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# Simultaneous Supplies of Dirty Energy and Capacity Constrained Clean Energy: Is there a Green Paradox?*

*Marc Gronwald*[†], *Ngo Van Long*[‡], *Luise Roepke*[§]


## Abstract

We analyze the effects of two popular second-best clean energy policies, using an extended resource extraction framework. This model features, first, heterogeneous energy sources and, second, a capacity-constrained backstop technology. This setup allows for capturing the following two empirical observations. First, different types of energy sources are used simultaneously despite different production costs. Second, experiences from various European countries show that a further expansion of the use of climate friendly technologies faces substantial technological as well as political constraints. We use this framework to analyze if under two policy scenarios a so-called "Green Paradox" occurs. A subsidy for the clean energy as well as an expansion of the capacity of the clean energy are considered. The analysis shows that while both policy measures lead to a weak Green Paradox, a strong Green Paradox is only found for the capacity expansion scenario. In addition, the subsidy is found to be welfare enhancing while the capacity increase is welfare enhancing only if the cost of adding the capacity is sufficiently small.


**Keywords**: Capacity constraints, Green Paradox, Climate change, Simultaneous resource use


## Résumé

Nous analysons les effets de deux politiques encourageant l'énergie verte, en utilisant un cadre élargi d'extraction des ressources. Ce modèle comporte, d'une part, des sources d'énergie hétérogènes et, d'autre part, une technologie verte dont l'exploitation est sous une contrainte de capacité. Cette configuration permet de capturer les deux observations empiriques suivantes. Tout d'abord, plusieurs sources d'énergie sont utilisées simultanément malgré l'écart de coûts de production. Deuxièmement, les expériences de divers pays européens montrent qu'une expansion accrue de l'utilisation de technologies respectueuses du climat fait face à des contraintes technologiques et politiques importantes. Nous utilisons ce cadre pour analyser si sous deux scénarios de politique un soi-disant « Paradoxe Vert » se produit. Une subvention sur le coût de l'énergie verte ainsi qu'une expansion de la capacité de l'énergie verte sont prises en considération. L'analyse montre que tandis que les deux mesures politiques conduisent à un Paradoxe Vert faible, un Paradoxe Vert fort est seulement trouvé pour le scénario d'expansion de la capacité. En outre, la subvention améliore le bien-être, alors que l'accroissement de la capacité ne favorise le bien-être que si le coût d'ajout de la capacité est suffisamment faible.


**Mots clés** : Contrainte de capacité, Paradoxe Vert, Changements climatiques, Utilisation simultanée des ressources

**Codes JEL/JEL Codes:** Q38, Q54, H23

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# 1 Introduction

The decarbonisation of the global economy is very high on the global political agenda. As various types of clean technologies are available, the situation looks generally promising: Wind as well as solar energy generally could replace conventional fossil fuel power plants; thus, electricity generation potentially could become considerably cleaner. The situation in the transport sector is similar: biofuels have the potential to replace conventional fuels. What is more, a considerable political will is evident, and has manifested itself in various types of policy measures such as feed-in tariffs for renewable energy or biofuel mandates.

However, this decarbonisation process necessarily involves nothing short of an entire reconstruction of the global energy sector. Thus, it is clearly not an easy task. It is not just that this process is of large scale and involves complex investment projects with very long horizons, it also seems to meet increasing resistance in the population. To name just a few examples, land used for production of biofuels reduces the land areas for growing food. This results in concerns about food security and sustainability. As long as more advanced technologies such as second generation biofuels are not yet available, the situation looks difficult. On the electricity generation front, the situation is similar: in countries such as Germany and the United Kingdom, the installation of additional wind generation capacities becomes increasingly difficult as it finds insufficient support in many local communities. An additional challenge in this context is the installation of enormous amounts of electricity transmission capacities - this is often equally unwelcome. Greatly improved energy storage technologies would certainly be very helpful in this regard but are not available yet.

The consequence of these challenges is that while clean technologies are certainly used in various countries, in particular in Europe, the available capacities are not sufficient for meeting the complete energy demand and, in addition, expanding the use of clean energies is getting increasingly complicated. This constitutes a major explanation of two empirical facts. First, both fossil and clean energy is produced simultaneously even though the latter is still considerably more expensive than the former. Second, climate friendly energy is used but its capacity is severely constrained. Further expansions are challenging because of technological and/or political constraints. To capture these two empirical observations, our paper proposes an extended Hotelling-type resource extraction model with two exhaustible resources and one capacity-constrained clean backstop. In addition, the model assumes heterogeneous dirty resources; this reflects the simultaneous use of e.g. both conventional and unconventional oil. By allowing heterogeneity in the pollution contents of dirty resources, this model extends Holland's (2003) analysis of constrained extraction capacities and the order of extraction, allowing us to evaluate second-best climate policies.

This framework is then used to analyse two different scenarios. First, assuming that the first-best carbon tax is not politically feasible, we consider the introduction of a subsidy on the clean technology. Subsidizing the clean energy sector is a very common and popular second-best policy measure. Second, the effect of an expansion of the capacity - an increase in the availability - of the backstop is analysed. This can be more broadly interpreted, to include the sudden availability of a new technology which allows using clean technologies to a much larger extent, e.g. a breakthrough in areas such as advanced biofuels or energy storage. The latter would allow a massive increase in the use of renewable electricity. Technological breakthroughs of this type may be the result of a public policy such as research and development subsidies. The effects on both extraction paths of the dirty exhaustible resources and the total welfare are analysed. Specifically, we ask if there are negative consequences for the climate when second-best policies are implemented. As Sinn (2008) puts it: is there a Green Paradox? The analysis conducted in this paper involves both analytical and numerical parts; the calibration of the numerical part is based on empirical data on the global crude oil market. The analysis, finally, employs the notions of a "weak Green Paradox" and a "strong Green Paradox" introduced by Gerlagh (2011). The former describes a short-term increase of anthropogenic emissions in response to a policy measure, the latter an increase in cumulative damages.

Our analysis shows that whereas both policy measures lead to a weak Green Paradox, a strong Green Paradox is only found for the capacity expansion scenario. In addition, the subsidy is found to be welfare enhancing while the capacity increase is welfare enhancing only if the cost of adding the capacity is sufficiently small. In terms of the present value of the stream of damage costs, we find that a subsidy of 25% on the clean energy will reduce total damage costs by about 10%, while a capacity expansion of 20% will increase total damage costs by about 5%. The reason is that a subsidy makes clean energy production profitable at an earlier date, resulting in pushing the fossil resource exhaustion dates further into the future, so that the pollution stock peaks at a later date. In contrast, a capacity expansion reduces the maximum price that the last drop of oil would earn. This results in a strong incentive for fossil resource owners to start their extraction earlier.

Assuming that the climate friendly backstop technology is capacity constrained makes a significant contribution to the Green Paradox literature.[1] Up to now, all papers which contain a backstop technology assume that at some point a backstop technology becomes (economically) available in unlimited amounts and replaces conventional energy sources completely.[2] Our assumption of a capacity constrained backstop technology allows for the analy-

[1] Ploeg and Withagen (2015) as well as Jensen et al. (2015) provide excellent overviews of the Green Paradox literature.

[2] The more the capacity constraint is relaxed, the more the clean substitute becomes a "classic" backstop

sis of a completely new scenario: what are the consequences of an increase in the availability of clean energy? Hoel (2011) contributes to the "traditional" backstop technology using a two-country model. His analysis shows that the degree of country heterogeneity has significant effects on how subsidies or taxes on the one hand and emissions paths on the other are related. Ploeg and Withagen (2012) offer an interesting refinement: they show that the cost of a backstop technology are essential for the existence or non-existence of a Green Paradox outcome. If the backstop is relatively expensive and, thus, full exhaustion of the non-renewable resource is optimal, a Green Paradox occurs. However, if the backstop is sufficiently cheap this finding is reversed. Hoel and Jensen's (2012) paper also consider different types of climate friendly technologies and show e.g. that CCS can have different effects than renewable energies. Michielsen (2014), in contrast, considers a more refined dirty resource sector. His paper shows that, under certain conditions, the anticipation of a climate policy can actually reduce current emissions: a so-called Green Orthodox occurs. A key factor of his model is the degree of substitutability between the dirty resources. Grafton et al. (2012) analyse the effects of biofuel subsidies. Their paper shows that whether or not a Green Paradox occurs depends on factors such as the extraction cost of the fossil resource and/or marginal cost of using biofuels.

It is worth noting that Holland's (2003) original model contributes to the optimal order of extraction literature.[3] The key feature of Holland's model is that some extraction capacities are limited, which has important implications for the optimal order of resource extraction: some high cost resources may be exploited simultaneously with (or even strictly before) other resources with lower marginal extraction costs. The resulting extraction patterns are similar to the ones that can be observed empirically. Holland (2003) argues that resource owners base their extraction decision not only on marginal extraction costs, but also on the scarcity rent of the resources. This paper along with contributions such as Grafton et al. (2012) vividly illustrate the usefulness of allowing for simultaneous use of energy resources. Thus, it is overall useful to reactivate this literature.

