  and  $SA$  and where  $y_t^{NS}$  and  $x_t^{NS}$  are *independently* generated processes mean zero, so that  $\beta_0 = \beta_1 = 0$  in our simulations. For the additive version of the X-11 program, the processes appearing in the regression model (3.7) were defined as follows:

$$y_t^i = (1 - L) Y_t^i \text{ and } x_t^i = (1 - L) X_t^i \quad (3.8)$$

for  $i = NS, LSA$  and  $SA$  while for the multiplicative version it was:

$$y_t^i = (1 - L) \log Y_t^i \text{ and } x_t^i = (1 - L) \log X_t^i. \quad (3.9)$$

To tackle immediately on the most practical question, we focus on testing the null hypothesis  $\beta_1 = 0$ , i.e., examine how spurious relationships can emerge from departures from linear filtering in a linear regression model. Obviously, since the error process in equation (3.7) will not be i.i.d. we need to correct for the serial dependence. This will be done in the nowadays established tradition among econometricians by using nonparametric procedures often referred to as heteroscedastic and autocorrelation consistent estimators for the variance of the residual process. The details are described in the next section. To conclude, we would like to note that to simplify the design, we will adopt a strategy similar to the one used in the construction of the aggregate process  $Z_t$  described in the previous section. In particular, the series  $X_t$  and  $Y_t$  used to run the regression in (3.7) will be independent draws from the same process structure.

### 3.4. Technical details

Several technical details need to be explained regarding the actual simulation setup. We will, in particular, describe the choice of parameter values to generate

the data. Next, we will explain how we conducted the statistical inference regarding the properties described in section 3.2. Then, we turn our attention to the specifics about the linear regression model of section 3.3. Finally, we conclude with some information of the software used in the computations.

**(a) Parameters and DGP's**

We have tried to cover a reasonably wide class of processes. A total of forty eight cases were considered, that is, sixteen model configurations with three different settings for the innovation variances  $\sigma_{NS}^2$  and  $\sigma_S^2$ . The parameter settings appear in Table 3.1. All data series were generated independently.

We first considered what will be called small-variance cases which correspond to  $\sigma_{NS}^2 = \sigma_S^2 = 1$ . The “large” standard error was chosen three times larger and hence a nine-times larger variance, i.e.,  $\sigma_{NS}^2 = \sigma_S^2 = 9$ . Cases 1 through 16 have a small variance, while cases 17 through 32 cover the large-variance configuration. Obviously, it is often the case that the seasonal component has a much larger innovation variance than the nonseasonal. This lead us to consider an intermediate case  $\sigma_{NS} = 1$  and  $\sigma_S = 3$ .

**Table 3.1: Data Generating Processes**

Cases	$\alpha_{NS}$	$\alpha_S$	Cases	$\alpha_{NS}$	$\alpha_S$
1/17/33	0.0	0.0	9/25/41	0.0	0.5
2/18/34	-0.5	0.0	10/26/42	-0.5	0.5
3/19/35	0.5	0.0	11/27/43	0.5	0.5
4/20/36	0.9	0.0	12/28/44	0.9	0.5
5/21/37	0.0	-0.5	13/29/45	0.0	0.9
6/22/38	-0.5	-0.5	14/30/46	-0.5	0.9
7/23/39	0.5	-0.5	15/31/47	0.5	0.9
8/24/40	0.9	-0.5	16/32/48	0.9	0.9

Cases 1-16:  $\sigma_{NS} = \sigma_S = 1$  / Cases 17-32:  $\sigma_{NS} = \sigma_S = 3$  /  
Cases 33-48:  $\sigma_{NS} = 1, \sigma_S = 3$ .

For the regression model (3.7), we conducted an extensive Monte Carlo study to examine the distribution of the t statistic for  $\beta_1 = 0$  when the actual

(unobserved component) series are used versus the linear and X-11 filtered series. The number of replications was 500, which is low by the usual standards, but the X-11 program was not available to us in a convenient format to construct a computationally efficient simulation setup. Even a stripped down version of the X-11 program would still be very demanding in terms of CPU time. At the end of the section, we will provide more details regarding software use. For the regression model, we investigated both a “small” sample which amounted to ten years of monthly data, i.e., 120 observations, in fact, 83 years or 996 data points to be more precise. The properties 1 through 6 were not studied via Monte Carlo, but instead for a single large sample. Conducting all the tests associated with the properties, which will be discussed in just a moment, in association with the X-11 program in a Monte Carlo experiment was simply beyond our human and computational resources.