The remainder of the paper is organised as follows. In the next section, we derive a model of substitute production under a capacity constraint. Section 3 describes the first-best solution; Section 4 discusses two feasible policy scenarios. Section 5 illustrates the policy

technology. We model the backstop technology in line with Dasgupta and Heal (1974), as a "perfectly durable commodity, which provides a flow of services at constant rate."

[3] This literature has its origin in Herfindahl's (1967) seminal paper accoring to which resources with different constant marginal extraction costs are extracted in strict order from low to high-cost. This so-called Herfindahl rule, however, has been repeatedly disputed; see, for example, Kemp and Long (1980) and Amigues et al. (1998). Chakravorty et al. (2008) have extended the order of extraction literature to the case where different resources have different pollution contents, assuming that the planner is constrained by a self-imposed ceiling on the pollution stock.

relevance of this paper; Section 6 offers some concluding remarks.

## 2 A model of substitute production under capacity constraint

Assume that there are two deposits of fossil fuels, $S_1$ and $S_2$.[4] The constant per unit extraction costs for these deposits are $c_1$ and $c_2$, respectively. There are no capacity constraints on the amount of extraction at any given point of time $t$. The cumulative extraction constraints are

$$\int_0^\infty q_i(t)dt \le S_i \text{ for } i = 1, 2.$$

The initial stocks of fossil fuels are $S_1(0) = S_{10}$ and $S_2(0) = S_{20}$. The rates of extractions from the two stocks are denoted by $q_1$ and $q_2$. Then we have

$$\dot{S}_i(t) = -q_i(t), \text{ with } S_i(0) = S_{i0}$$

Following Ploeg and Withagen (2012), we assume a pollution decay rate of zero. Then the stock of pollution, denoted by $X$, evolves according to the rule

$$\dot{X} = \eta_1 q_1 + \eta_2 q_2, \text{ with } X(0) = X_0, \text{ given.}$$

Here $\eta_1$ and $\eta_2$ are the pollution contents per unit, and we assume that $\eta_2 > \eta_1 > 0$.

The maximum possible stock of pollution is $\overline{X}$, where $\overline{X} = X_0 + \eta_1 S_{10} + \eta_2 S_{20}$. The damage cost at time $t$ depends on the stock $X(t)$. The damage function is denoted by $G(X)$. We assume that $G'(X) > 0$ and $G''(X) \ge 0$.

There is a clean energy that is a perfect substitute for the fossil fuels. Let $q_3(t)$ be the amount of clean energy produced at time $t$. The key contribution of this paper is the assumption that there is a capacity constraint on clean energy production: $q_3(t) \le \overline{q}_3$. This means that at each point of time, the amount of clean energy that can be produced is exogenously determined by the capacity constraint. Let $c_3$ be the constant unit cost of production of the clean energy.

Let $Q(t) = q_1(t) + q_2(t) + q_3(t)$ denote the aggregate supply of energy from the three resources at time $t$, where some of these $q_i(t)$ may be zero. The utility of consuming $Q(t)$ is

[4] Please note that this basic model is borrowed from Holland (2003) who analyses the simultaneous use of exhaustible resources with different marginal extraction cost from the social planner's perspective. Holland (2003) does not distinguish between dirty and clean energy and does not undertake any comparative static or welfare analysis.

$U[Q(t)]$, where $U(\cdot)$ is a strictly concave and increasing function and $U'(0)$ can be finite or infinite. We assume that

$$U'(\overline{q}_3) > c_3 > c_2 > c_1 > 0$$

The instantaneous welfare at time $t$ is

$$W(t) = U(Q(t)) - \sum_{i=1}^{3} q_i(t) - G(X(t)) \quad (1)$$

Our first task is to characterize the equilibrium in the *perfect competition situation, in the absence of a carbon tax.* Consumer' demand is represented by the condition $p = U'(Q)$. Inverting this function, we obtain the demand function $Q = D(p)$, $D'(p) < 0$.

The resource owners follow a Hotelling-type extraction path, maximizing the value of the resource stocks such that the resource rent increases at the rate of interest. The extraction order of the exhaustible resource stocks is based on the Herfindahl rule: The low-cost resource stock is strictly exhausted before the high-cost resource stock is extracted. Since the renewable resource owners do not have to optimise intertemporarily, their supply behaviour is different from that of the exhaustible resource owners. The next subsection presents two conditions which guarantee that the high-cost renewable energy will be produced simultaneously with extraction of the lowest cost deposit, and well before the intermediate cost stock $S_2$ enters into production. These conditions were first identified by Holland (2003).

## 2.1 Extraction capacity and cost reversal in the absence of a carbon tax

Based on Holland (2003), two conditions are imposed to ensure that both a binding capacity constraint of the renewable energy, as well as the cost reversal phenomenon, can be illustrated in the model. By "cost reversal", we mean that the higher cost renewable resource is produced well before the intermediate cost exhaustible resource begins to be extracted. In specifying the capacity constraint, we describe the real-world situation where even though in theory we have enough renewable energy resources, only a limited amount of that energy is practically available due to technological and economic constraints. To sharpen the consequences of this situation, we focus in the following analysis on the case where the capacity constraint is binding when clean energy is produced. Then, at price $p = c_3$, the market demand $D(c_3)$ for energy exceeds the capacity output of the clean energy sector $\overline{q}_3$. This is stated in the following condition.

**Condition 1:** $D(c_3) > \overline{q}_3$

It follows that when $p(t)$ reaches $c_3$, the market demand must be met from both the clean energy sector and fossil fuel extraction.

Since the demand curve is downward sloping, Condition 1 implies that there exists a value $\overline{p} > c_3$ such that $D(\overline{p}) = \overline{q}_3$. Therefore, for all $p$ in the range $[c_3, \overline{p}]$, the clean resource will always be produced at maximum capacity. The equilibrium price of energy can never exceed $\overline{p}$.

The second condition is that the size of the high-cost exhaustible resource must be small enough such that the cost reversal of resource use described in the introduction can be illustrated with the present model. An analytical derivation of this condition can be found in Appendix A.

$$\textbf{Condition 2: } S_{20} < S_{20}^{\max} \equiv \int_0^x D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau - \frac{\overline{q}_3}{r} \ln\left[\frac{\overline{p} - c_2}{c_3 - c_2}\right]$$

where we define $x$ by

$$x = \frac{1}{r} \ln\left[\frac{\overline{p} - c_2}{c_3 - c_2}\right].$$

From condition 2, we can show that if the size of deposit 2 is smaller than the threshold value $S_{20}^{\max}$, the equilibrium time path of extraction is continuous and production of clean energy starts strictly before the extraction of the high-cost resource deposit $S_2$ begins (Holland 2003).[5]

## 2.2 Four phases of resource utilization and the price path

Based on Conditions 1 and 2, the equilibrium path of the energy price is continuous and the resource use pattern can be described as follows.

*Phase 1*: Energy is supplied only by extraction from the low-cost deposit. This phase begins at time 0 and ends at an endogenously determined time $t_3 > 0$, such that the equilibrium price at time $t_3$ is equal to $c_3$. During this phase, the net price of the low-cost resource, $p(t) - c_1$, rises at a rate equal to the interest rate $r$.

*Phase 2*: Energy is simultaneously supplied by both extraction from the low-cost resource deposit $S_1$ and the (more costly) renewable energy running at its capacity level $\overline{q}_3$. This phase begins at time $t_3$ and ends at an endogenously determined time $T > t_3$. The low-cost resource

[5] The situations we analyze, based on the stated conditions, must be viewed as extreme cases. The model could also be designed to lead to a smooth increase in the production of clean energy until the constraint is reached (which would be in accordance with actual observations in, for example, Germany). For simplicity and to sharpen our results, we believe it is useful to retain the strong assumptions. Determining a "dynamic capacity increase" would allow differentiating between constraints on existing production and natural capacity restrictions. Modeling such a differentiation would allow us to show a smooth and increasing use of clean energy while maintaining the constrained situation.

stock $S_1$ is entirely exhausted at time $T$. During this phase, the net price of the low-cost exhaustible resource, $p(t) - c_1$, also rises at a rate equal to the interest rate $r$.

*Phase 3:* Energy is simultaneously supplied by both extraction from the intermediate-cost resource deposit $S_2$ and the (more costly) renewable energy running at its capacity level $\overline{q}_3$. This phase begins at time $T$ and ends at an endogenously determined time $\overline{T}$. At time $\overline{T}$, the stock $S_2$ is completely exhausted. During this phase the net price of the higher cost exhaustible resource, $p(t) - c_2$, rises at a rate equal to the interest rate $r$. At time $\overline{T}$, the energy price reaches $\overline{p}$ (where $\overline{p}$ is defined by $D(\overline{p}) = \overline{q}_3$).