#### **(b) Test statistics**

In the section 3.2, we formulated several properties which we expect to hold if no significant nonlinearities occur in the X-11 program. We now turn our attention to the analysis of these properties via statistical hypothesis testing. The null hypothesis of the test statistics will correspond to a situation where a property of interest holds whenever it relates to linearity and stationarity conditions, i.e., Properties 2L, 2X, 4 and 5. Because of the structure of cointegration tests, the situation will be slightly different with such tests. Indeed, the null hypothesis will correspond to a lack of cointegration and hence Properties 1L, 1X, 3 and 5 will be violated. The testing procedure proposed by Engle and Granger (1987) and Johansen (1991) were used to test the cointegration hypothesis. Since both procedures are by now widely known and applied, we refrain here from formally representing the tests. Instead, in the remainder of this section, we shall focus on the tests for nonlinearity in time series and conclude with observations regarding the t statistics in the linear regression model.

Obviously, there are many tests for nonlinearity in time series. The size and power properties against specific alternatives have been the subject of several Monte Carlo studies, including, most recently, Lee, White and Granger (1993). With 48 series and several properties to investigate, we were forced to make a very restrained and selective choice. Tests proposed by Tsay (1986), Luukkonen, Saikkonan and Teräsvirta (1988), and Tsay (1988) were used in our investigation.

Tests in this class are all designed according to a unifying principle, namely, they are all of the same form and yield an  $F$ -test.

The first step in all  $F$ -type tests consists of extracting a linear structure via an  $AR(p)$  model. Let the fitted value be denoted  $\hat{x}_t$  and the residual  $\hat{a}_t$ , while the original series is denoted  $x_t$ . Obviously,  $x_t$  will be a stand-in series for any of the series involved in testing the properties of interest formulated in the preceding section. The second step consists of regressing  $\hat{a}_t$  onto  $p$  lags of  $x_t$ , a constant and a set of nonlinear functions of past realizations of the  $x_t$  process. This operation yields a residual denoted  $\hat{e}_t$ . Finally, a  $F$ -test is computed from the sum of squared residuals obtained from both regressions. The tests differ in terms of the choice of nonlinear functionals used to form the regression producing the  $\hat{e}_t$  residuals. Tsay (1986) proposed to use the  $\{x_{t-1}^2, x_{t-1}x_{t-2}, \dots, x_{t-1}x_{t-p}, x_{t-2}^2, x_{t-2}x_{t-3}, \dots, x_{t-p}^2\}$  regressors. Luukkonen, Saikkonen and Teräsvirta added cubic terms to Tsay's test, namely,  $\{x_{t-1}^3, \dots, x_{t-p}^3\}$ . Finally, the second test proposed by Tsay (1988) is designed to test linearity against threshold nonlinearity, exponential nonlinearity and bilinearity. The fact that the test is designed against threshold nonlinearity may be of value, as the outlier detection schemes described in section 2 may result in threshold behavior of the linearly filtered versus X-11 filtered series. To conduct the test, one selects a threshold lag, say,  $x_{t-d}$ . Again, an  $AR(p)$  regression is fit to compute normalized predictive residuals  $\hat{e}_t$  similar to a CUSUM test. Then one regresses  $\hat{e}_t$  onto  $p$  lags of  $x_t$ , a constant, the regressor sets  $\{x_{t-1}\hat{e}_{t-1}, \dots, x_{t-p}\hat{e}_{t-p}\}$ ,  $\{\hat{e}_{t-1}\hat{e}_{t-2}, \dots, \hat{e}_{t-p}\hat{e}_{t-p-1}\}$ ,  $\{x_{t-1}\exp(-x_{t-1}^2/\gamma), G(z_{t-d}), x_{t-1}G(z_{t-d})\}$  where  $\gamma = \max |x_{t-1}|$  and  $z_{t-d} = (x_{t-d} - \bar{x}_d)/S_d$  with  $\bar{x}_d$  and  $S_d$  being the sample mean and standard deviation of  $x_{t-d}$  while  $G(\cdot)$  is the CDF of the standard normal distribution. One proceeds in the same manner as in the other two  $F$ -tests. In all our computations, we let  $p = 12$  and  $d = 1$  and 2.