*Phase 4:* The only source of energy is clean energy, available at capacity level $\overline{q}_3$. The price is constant at $\overline{p}$. This phase begins at time $\overline{T}$ and continues for ever.

Note that from time $t_3$ on, where $p(t_3) = c_3$, the clean energy sector will supply $\overline{q}_3$ without any intertemporal considerations, and due to the assumption stated in Condition 1, there will not be enough energy to meet the demand $D(c_3)$. The shortfall, or residual demand, is met by extraction from the lowest-cost deposit available such that at $t_3$,

$$\overline{q}_3 + q_1(t_3) = D(c_3).$$

In other words, only the residual demand must be met by the exhaustible resource, indicating that the existence of a constrained renewable resource alleviates the scarcity problem of the exhaustible resources.[6]

Holland (2003) did not provide explicit equations that specify how the length of various phases depends on parameters such as $c_1, c_2, c_3, \overline{q}_3, S_{10}$ and $S_{20}$. In what follows, we derive such equations, which help us obtain insightful comparative static results.

## 2.3 Numerical analysis

In addition to theoretical analyses, this paper also uses a numerical illustration. This section briefly summarises parameter choices. The general aim is to capture relationships observable in the global crude oil market. We set $c_1 = 0.75, c_2 = 1.75$, and $c_3 = 4$. Moreover, we assume linear demand, $D(p) = A - p$. We choose $A = 20, \overline{p} = 15$, $r = 0.01$. Then $\overline{q}_3 = A - \overline{p} = 5$. To compute the pollution stock, we specify the stock sizes $S_{10}$ and $S_{20}$. We assume the following: $S_{20} = 900$ and $S_{10} = 700$. According to International Energy Agency (2015, Table 3.4), remaining conventional and unconventional oil resources are 2,787 billion barrels and 3,298 billion barrels, respectively. Thus, the ratio between these two types of resources

[6] The reason deposit 2 is not extracted during the time interval $[t_3, T)$ is that any attempt to move extraction from $S_2$ to that interval to replace the high-cost clean energy would require curtailing consumption during the phase $[T, \overline{T})$, which implies costs in terms of foregoing consumption smoothing.

approximately matches with the two fossil resources $S_1$ and $S_2$ in this paper. In addition, it would be plausible to assume the following marginal extraction cost: conventional oil 30 USD per barrel, unconventional oil 70 USD per barrel, advanced biofuels 160USD per barrel. The ratios between these marginal costs also approximately match the ratios between $c_1 = 0.75, c_2 = 1.75$, and $c_3 = 4$ used in this paper. The remaining parameters are chosen arbitrarily.

First, we need to make sure that $S_{20} < S_{20}^{\max}$. This means that we first have to compute the value $S_2^{\max}$ from our specifications of the cost parameters $c_1$, $c_2$, and $c_3$ and of capacity $\overline{q}_3$. We finds that $S_{20}^{\max}$ equals approximately 1249, hence $S_{20} = 900$ does indeed satisfy the condition $S_{20} < S_{20}^{\max}$.[7]

# 3 The first-best scenario

In this section, we consider the first-best scenario. We assume that the social planner chooses the time path of extractions and supply of renewable energy to maximize the integral of the discounted stream of instantaneous welfare

$$\int_0^\infty e^{-rt} W(t) dt$$

where $W(t)$ is given by (1), subject to

$$\dot{S}_i(t) = -q_i(t),\, S_i(0) = S_{i0},\;\; S_i(t) \geq 0,\, i = 1, 2$$

$$\dot{X}(t) = \eta_1 q_1(t) + \eta_2 q_2(t),\, X(0) = X_0$$

$$q_i(t) \geq 0,\, i = 1, 2, 3$$

$$\overline{q}_3 - q_3(t) \geq 0.$$

## 3.1 Characterizing the planner's solution

### 3.1.1 The necessary conditions

Let $\phi(t)$ denote the shadow price of the pollution stock $X(t)$ and $\psi_i(t)$ the shadow price of the resource stock $S_i(t)$. We form the Hamiltonian function

$$H = U(Q(t)) - \sum_{i=1}^{3} c_i q_i(t) - G(X(t)) - \sum_{i=1}^{2} \psi_i(t) q_i(t) + \phi(t) \sum_{i=1}^{2} \eta_i q_i(t)$$

[7] This condition is also fulfilled for all the following model specifications.

Let $\lambda_i(t) \geq 0$ be the Lagrange multiplier associated with the non-negativity constraints on $q_i(t)$ and $\rho(t) \geq 0$ be the Lagrange multiplier associated with the capacity constraint $\overline{q}_3 - q_3(t) \geq 0$. Form the Lagragian function

$$L = H + \sum_{i=1}^{3} \lambda_i q_i + \rho \left[\overline{q}_3 - q_3\right]$$

The necessary conditions are

$$\frac{\partial L}{\partial q_i} = U'(Q) - (c_i - \phi\eta_i) - \psi_i + \lambda_i = 0, \; i = 1, 2, \tag{2}$$

$$\frac{\partial L}{\partial q_3} = U'(Q) - \rho + \lambda_3 = 0 \tag{3}$$

$$q_i \geq 0, \; \lambda_i \geq 0, \; \lambda_i q_i = 0, \; i = 1, 2, 3$$

$$\overline{q}_3 - q_3 \geq 0, \; \rho \geq 0, \; \rho\left[\overline{q}_3 - q_3\right] = 0$$

$$\dot{\psi}_i = r\psi_i, \; i = 1, 2 \tag{4}$$

$$\dot{\phi} = r\phi - \frac{\partial H}{\partial X} = r\phi + G'(X)$$

The transversality conditions are

$$\lim_{t\to\infty} e^{-rt}\psi_i(t) \geq 0, \;\; \lim_{t\to\infty} S_i(t) \geq 0, \; \lim_{t\to\infty} e^{-rt}\psi_i(t)S_i(t) = 0 \tag{5}$$

$$\lim_{t\to\infty} e^{-rt}\phi(t) = 0 \tag{6}$$

It will be convenient to define the social cost of carbon as $\mu(t) = -\phi(t)$. Then

$$\dot{\mu} = r\mu - G'(X) \tag{7}$$

From equation (7),

$$(\dot{\mu} - r\mu)e^{-rt} = -G'(X)e^{-rt}$$

Integrating, and using the transversality condition (6), we get

$$\mu(t) = \int_t^{\infty} e^{-r(\tau - t)} G'(X(\tau))d\tau \tag{8}$$

This shows that the social cost of carbon at time $t$ is the present value of the stream of marginal damage costs.

Since $\mu(t) = -\phi(t)$, condition (2) may be written as

$$U'(Q(t)) - (c_i + \mu(t)\eta_i) - \psi_i(t) + \lambda_i(t) = 0,\ i = 1, 2, \tag{9}$$

Interpreting $U'(Q(t))$ as the price of energy at time $t$, denoted by $p(t)$, and $(c_i + \mu(t)\eta_i)$ as the extraction cost plus the optimal carbon tax, equation (9) means that the resource rent, $\psi_i(t)$, is equal to the price minus the marginal tax-inclusive cost of extraction. From equation (4) the resource rent must rise at a rate equal to the rate of interest. Thus, if resource $i$ is extracted at any two dates $t$ and $t'$ then it must hold that the present value of the resource rents at these dates are equalized:

$$[p(t) - (c_i + \mu(t)\eta_i)]\, e^{-rt} = [p(t') - (c_i + \mu(t')\eta_i)]\, e^{-rt'} \tag{10}$$

This is the Hotelling rule when the optimal carbon tax is levied on extraction. Furthermore, since $c_2 + \mu(t)\eta_2 > c_1 + \mu(t)\eta_1$ for all $\mu(t) \geq 0$, the deposit with the low tax-inclusive cost must be exhausted before the extraction of the higher tax-inclusive cost deposit begins. This is in accordance with Herfindahl's rule.

### 3.1.2 Some general results concerning the first-best solution

The following assumption will ensure that both deposits will be exhausted at some finite time:

**Assumption A1:** *The marginal damage cost when the pollution stock is at its maximum level, $G'(\overline{X})$, is small enough so that*

$$\eta_2 \frac{G'(\overline{X})}{r} + c_2 < c_3 \tag{11}$$

This assumption means that even the last drop of oil has a positive marginal contribution to social welfare.

Under Assumption A1, the economy will eventually reach a steady state when both deposits have been exhausted. The steady-state instantaneous welfare level is

$$\overline{W} = U(\overline{q}_3) - c_3\overline{q}_3 - G\left(\overline{X}\right)$$

At the steady state, the social cost of carbon is equal to the present value of the marginal damage flow:

$$\overline{\mu} = \frac{G'(\overline{X})}{r}$$

The order of exploitation of the two deposits is as follows: the low-cost and less-polluting deposit $S_1$ must be exhausted before extraction begins for the high-cost and more-polluting deposit $S_2$. At the steady state, the price of energy is $\overline{p} \equiv U'(\overline{q}_3)$ while the marginal production cost of clean energy is $c_3 < U'(\overline{q}_3)$ (by assumption). The "profit" flow to the clean energy producers is $[\overline{p} - c_3]\,\overline{q}_3$. This "profit" is the quasi-rent earned by owners of the fixed capacity $\overline{q}_3$. Clearly, because $\overline{p} > c_3$, it is optimal to use the clean energy source before the exhaustion of the high-cost deposit.

### 3.1.3 First-best solution when the damage cost is linear in the stock of pollution

In this subsection, we consider a special case of the model: we assume that the damage cost function is linear in the pollution stock: $G(X) = \beta X$ where $\beta > 0$. This assumption allows us to have an explicit solution of the model.

Using $G'(X) = \beta$, equation (8) gives

$$\mu(t) = \frac{\beta}{r} \equiv \overline{\mu}$$

Then using equation (10), we obtain the result that for any two dates $t$ and $t'$ such the extraction from deposit $i$ is strictly positive $q_i$, it holds that

$$[p(t) - (c_i + \overline{\mu}\eta_i)]\, e^{-rt} = [p(t') - (c_i + \overline{\mu}\eta_i)]\, e^{-rt'} \tag{12}$$

It follows that the analysis of the four phases of the BAU scenario applies also to the first-best scenario, provided we replace $c_i$ with $(c_i + \overline{\mu}\eta_i)$. Condition 1 would then be replaced by

$$S_2(0) \leq S_2^* \equiv \int_0^x D\left[c_2 + \overline{\mu}\eta_2 + (c_3 - (c_2 + \overline{\mu}\eta_2))e^{r\tau}\right] d\tau - x\overline{q}_3$$

where $x$ now stands for

$$x \equiv \frac{1}{r} \ln \left[\frac{\overline{p} - (c_2 + \overline{\mu}\eta_2)}{c_3 - (c_2 + \overline{\mu}\eta_2)}\right]$$

For details, please refer to Appendix A.

## 3.2 Calculation of welfare in the first-best scenario

The social welfare in the first-best scenario is the sum of welfare levels of the four successive phases:

$$W = W_1 + W_2 + W_3 + W_4$$

where $W_i$ $(i = 1, 2, 3, 4)$ are defined as follows:

$$W_1 = \int_0^{t_3} e^{-rt} \left[ U(q_1(t)) - c_1 q_1(t) - \beta X(t) \right] dt$$

$$W_2 = \int_{t_3}^{T} e^{-rt} \left[ U(\overline{q}_3 + q_1(t)) - c_3 \overline{q}_3 - c_1 q_1(t) - \beta X(t) \right] dt$$

$$W_3 = \int_{T}^{\overline{T}} e^{-rt} \left[ U(\overline{q}_3 + q_2(t)) - c_3 \overline{q}_3 - c_2 q_2(t) - \beta X(t) \right] dt$$

and

$$W_4 = e^{-r\overline{T}} \left[ \frac{U(\overline{q}_3) - c_3 \overline{q}_3 - \beta \overline{X}}{r} \right]$$

#### 3.2.1 Numerical results and calculation of welfare under the first-best carbon tax

For ease of computation, we assume a quadratic utility function:

$$U(Q) = AQ - \frac{1}{2} Q^2$$

We set $r = 0.01$, $c_1 = 0.75$, $c_2 = 1.75$, $c_3 = 4$, $A = 20$ , $\overline{q}_3 = 5$. Assume $S_{10} = 700$ and $S_{20} = 900$ and $X_0 = 100$. Concerning pollution, we assume that $\beta = 0.01, \overline{\mu} = \frac{\beta}{r} = 1, \eta_1 = 1, \eta_2 = 2$.

We find that

(1) Phase 1 starts at $t_0 = 0$ , with the initial price $p(0) = 3.92$ and ends at $t_3 = 3.31$, when the price reaches $c_3 = 4$.

(2) Phase 2 starts at time $t_3 = 3.31$ and lasts 64.07 years. It ends at time $T = 67.38$, when the price reaches $p(T) = 6.02$.

(3) Phase 3 starts at time $T = 67.38$ and lasts 160 years. It ends at time $\overline{T} = 227.38$, when the price reaches $\overline{p} = 15$.

The welfare calculations reveal that

$$W_1 = 582.4,\ W_2 = 7145.9,\ W_3 = 4660.8 \text{ and } W_4 = 427.12; \quad \sum W_i = 12,816.22$$

Concerning pollution damages, let us define

$$\Omega_1 = \int_0^{t_3} e^{-rt} \left[ \beta X(t) \right] dt,\ \Omega_2 = \int_{t_3}^{T} e^{-rt} \left[ \beta X(t) \right] dt,$$

$$\Omega_3 = \int_T^{\overline{T}} e^{-rt} \left[\beta X(t)\right] dt, \; \Omega_4 = \int_{\overline{T}}^{\infty} e^{-rt} \left[\beta X(t)\right] dt.$$

We find that $\Omega_1 = 412$, $\Omega_2 = 207.12$, $\Omega_3 = 684.81$ and $\Omega_4 = 267.59$. Thus, the total damage over the entire program is $1,163$. The ratios of damages to welfare in the four phases are

$$\frac{\Omega_1}{W_1} = 0.7\%, \; \frac{\Omega_2}{W_2} = 2.9\%, \; \frac{\Omega_3}{W_3} = 14.7\%, \frac{\Omega_4}{W_4} = 62\%,$$

and the overall ratio is

$$\frac{\sum \Omega_i}{\sum W_i} = 9\%$$

# 4 Policy scenario analysis

This section moves to the second-best world. We intially describe in more details the so-called Business-as-usual (BAU) scenario - i.e., assuming that there is no government intervention (the carbon tax is zero identically). Next, we build on this situation to analyse the effects of the two second best policies: (i) subsidizing the clean energy and (ii) expansion of the clean energy capacity.

## 4.1 Sequential determination of the key variables

**First, we determine the length of Phase 3**. Recall that $y \equiv \overline{T} - T$. Since total demand must equal total supply during $\left[T, \overline{T}\right)$ and deposit 2 must be exhausted during this interval, we can solve for $y$ from the following equation

$$\int_T^{\overline{T}} D[p(t)]dt = S_{20} + \left(\overline{T} - T\right) \overline{q}_3. \tag{13}$$

Since $q_2(t) > 0$ over the time interval $\left[T, \overline{T}\right)$, the Hotelling rule applied to deposit 2 must hold with equality such that

$$p(t) = c_2 + \left(\overline{p} - c_2\right) e^{r(t-T-y)} \tag{14}$$

with $t - \overline{T} = t - T - \left(\overline{T} - T\right)$.[8] Inserting this into Equation (13), together with $\tau = t - T$, and noting that $\overline{p}$ and $\overline{q}_3$ are related through the equation $\overline{q}_3 = D\left(\overline{p}\right)$, then $y$ is the solution

[8] Analogous to the Appendix, $p(t)$ can be derived from the condition $(p(t) - c_2)e^{-rt} = (p(T) - c_2)e^{-rT} = (p(\overline{T}) - c_2)e^{-r\overline{T}}$.

of the following equation

$$0 = F(S_2, \overline{p}, c_2) = \int_0^y D[c_2 + (\overline{p} - c_2)\, e^{r(\tau - y)}]d\tau - yD(\overline{p}) - S_{20} \tag{15}$$

where $S_{20} < S_{20}^{\max}$ as stated in Condition 2.

**Effect of the size of $S_2$ on the length of Phase 3:** Keeping $\overline{p}$ and $c_2$ constant, and differentiating the previous equation totally, we obtain

$$\left\{[D(c_2) - D(\overline{p})] - r\,(\overline{p} - c_2)\int_0^y (e^{r(\tau-y)})D'[c_2 + (\overline{p} - c_2)\, e^{r(\tau-y)}]d\tau\right\} dy = dS_{20}.$$

Thus

$$\frac{\partial y}{\partial S_{20}} > 0. \tag{16}$$

Having solved for $y$, we can determine the price at time $T$, when the high-cost deposit begins being extracted, as

$$p(T) = c_2 + (\overline{p} - c_2)\, e^{-ry} \tag{17}$$

**Determination of the length of Phase 2**

Next, we can determine $z$, i.e., the length of the time interval $[t_3, T]$ over which energy demand is met by both extraction from the lowest cost deposit and via production of renewable energy at capacity level. Then, since $p(t_3) = c_3$ by definition, the Hotelling rule gives

$$z = \frac{1}{r}\ln\left[\frac{p(T) - c_1}{c_3 - c_1}\right].$$

Substituting for $p(T)$, we obtain the following equation, which determines the length $z$ of the time interval $[t_3, T]$

$$0 = G(y, c_1, c_2, c_3, \overline{p}) = (c_3 - c_1)e^{rz} - (c_2 - c_1) - (\overline{p} - c_2)\, e^{-ry}. \tag{18}$$

This equation yields

$$\frac{\partial z}{\partial y} < 0. \tag{19}$$

From Equations (16) and (19), we conclude that an increase in $S_{20}$ will reduce $z$. Specifically, as $S_{20}$ approaches $S_{20}^{\max}$, $z$ approaches zero.