We now turn our attention to the regression model. Since the series in equation (3.7) were generated independently, we are interested in testing the null hypothesis  $\beta_1 = 0$  knowing that the errors are not i.i.d. We followed the customary practice in econometrics of dealing with the temporal dependence in the residuals via a nonparametric estimator. The weights were those of the Bartlett window using 12 lags in the small sample and 24 in the large one [see for instance Andrews (1992) for a more elaborate discussion].

### (c) The Monte Carlo simulations and X-11 procedure

The original creators of the X-11 procedure probably never meant it to be inserted in a Monte Carlo simulation. The program is structured to be used on a case by case basis leaving many choices open to the discretion of the user. It would be impossible to simulate this elaborate day to day implementation of the procedure in dozens of statistical agencies around the globe. Such “judgemental corrections” are omnipresent, but they are most likely going to aggravate rather than attenuate the nonlinearities we will investigate. In our paper, we aimed co-apply the X-11 procedure without any active intervention on the part of the user. Doing otherwise, at least in a Monte Carlo setup, would simply be impossible. All calculations were done with the SAS version 6.01 PROC X-11 procedure. While we created samples of 120 monthly observations and 996 data points, we actually simulated longer samples which were shortened at both ends. This was done primarily for two reasons: (1) to be able to compute the two sided linear filter estimates requiring data points beyond the actual sample, and (2) because we wanted to a certain degree reduce the effect of starting values. Since all the time series generated are nonstationary, we have to be careful regarding the effect of starting value. (3) In a sense, the question of starting values is quite closely related to many of the questions regarding nonlinearities in X-11. There is, however, no obvious choice for these values. This implies a certain degree of arbitrariness in dealing with the problem. In our simulations, we took ten years of pre-sample data points while all components started at zero initial values. This can be criticized, but any other choice could be subjected to criticism as well because of the arbitrariness of the issue.

## 4. SIMULATION AND EMPIRICAL RESULTS

We have identified a set of properties and regression statistics. In this section, we summarize the findings of the simulation study and we complement them with empirical evidence drawn from actual economic time series. In a first subsection, we describe the results pertaining to the properties of a linear approximation described in section 3.2. The results reported in section 4.1 relate to the simulated data while the next section contains the empirical results. Section 4.3 concludes with a summary of the regression evidence.

#### 4.1. Simulation evidence on properties of linear approximation

We shall first report the results regarding cointegration tests and then proceed with the tests for nonlinearity. We report only the cases of the additive decomposition. The multiplicative decomposition yielded essentially the same results. Table 4.1 summarizes the cointegration test for the 48 model specifications for each of the four properties of interest. A lag length of 21 was selected and a constant and trend were included in all the test regressions. The top panel of Table 4.1 covers all “small” variance cases, while the middle part covers the equivalent parameter settings but with a larger innovation variance. The mixed variance cases, small for NS and large for S, appear in the bottom part. Whenever the null hypothesis is rejected, we find supporting evidence for the property of interest. For instance, Property 1L holds, regardless of the model specification. This is reassuring, of course, as we expect the linear filter to yield an extracted series which is cointegrated with the unobserved component process. The situation is quite different though for Property 1X. Indeed, with a small innovation variance, most cases yielded cointegrated processes. Two exceptions are models 11 and 12. The situation is completely different though when we increase the innovation variance either for both components together or for the seasonal only. Here, the extracted series and the target process are never cointegrated. This is obviously quite problematic and can only be attributed to the nonlinear properties of the X-11 program which come seriously into play. Since the mixed variance case is probably the most relevant for practical purpose, it appears from the results in Table 4.1 that what was identified as a *weak* property regarding the linear approximation does not seem to hold. Before turning to the stronger properties of linearity, let us briefly look at the aggregation results and property 3. The latter property only involves observed processes, namely  $X_t^{LSA}$  and  $X_t^{SA}$ , and is therefore more useful as it can be verified empirically. Generally speaking, the results in Table 4.1 show the same pattern as with properties 1X and 1L. This should not come as a surprise, since Property 3 is essentially a combination of the two. The results do not exactly conform with the combination of properties 1X and 1L, but the minor differences which occur can be attributed to statistical arguments about the sampling properties of tests. Finally, we turn our attention to the last property of interest. Here, as noted in section 2, we no longer investigate the internal *modus operandi* of the program, but we also consider the combined effects of seasonal adjustment and aggregation. Property 5 yields rather strong results and shows that aggregation adds a potentially important

source of nonlinearity to the data-adjustment process. Only less than a third of all cases yield a cointegration relationship between  $Z_t^{SAA}$  and  $Z_t^{SAD}$ . Clearly, all the potential sources of nonlinearity in  $Z_t^{SA}$ ,  $X_t^{SA}$  and  $Y_t^{SA}$  combined make it quite likely that the linear approximation will not be adequate in the sense that seasonal adjustment and aggregation are not interchangeable.