**Determination of the exhaustion time of the low cost stock**

Over the period $[0, T]$ the total demand for energy must equal total supply that comes from deposit 1 and from renewable energy produced at capacity after time $t_3$. Thus $T$ must satisfy the equation

$$\int_0^T D[p(t)]dt = S_1 + [T - t_3]\,\bar{q}_3, \tag{20}$$

where, since deposit 1 is extracted over the interval $[0, T)$, the Hotelling rule applies to this deposit over that period such that

$$p(t) = c_1 + (c_2 - c_1)e^{r(t-T)} + (\bar{p} - c_2)\,e^{r(t-T-y)}. \tag{21}$$

Finally, from inserting Equation (21) into (20), the following equation determines $T$ as

$$0 = H(y, z, T, c_1, c_2, \bar{p}) = \int_0^T D\left[c_1 + \left(c_2 + (\bar{p} - c_2)\,e^{-ry} - c_1\right) e^{r(t-T)}\right] dt - S_{10} - zD(\bar{p}). \tag{22}$$

**Determination of the time at which clean energy production become profitable**

Having determined $T$ from equation (22) and $z$ from equation (18), we can compute the time at which clean energy production becomes profitable, $t_3 = T - z$.

## 4.2 Comparative statics

This section analyses policy scenarios in which policies aimed at reducing anthropogenic carbon emissions may lead to a Green Paradox.

Following Gerlagh (2011), we say a "weak Green Paradox" is found if there is an increase in current emissions in response to a policy measure whereas a "strong Green Paradox" is associated with an increase in cumulative damages. In our analysis, a weak Green Paradox can be identified as a decrease of $p(0)$, which indicates higher initial resource extraction and/or a decrease in $T$.

To assess the possibility of a Green Paradox, we apply the implicit function theorem to the system of Equations (15), (18), and (22) to determine the response of the endogenous variables $(y, z, T)$ as well as of price behavior, to changes in the exogenous parameters reflecting the two policy scenarios. Subsidizing the backstop technology is captured by a decrease in $c_3$ whereas an increase in $\bar{q}$ reflects the exogenous increase in capacity.

### 4.2.1 Effect of a subsidy for renewable energy

In the first part of our comparative static analysis, we investigate how subsidizing clean energy affects the extraction speed of the exhaustible resources. It is well known that a subsidy can have detrimental effects on the environment if the clean energy is available at a constant cost without capacity constraint (Strand, 2007, Hoel, 2011). Our paper, however, assumes that the backstop technology is capacity constrained. Various examples for such subsidy systems exist: the renewable energy feed-in tariffs in Germany and Sweden or the exemption of biofuels from taxation, to name just two. Subsidizing the clean energy is captured in this paper in form of a decrease of the constant marginal production cost, $c_3$. The following proposition summarises the effect of a change in $c_3$ on the endogenous variables $(y, z, T)$, see Appendix C for details.

**Proposition 1:** *Subsidizing the clean energy product results in a lower initial price of energy. This leads to a faster extraction of the lowest-cost exhaustible resource during the initial phase*$[0, t_3)$: *there is a weak Green Paradox effect. However, this phase itself is shortened (*$t_3$ *is brought closer to time* $0$*), and thus clean energy production will begin earlier. This effect allows deposit 1 to be extracted over a longer period..*

This first result can be explained as follows: Subsidizing the renewable energy is equivalent to a decrease in $c_3$. From $dy/dc_3 = 0$ (Equation (C.11) in the Appendix), we know that subsidizing the renewable backstop has no effect on how long it will take to exhaust $S_2$. For illustration purposes, let $T^*$ denote the time of exhaustion of $S_1$ when the renewable technology is subsidized. Let the equilibrium price path that results from the subsidy be denoted by $\widetilde{p}(t)$. From the invariance of $y$, it follows that $\widetilde{p}(T^*) = p(T)$. This in turn ensures that the aggregated supply of energy over the length of time $y$ equals the demand. Moreover, from Equation (C.13) follows that subsidising the renewable resource increases the time span of extraction of $S_1$ by $(T^* - T)$. This means that resource stock $S_1$ is available for longer and the price level $p(T) = \widetilde{p}(T^*)$ is reached later.

The following serves as an additional intuitive explanation of the effect of a clean-energy subsidy on the extraction $q_1$ at the production start date of the renewable energy and, therefore, on $z$. If the price path were not affected, subsidising the backstop would lead to earlier production of the renewable energy, implying that, given the unchanged time path of price, the supply of energy is greater than demand. Since this situation would be a disequilibrium, the price path must change. In consequence, $p(0)$ declines, as seen in Equation (C.14). This decrease moderates the decline in $t_3$, restoring the balance between supply and demand; still, the analytical results show that $t_3^* < t_3$ (Equation (C.15)).

These considerations show that two opposed effects work on $T^*$ and $z$. (1) Due to the decrease of $c_3$, $t_3$ decreases (Equation (C.15)), which increases $T$ since, as $q_3$ is available earlier, it can alleviate the demand for $q_1$ sooner. This effect tends to increase $z$. (2) To equalize demand and supply at $t_3$, $p(0)$ decreases, as explained previously (see Equation (C.14)). This second effect works in a direction opposite to the first effect and tends to postpone $t_3$ and also to shorten $z$. Moreover, due to a lower initial price level, the demand for energy increases and is satisfied by an increase in $q_1$ in period $[0, t_3)$. Which of the two effects dominates depends on their relative strength, which has been analyzed analytically.

From $dT/dc_3 < 0$ and $dz/dc_3 < 0$ (Equations (C.13) and (C.12)), we find that the first effect is stronger than the second. This means that the exhaustible-resource-saving effect (of the subsidy on renewable energy) on $S_1$ dominates the demand-increasing effect of the price decrease (the effect of $dT/dc_3 + dy/dc_3$ is unambiguous).

### 4.2.2 Effect of an increase in capacity

We now investigate the effect of an increase in capacity $\overline{q}_3$. This can occur as a result of a technological innovation such as the introduction of electricity storage which allows massive expansions of wind generating capacities or a move from first-generation to second-generation biofuels. An increase in capacity is equivalent to a decrease in the capacity-induced choke price ($\overline{p}$). Both the general case and the special case of a linear demand function, $D(p) = A - p$, are considered.[9] The results for these two cases are summarised in the following two propositions:

**Proposition 2 (general case):** *An increase in the capacity of the clean energy sector has an ambiguous effect on the life of the aggregate resource stock, and it lowers the scarcity rent of both exhaustible resource stocks.*

**Proposition 3 (linear demand):** *Under linear demand, an increase in the capacity of the clean energy sector (i.e., a decrease in $\overline{p}$) will lengthen the life of deposit 2, shorten the interval of simultaneous supply of $q_1$ and $q_3$, and has an ambiguous effect on the life of deposit 1 and of the aggregate resource stock. In the special case where the linear demand function parameterA is large and $z$ is very small (i.e., $S_{20}$ approaches $S_{20}^{\max}$ from below), an increase in capacity will shorten the life of deposit 1:*

$$\frac{dT}{d\overline{p}} > 0. \tag{23}$$

An increase in the capacity of the renewable resource increases the extraction duration of the second exhaustible resource: $dy/d\overline{p} < 0$ (Equation (C.20)). This indicates that a capacity expansion of the renewable resource sector allows the stock of higher-cost resource $S_2$ to be spread over a longer period. In contrast, if $z$ is small and $A$ is large, we can state that $dT/d\overline{p} > 0$ (Equation (23)), and the effect of a capacity increase on the extraction duration of the low-cost stock $S_1$ is negative. This case is especially plausible since we know that a capacity expansion reduces the energy price at the exhaustion point of $S_1$ (Equation (C.18)), which indicates a faster extraction of $q_1$. Additionally, as with the subsidy, the capacity

[9] Modeling a dynamic capacity constraint would complicate the analysis and potentially induces additional extraction and production phases. For the sake of simplicity, the present paper abstracts from any kind of dynamic transition process in the supply of renewable energy.

increase induces a reduction in the initial energy price, which also accelerates exhaustion (Equation (C.17)). Moreover, increased capacity shortens the period of simultaneous supply of $q_1$ and $q_3$: $dz/d\overline{p} > 0$ (Equation (C.21)). Therefore, the capacity increase cannot alleviate the demand for $S_1$ and, consequently, weakening the capacity constraints leads to at least a weak Green Paradox with regard to the cheaper exhaustible resource.

Having completed the analytical exercise, the following section now presents a numerical analysis. This will further illustrate the usefulness of our model and some more refined results will be derived. Furthermore, we conduct a welfare analysis of the second-best policies.

## 4.3 Numerical Calculation for the Business-As-Usual Scenario

The analysis is the same as in the first-best case, except the carbon tax is not equal to $\overline{\mu}$.

### 4.3.1 Lengths of phases 1,2 and 3 under the BAU scenario and the initial prices for phases 1 and 3

The parameter values are $r = 0.01$, $c_1 = 0.75$, $c_2 = 1.75$, $c_3 = 4$, $A = 20$ , $\overline{q}_3 = 5$. We find that

$$y = 144.30,\ z = 23.96,\ T = 51.18,\ t_3 = 27.22,\ \overline{T} = 195.48$$

Then it follows that

$$p(0) = 3.23 \text{ and } p(T) = 4.88$$

### 4.3.2 Welfare in the BAU scenario

To calculate welfare under the BAU scenario, we calculate welfare in each of the 4 phases, and add them up.

**Welfare in Phase 1:** (The length of this phase is $t_3 = 27.22$.)

We find that $W_1 = 4244$ and the present value of damages in this phase is $\Omega_1 = 75$. The ratio of damage costs to welfare is

$$\frac{\Omega_1}{W_1} = 1.8\%$$

**Welfare in Phase 2:** (From time $t_3$ to time $T$, where $t_3 = 27.22$ and $T = 51.18$)

For this phase, we obtain $W_2 = 2525$ and $\Omega_2 = 108.67$. Thus

$$\frac{\Omega_2}{W_2} = 4.3\%$$

**Welfare in Phase 3:** (From time$T = 51.18$ to time $\overline{T} = 195.48$)

We obtain $W_3 = 5373.2$ and $\Omega_3 = 782.66$. The ratio of damage costs to welfare is

$$\frac{\Omega_3}{W_3} = 14.5\%$$

**Welfare in Phase 4:** The present value of welfare for this terminal phase is $W_4 = 587.61$, and the present value of damages is $\Omega_4 = 368.14$. The ratio of damage costs to welfare is

$$\frac{\Omega_4}{W_4} = 62\%$$

Notice that the total damage cost in the BAU scenario is $1,334.5$ while under the first-best policy scenario the figure is $1,163.7$; Table 1 summarizes these results.

Thus the first-best climate policy reduces total damage cost by about 13%.

Table 1: Summary of numerical results

| | first-best | business-as-usual | subsidy scenario | capacity increase |
|---|---|---|---|---|
| y | 160.00 | 144.30 | 144.3 | 151.49 |
| z | 64.07 | 23.96 | 60.73 | 12.70 |
| T | 67.38 | 51.18 | 62.06 | 47.58 |
| $t_3$ | 3.31 | 27.22 | 1.33 | 34.88 |
| $\overline{T}$ | 227.38 | 195.48 | 206.36 | 199.07 |
| p(0) | 3.92 | 3.23 | 2.97 | 3.04 |
| p(T) | 6.02 | 4.88 | 4.88 | 4.44 |
| $\sum_i \Omega_i$ | 1,163 | 1,334.5 | 1,203.4 | 1,371.5 |
| $\sum_i W_i$ | 12,816.22 | 12,729.81 | 12.807 | 12,923 |
| $\frac{\sum_i \Omega_i}{\sum_i W_i}$ | 9% | 10% | 9% | 11% |

## 4.4 A Politically Feasible Scenario: Subsidy on Clean Energy

Now suppose that the government cannot introduce the carbon tax. Instead, suppose the government introduces a subsidy $s = 1$ per unit of clean energy, without affecting the capacity. (We assume that this subsidy has no effect on the capacity.) Then clean energy will be produced as soon as the price of oil reaches $p = 3$ (instead of 4 as under the BAU scenario).This represents a subsidy rate of 25%.

### 4.4.1 Computing the length of the three phases and the initial prices for phases 1 and 3

We must redo the calculation with $r = 0.01$, $c_1 = 0.75$, $c_2 = 1.75$, $c_3^s = 4 - 1 = 3$, $A = 20$, $\overline{q}_3 = 5$. We numerically compute the length of each of the four phases. We obtain the following: The length of phase 3 is $y = 144.3$, the length of phase 2 is $z = 60.73$, $T = 62.06$. Phase 1 becomes much shorter, $t_3 = 1.33$, because the clean energy is made available as soon as the price reaches 3. Finally, $\overline{T} = 206.36$. The key prices are $p(0) = 2.97$ and $p(T) = 4.88$. Since $p(0)$ under the subsidy scenario is smaller than $p(0)$ under the BAU scenario (see Table 1), there is a short-term increase in extraction and hence short-term increase in damage costs. This is a weak Green Paradox result.

### 4.4.2 Welfare in the subsidy scenario

To calculate welfare under the subsidy scenario, we calculate welfare in each of the 4 phases, and add them up.

**Welfare in Phase 1:** Since this phase is now very short due to the subsidy on the clean energy, we find that the welfare for this phase is $W_1 = 240.02$. The present value of damages in this phase is $\Omega_1 = 1.47$. The ratio of damage costs to welfare is

$$\frac{\Omega_1}{W_1} = 0.6\%$$

**Welfare in Phase 2:** We find that $W_2 = 7187.6$ and the present value of damages in this phase is $\Omega_2 = 196.11$. The ratio of damage costs to welfare is

$$\frac{\Omega_2}{W_2} = 2.72\%$$

**Welfare in Phase 3:** We obtain $W_3 = 4851.9$ and $\Omega_3 = 675.65$. The ratio of damage costs to welfare is

$$\frac{\Omega_3}{W_3} = 14\%$$

**Welfare in Phase 4:** For this phase, $W_4 = 527.03$, $\Omega_4 = 330.19$, and the ratio of damage costs to welfare is

$$\frac{\Omega_4}{W_4} = 62\%$$

Total welfare under the subsidy is

$$\sum_i W_i = 12,807$$

This can be compared to the first best welfare of $12,816$ and the BAU welfare of $12,730$. Thus the subsidy on clean energy raises welfare above the BAU welfare by about 0.6%; there is no strong Green Paradox. The welfare gains relative to the BAU scenario is largely driven by delaying the extraction of the more dirty deposit, $S_2$. In fact, the total damage cost under the subsidy scenario is

$$\sum_i \Omega_i = 1203.4$$

Thus the subsidy policy reduces total damage cost by about 9.8%.

## 4.5 Politically feasible scenario 2: capacity increase

We now consider an increase in capacity from $\overline{q}_3 = 5$ to $\overline{q}_3' = 6$. This represents a capacity expansion of 20%. Assume that this involves an investment cost, denoted by $K$.

### 4.5.1 Computing the length of the three phases and the initial prices for phases 1 and 3

The length of phase 3 is $y = 151.49$ and the length of phase 2 is $z = 12.70$.And $T = 47.58$, $t_3 = 34.88$ and $\overline{T} = 199.07$. The key prices are $p(0) = 3.04$ and $p(T) = 4.44$. Since $p(0)$ under the capacity expansion scenario is smaller than $p(0)$ under the BAU scenario (see Table 1), there is a short-term increase in extraction and hence short-term increase in damage costs. This is a weak Green Paradox result. We will see below that there is also a strong Green Paradox in this case.

### 4.5.2 Welfare in the capacity expansion scenario

Under the capacity expansion scenario, we obtain the following numerical results for damage costs and welfare.

**Welfare in Phase 1:** We find that $W_1 = 5236.3$, and $\Omega_1 = 110.13$. The ratio of damage costs to welfare is

$$\frac{\Omega_1}{W_1} = 2\%$$

**Welfare in Phase 2:** We obtain $W_2 = 1317.3$ and $\Omega_2 = 27.45$. The ratio of damage costs to welfare is

$$\frac{\Omega_2}{W_2} = 2.08\%$$

**Welfare in Phase 3:** In this phase, $W_3 = 5658.8$ and $\Omega_3 = 878.74$. Thus

$$\frac{\Omega_3}{W_3} = 12\%$$

**Welfare in Phase 4:** We find that $W_4 = 710.32$ and $\Omega_4 = 355.16$. The ratio of damage costs to welfare is

$$\frac{\Omega_4}{W_4} = 50\%$$

Total welfare (*before subtracting* $K$, *the cost of the capacity expansion*) under the capacity expansion is

$$\sum_i W_i = 12,923$$

Note that the welfare under this scenario is higher than the first-best welfare of $12,816$, the BAU welfare of $12,730$ and the subsidy welfare of $12,807$. Whether or not the expansion of the capacity is welfare enhancing, thus, depends on the cost, $K$, associated with this.