Next, we turn our attention to tests for nonlinearity. Strictly speaking, the distribution theory for such tests applies to stationary time series only. Therefore, we have limited our analysis to the cases where cointegrating relationships were found and ignored all other cases. To keep matters simple, however, we focused on all the small-variance cases, i.e., models 1 through 16, and deleted individual cases which, according to Table 4.1, did not support the cointegration hypothesis from the selection of models. Consequently, Tables 4.2 through 4.4 contain some missing values which correspond to the position in Table 4.1 where the hypothesis of no cointegration could not be rejected. Hence, conditional on having found cointegration, we investigate the stronger nonlinear properties.

For sake of simplicity, we use *Ori-F* for Tsay's original test, *Aug-F* for Luukkonen et al. test and *New-F* for Tsay's threshold test. The null hypothesis of linearity is almost always rejected for properties 4 and 6, regardless of the test statistic and model specification. Both properties are quite important since they have an empirical content, i.e., involve series that can be constructed from data. The results for properties 2L and 2X are mixed and depend on the test being used. For property 2L, we should not find nonlinearity and indeed most often we don't, but size distortions seem to be present in quite a few cases. For property 2X, we also find a mixture of results. It is interesting to note, however, that whenever we do not reject the null for property 2L, hence there is no size distortion, we tend to reject the null of linearity for property 2X.

## 4.2. An empirical investigation

The empirical investigation reported in this section is meant to match the simulations of the previous section. In particular, we investigated the properties 3, 4, 5 and 6 with actual data. The data do not involve corrections for trading-day variations and holidays. Hence, we tried to have the data conform with some of the assumptions made in the simulation experiments. A total of 39 series were investigated with some of the series being aggregates of several series.

According to our information, they are all treated with the multiplicative X-11 program. Such aggregate series were included to address the empirical evidence regarding properties 4 and 6. To construct  $X_t^{LSA}$  in each case, we used the two-sided symmetric filter applied to the logs of unadjusted data. Obviously, because of the number of leads and lags, a fair number of data points were lost at each end of the sample of unadjusted data. In all cases, data covered ten to fifteen years of monthly time series. Obviously, such sample sizes were much smaller than the simulated series. For  $X_t^{SA}$ , we took the officially adjusted series provided by the US Census Bureau or Federal Reserve (for monetary data). This may also be considered as a deviation from the simulation where the SAS X-11 procedure was used.

Table 4.5 summarizes the results of the Engle-Granger cointegration tests applied to  $X_t^{SA}$  and  $X_t^{LSA}$  for each of the 39 series listed. The BR, NBR and TR series are borrowed, nonborrowed and total reserve series of the US money supply. The BA extension is a break adjusted version of those series. All other series are drawn from the US Census manufacturing data bank, including industrial production IP, finished goods inventories FI, work in progress, WI for several two-digit SIC classification industries, and finally, total inventories TI for five subcategories of the SIC 20 sector (food). In all cases, the aggregate or TOT was also considered. In quite many cases, we do not reject the null hypothesis, implying that  $X_t^{LSA}$  and  $X_t^{SA}$  are not cointegrated. In 17 out of the 39 cases, or almost 50%, we find no cointegration at 10%, and in 21 out of the 39 cases, we find no cointegration at 5%. Obviously, the sample sizes are smaller compared to the results reported in Table 4.2, but still more than half of the series confirm the results found by simulation.

The empirical evidence with respect to the other properties, i.e., nonlinearity of  $X_t^{LSA} - X_t^{SA}$  and properties regarding  $Z_t^{SAD}$  and  $Z_t^{SAA}$  are not reported via tables, as they are relatively easy to summarize. All  $X_t^{LSA} - X_t^{SA}$  series were found to have nonlinearities. The rejections of the null hypothesis were very strong without any exception. Of course, unlike the simulated data which are by construction linear, an important caveat must be made regarding the interpretation of this kind of nonlinearity. Indeed, the individual series may very well be nonlinear, and we therefore find their difference to be nonlinear as well. For the TRBA, TR, FITOT, IPTOT, WITOT and TI20TOT series, we analyzed the nonlinearities via cointegration properties 5 and 6, since they involved a combination of aggregation

and seasonal adjustment. We found no cointegration and evidence of nonlinearity, though evidence regarding the latter is difficult to interpret because of lack of cointegration, of course.