The total damage cost under the capacity expansion scenario is

$$\sum_i \Omega_i = 1371.5$$

The cumulative damages in this scenario are higher than under the subsidy scenario (1203.4) and the BAU scenario (1334.5). Thus, a strong Green Paradox occurs. In terms of the present value of the stream of damage costs, Table 1 shows that a subsidy of 25% on the clean energy will reduce total damage costs by about 10%, while a capacity expansion of 20% will increase total damage costs by about 5%. The reason is that in our model with a binding capacity constraint, a subsidy makes clean energy production profitable at an earlier date, without changing the peak price of oil. These effects result in pushing the fossil resource exhaustion date further into the future, so that the pollution stock peaks at a later date. In contrast, a capacity expansion reduces the maximum price that the last drop of oil would earn ($\overline{p}$ falls from 5 to 4).This results in a strong incentive for fossil resource owners to start their extraction earlier: the dirty deposit 2 begins to be extracted at $T = 47.58$ instead of $T = 51.8$.

# 5 Illustration of policy relevance

The model presented in this paper exhibits a considerable degree of flexibility and is able to capture various empirical observations as well as challenges policy makers currently face. To illustrate this wide applicability, this section provides (stylized) evidence that supports this paper's approach, showing that it is highly relevant. In addition to the crude oil market application introduced in Section 2 and analysed in detail in Section 4, this section illustrates additional applications for this paper's model for the analysis of the transformation of the electricity sector.

As already explained above, the natural application of our model is an oil market with conventional and unconventional oil as well as biofuels as a clean substitute. The parametrisation of the model generally reflects the cost structure and environmental impacts in this sector. The consideration of two rather than one "dirty" resource allows us to capture unconventional carbon resources such as extra heavy oil, oil sands, and oil shale; see Gordon (2012).[10] Extracting oil from unconventional sites is more costly as well as more energy intensive and, thus, unconventional oil has a higher $CO_2$ emission intensity and extraction cost than conventional oil. Specifically, in addition to various technological problems, biofuel production raises land use concerns as it cannot be ruled out that there is not be enough (suitable) land available for biofuel production and, even if there were, using it for that purpose might seriously compromise food production and raise sustainability concerns; see, e.g., Sinn (2012). Thus, it seems plausible to assume that there is a constraint imposed on the share of biofuels production. The share of biomass from global primary energy supply is currently about 15%. This, however, is to a very large extent attributable to so-called "traditional biomass" - the use of firewood, charcoal as well as agricultural residues; see International Energy Agency (2012). The share of biofuels in global road transport, however, is merely 3% and several problems indicate that it is more than reasonable to assume that biomass is not a backstop technology that can be used without constraints; see nternational Energy Agency (2011).[11] Our model not just allows us to capture this issue, it is furthermore possible to analyse the effect of changes in this capacity. In light of the finding of negative welfare effects under the capacity expansion scenario, the global biomass potential

[10] This might also be seen as an approximation of an increasing (instead of flat) marginal cost curve.

[11] Even though projections certainly indicate that there is a vast potential for biomass (for example, according to International Energy Agency (2011), unused and surplus land, has the potential of about 550-1,500 EJ biomass production in 2050, the way to exploit this potential is nevertheless long and stony. To mention just a few of the challenges, crop yields need to increase considerably, and substantial parts of land needs to be converted. In addition to that, International Energy Agency (2011) points to regulatory requirements and stresses the importance of ensuring that food security is not compromised; see also Sinn (2012).

that actually exists would have to be seen as a considerable problem.

Applications of our model are not restricted to the crude oil market: the electricity market is another possible application for our model. The overall situation there is a similar to the oil market example: Electricity is generated from both different "dirty" and exhaustible conventional resources as well as clean ones simultaneously - despite the fact that renewable energy is considerably more expensive than conventionally produced electricity. In order to fight climate change, decrease the dependency on imports of energy resources as well as the issue of resource scarcity contribute to the attractiveness of renewable energies. In consequence, wind or solar power is used instead of (or at least in addition to) coal or gas. As a result, policy instruments such as feed-in-tariffs or clean energy quotas are in place in many countries. For example, Germany today generates approximately 20% of total electricity from renewable sources such as wind and solar and the European Union aims at reaching this share at the European level until 2020. However, further increasing this share seems to be more challenging than originally expected. For example, substantial investments into the electricity transmission and distribution network are required. What is more, the problems of intermittent renewable energies and the considerable lack of storage facilities are still unresolved. In addition to these technological challenges, there are also important regulatory ones. The requirement of backup power plants to guarantee network stability sparked the debate on an entire redesign of electricity market - the introduction of so-called capacity markets is among the options. Finally, the requirements of the politically important so-called triangle of energy supply - energy is supposed to be sustainable, affordable, and reliable - effectively constrain the further development of renewable energies in electricity production. In short, assuming that a backstop resource for electricity generation is unconstrained unrestrictedly is highly unrealistic. In light of our findings policy instruments aiming at an increase of the capacity constraint of a renewable substitute are problematic. However, subsidizing this technology and, thus, develop it to market maturity earlier may have positive long-term effects. However, a detailed analysis of possible Green Paradox effects in the electricity market requires a corresponding calibration of the numerical model.

Finally, our model is also be able to capture the issue of nuclear energy. This "conventional," but carbon-free form of energy is constrained by regulatory, political, and maybe even (safety-related) technological restrictions.[12]

These reflections vividly illustrate the wide applicability of this paper's model. It is fairly obvious that applications of this model make an important contribution to current energy

[12] Finally even the assumption that the constrained backstop technology is clean could be relaxed. The case of a dirty backstop technology is studied in Ploeg and Withagen (2012a). Liquid fuels produced with coal-to-liquids technologies serve as one example for a dirty but certainly also constrained backstop technology.

policy debates. In a nutshell, the model applied in this paper is able to capture various empirical energy market observations and the results obtained in this paper clearly indicate that ignoring the important feature of capacity-constrained backstop technologies can lead to inappropriate policy recommendations.

# 6 Conclusions

It is no exaggeration to state that climate change is among the biggest challenges mankind has ever been faced. Thus, it is very important that we respond appropriately to this challenge. Perhaps for this very reason and perhaps just because we need to respond soon, this challenge is particularly difficult. Economic analyses identified various well-intentioned climate policy measures in the past which, at first glance, appeared useful, but after taking a closer look, turned out to be counterproductive. While this literature has a long history - older contributions date back to the 1980s and 1990s - there is a recent stream of literature sparked by Sinn's (2008) discovery of the so-called Green Paradox. The basic finding of that paper is that the owners of exhaustible fossil resources possibly bring forward extraction of their resources as a response to intensifying climate policies. Because of the importance of finding an appropriate answer, however, it is also necessary to use appropriate economic modelling frameworks. If these frameworks are not designed carefully enough there is a risk that inappropriate policy recommendations emerge from these research efforts.

Sinn's (2008) original findings are very elegantly derived and are also intuitively very convincing. However, in response to Sinn's paper, a large number of papers emerged which can be summarized as follows: the more realistic the modelling approach is the more detailed the results become. This paper's findings fit very well into this overall landscape. The model used here allows to capture two important empirical observations. First, even though clean technologies are generally available - wind and solar energy seem to be suitable for replacing coal and gas power plants, cars could very well be run on biofuels rather than conventional fuels, clean and dirty technologies are used simultaneously. Second, further expanding and implementing clean technologies increasingly meets resistance. In more and more countries, in particular Germany and the United Kingdom, there are significant local initiatives to oppose the installation of additional wind parks. Extending the use of biofuels is a major concern for organisations which care about food security and food prices. Thus, there is sufficient evidence to assume that the use of clean technologies is constrained. These very constraints are most likely the reasons why different types of energy sources are used simultaneously even though costs associated with their use differ dramatically: solar energy is still much more expensive than conventional electricity, to name just one example.

As currently existing models are not able to capture these two observations, this paper makes an important contribution to this literature. As stated above, it is important to use appropriate economic models in order to rule out the possibility of deriving inappropriate policy recommendations. In addition, our model allows for the analysis of an entirely new scenario: the expansion of the capacity constrained clean energy. The results indicate that this scenario is considerable more harmful than a subsidization of the clean energy.

Various channels through which a Green Paradox can occur have been discussed in the literature: intertemporal arbitrage, spatial, technological, or extraction order effects; see Ploeg and Withagen (2015) and Jensen et al. (2015). Intertemporal effects play a large role in Sinn's (2008) paper as well as in the earlier contribution by Long and Sinn (1985). A technology-induced Green Paradox has been identified by Strand (2007). In this paper the intertemporal channel is important but also the extraction order effect plays an important role. The theoretical framework used in this paper is based on Holland's (2003) analysis of extraction capacities and the optimal order of extraction of exhaustible resources. This model is re-interpreted and considerably extended. In order to operationalize the concept of the Green Paradox in greater detail, this paper, in addition, borrows from Gerlagh (2011) and considers different degrees of the Green Paradox.