### 4.3. Seasonal filtering and linear regression

We now turn our attention to a final question, which without any doubt is the most relevant for econometric practitioners: Are there spurious statistical relationships in linear regression models due to the nonlinear features in seasonal adjustment? We have computed a Monte Carlo simulation of the distribution of the  $t$  statistic in regression (3.7). There are 48 cases for the DGP and for each case, two filters (additive and multiplicative), as well as a large and small sample distribution for three regression  $t$  statistics with the true unobserved components and with the linearly filtered data and with X-11. Hence, we have a total 576 distributions. Reporting them all would of course be impossible. Fortunately, it was not very hard to select or choose some to report as there were remarkable similarities across the different cases. To illustrate this, we provide graphs of the distribution for cases 1 through 3 for “mixed” innovation variances both for a multiplicative and an additive X-11 filter setup. Each graph contains three plots of  $t$  distributions for the  $\beta_1$  coefficient simulated by Monte Carlo. The first is labeled “True” when the unobserved component series are used, a second is labeled “Linear” when the series are linearly filtered and a third is labeled “X-11”.

Before discussing the relative position of the three plots in each graph, we need to make some general observations. Because of the nonparametric correction of the residual variance estimator, the statistic is distributed as  $\chi^2(1)$ . There are clearly some minor size distortions since the 5% critical value does not yield a 5% rejection rate but instead a higher one in many cases, as will be reported later. The size distortion issue is not our main concern here, of course. In particular, it is interesting that while the “true” and “linear” regressions have very different dependences across their residuals, one observes that they have quite similar tail behavior for the  $t$  distribution. In contrast, the tail behavior of the “X-11” distribution in small samples almost always dominates that of the two other ones. This means that filtering with X-11 has spurious effects on finding significant relationships among independent series. To continue with the small sample case, we also notice that the multiplicative filter often causes more rejections in comparison to the additive decomposition though this is not

always the case. We will report this more explicitly with numerical results in Table 4.6. Before we do so, however, let us first turn our attention to the large sample cases. Here, we notice quite often a shift in the distribution of the “X-11” case relative to the others. It should parenthetically be noted that some caution is necessary when visually comparing the large and small sample plots as the scales of the two plots often are quite different. Moreover, when the peaks of the two distributions of the filtered cases coincide in large sample we still observe fatter tails for the X-11 case.

We turn our attention now to Table 4.6 where we report rejection rates obtained from the Monte Carlo simulations. Again, to avoid reporting 576 figures, we will focus on all DGP’s with a mixed variance covering both the additive and multiplicative filters in small and large samples. The figures reported in Table 4.6 confirm the size distortion issue which was already noted. In the large sample case with the “true” unobserved components, the distortions are minor, however. The results in the table quantify what the plot already revealed, namely that the rejections in the X columns are far higher than in the two other columns and that the “true” and “linear” cases are often very close. Moreover, the multiplicative filter often, though not always, leads to a higher rejection rate than the additive linear decomposition filter. For the X column, in large samples and using the multiplicative filter, the rejection rates range from 43.8% to 64.2%, while the T column ranges from 5.2% to 8.8% and the L column 6.8% to 13%. The results for the additive filter are equally dramatic for the X column, as rejection rates range from 47.8% to 63.2%. Finally, the rejection rates drop significantly from small to large samples in the T and L cases, but often they do not drop much in comparison with the X-11 filter.

It was noted in the previous section that we only can assess the effect of potential nonlinearities through simulation. Many more simulations were performed than are actually reported here. They clearly revealed the recurring pattern which was displayed Figures 4.1 and 4.2 and Table 4.6. There indeed appear to be departures from linearity that have serious effects on statistical inference in the practical circumstances which were simulated here.

**Figure 4.1: Density Plots Cases 1-3 for Additive X-11  
with Mixed Innovation Variances**

**Figure 4.2: Density Plots Cases 1-3 for Multiplicative X-11  
with Mixed Innovation Variances**

## 5. CONCLUSION

This paper probably raises more questions than it actually answers. There is indeed more research to be done on the topics which were discussed here. The issue of seasonality will never really easily be resolved and keeps intriguing generations of time series econometricians and statisticians. A quarter of a century after Young's paper was written with serious questions regarding the linearity of adjustment procedures, we find ourselves with the same question, but a different answer.

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