What is fascinating to observe is that a large number of papers emerged only in response to Sinn's (2008) discovery of the Green Paradox. The theoretical framework used in many if these papers is almost 100 years old and goes back to Hotelling (1930). For a surprisingly long period this literature remained is a hibernation-type state. The oil price hike in the 1970s provided impetus for deepening the Hotelling framework. In the 1990s, a number of papers addressed the issue of global warming. In terms of climate implications of feasible second-best policies, the big awakening came with Sinn (2008). What we can now hope is that the concerted research efforts that paper sparked helps identifying the responses we need to apply if we want to keep climate change under control.

# A Appendix: Four phases of the first-best scenario

Let $\overline{T}$ be the time at which the high-cost deposit is exhausted. From time $\overline{T}$ onwards, energy consumption is constant, at $\overline{q}_3$, and the price of energy is $\overline{p} = U'(\overline{q}_3) > c_3$. At time $\overline{T}$, the last drop of oil from deposit 2 earns the rent $\psi_2(\overline{T}) = \overline{p} - (c_2 + \overline{\mu}\eta_2) > \overline{p} - c_3 > 0$.

Let $T < \overline{T}$ be the time at which the extraction from deposit 2 begins. Then, from (12), for all $t \in [T, \overline{T}]$, we must have

$$[p(t) - (c_2 + \overline{\mu}\eta_2)] e^{-rt} = [p(T) - (c_2 + \overline{\mu}\eta_2)] e^{-rT} = \left[p(\overline{T}) - (c_2 + \overline{\mu}\eta_2)\right] e^{-r\overline{T}}$$

Hence,

$$p(t) = (c_2 + \overline{\mu}\eta_2) + [p(T) - (c_2 + \overline{\mu}\eta_2)] e^{-r(T-t)} \text{ for all } t \in \left[T, \overline{T}\right] \tag{A.1}$$

or, equivalently,

$$p(t) = (c_2 + \overline{\mu}\eta_2) + [\overline{p} - (c_2 + \overline{\mu}\eta_2)] e^{-r(\overline{T}-t)} \text{for all } t \in \left[T, \overline{T}\right] \tag{A.2}$$

It follows that

$$\overline{T} - T = \frac{1}{r} \ln \left[ \frac{p(\overline{T}) - (c_2 + \overline{\mu}\eta_2)}{p(T) - (c_2 + \overline{\mu}\eta_2)} \right] \tag{A.3}$$

If $p(T) \geq c_3$, then it must hold that the clean energy is produced at the capacity level $\overline{q}_3$ throughout the time interval $\left[T, \overline{T}\right]$, and consequently the market clearing condition over this time interval is

$$\int_T^{\overline{T}} D\left[c_2 + \overline{\mu}\eta_2 + (p(T) - (c_2 + \overline{\mu}\eta_2))e^{-r(T-t)}\right] dt = S_{20} + (\overline{T} - T)\overline{q}_3 \tag{A.4}$$

Let $S_2^*$ denote the critical value of the stock size $S_{20}$ such that $p(T)$ is exactly equal to $c_3$. Let $\tau = t - T$. From equations (A.3) and (A.4), we find that $S_2^*$ is given by

$$S_2^* = \int_0^x D\left[c_2 + \overline{\mu}\eta_2 + (c_3 - (c_2 + \overline{\mu}\eta_2))e^{r\tau}\right] d\tau - x\overline{q}_3 \tag{A.5}$$

where

$$x \equiv \frac{1}{r} \ln \left[ \frac{\overline{p} - (c_2 + \overline{\mu}\eta_2)}{c_3 - (c_2 + \overline{\mu}\eta_2)} \right] \tag{A.6}$$

To focus on the interesting and relevant case (where the clean energy is supplied well before the deposit $S_2$ is exploited), we make the following Assumption:

**Assumption A2:** The initial stock size $S_{20}$ is strictly smaller than the critical size $S_2^*$, defined by (A.5).

Under Assumptions A1 and A2, the first-best solution consists of four phases, just as under the BAU scenario. In what follows, we refer to $\overline{\mu}$ as the optimal carbon tax, and interpret $p(t) - (c_i + \overline{\mu}\eta_i)$ as the "net price" (net of extraction cost and carbon tax) of the resource extarcted from deposit $i$.

**Phase 1:** Energy is supplied only by extraction from the low-cost (and less-polluting) deposit, $S_1$. This phase begins at time 0 and ends at some time $t_3$ such that the equilibrium price at $t_3$ is equal to $c_3$. During this phase, the net price of the low-cost (and less-polluting) resource, $p(t) - (c_1 + \overline{\mu}\eta_1)$, rises at a rate equal to the rate of interest.

**Phase 2:** Energy is simultaneously supplied by both extraction from the low-cost (and less-polluting) resource stock $S_1$ and the more costly renewable energy running at its capacity level $\overline{q}_3$. The stock $S_1$ will be exhausted at some time $T$. During this phase, the net price $p(t) - (c_1 + \overline{\mu}\eta_1)$ rises at a rate equal to the rate of interest. Thus

$$[p(t) - (c_1 + \overline{\mu}\eta_1)]^{-rt} = [p(T) - (c_1 + \overline{\mu}\eta_1)]\, e^{-rT} \text{ for } 0 \leq t \leq T \tag{A.7}$$

The *length* of this phase is denoted by $z \equiv T - t_3$.

**Phase 3:** This phase begins at $T$ and ends at some time $\overline{T}$. The deposit $S_2$ is exhausted at time $\overline{T}$. During this phase, energy comes from two sources: (i) the high-cost (and more polluting) deposit $S_2$, and (ii) the renewable energy, operating at its capacity level $\overline{q}_3$. The net price $p(t) - (c_2 + \overline{\mu}\eta_2)$ rises at a rate equal to the rate of interest. Thus

$$[p(T) - (c_2 + \overline{\mu}\eta_2)]\, e^{-rT} = [p(t) - (c_2 + \overline{\mu}\eta_2)]\, e^{-rt} = [\overline{p} - (c_2 + \overline{\mu}\eta_2)]\, e^{-r\overline{T}} \text{ for } T \leq t \leq \overline{T} \tag{A.8}$$

The length of this phase is denoted by $y \equiv \overline{T} - T$.

**Phase 4:** This is the final phase. The only source of energy in this phase is the clean energy available at the capacity level $\overline{q}_3$. The pollution stock in this phase is a constant, equal to $\overline{X} \equiv X_0 + \eta_1 S_{10} + \eta_2 S_{20}$.

# B Appendix: Special cases of the BAU scenario

In this Appendix, we identify conditions for the parameter values such that $T$ is exactly equal to $t_3$, such that Phase 2 collapses to a single point. If $T = t_3$, then from time $t_3$, energy supply comes both from deposit 2 and from the clean energy sector (deposit 1 having been exhausted, we have identical starting-times of clean energy production and extraction from the high-cost deposit with $T = t_3$). As defined before, the time at which deposit 2 is exhausted is called $\overline{T}$. At $\overline{T}$ and from then on, the price of energy must equal $\overline{p} \equiv U'(\overline{q}_3) \equiv \phi(\overline{q}_3)$. During the time interval $t \in [t_3, \overline{T})$, the Hotelling rule must hold for deposit 2:

$$(p(t) - c_2)\, e^{-rt} = (p(t_3) - c_2)\, e^{-rt_3} = \left(p\left(\overline{T}\right) - c_2\right) e^{-r\overline{T}} \equiv (\overline{p} - c_2)\, e^{-r\overline{T}}.$$

From this equation, the explicit price path between $t_3$ and $\overline{T}$ as well as the extraction duration can be determined. With $p(t_3) = c_3$, it follows that the length of time it takes for the price to rise from $c_3$ to $\overline{p}$ is

$$x = \frac{1}{r} \ln \left[ \frac{\overline{p} - c_2}{c_3 - c_2} \right]$$

where $x$ is defined as

$$x \equiv \overline{T} - t_3.$$

Moreover, for all $t \in [t_3, \overline{T})$, the price path is

$$p(t) = c_2 + \frac{(p(t_3) - c_2)\, e^{-rt_3}}{e^{-rt}} = c_2 + (c_3 - c_2)\, e^{r(t - t_3)}.$$

From this, total demand for energy over the time interval $[t_3, \overline{T})$ can be determined as

$$\int_{t_3}^{\overline{T}} D\left[p(t)\right] dt = \int_{t_3}^{\overline{T}} D\left[c_2 + (c_3 - c_2)\, e^{r(t - t_3)}\right] dt.$$

Then, we use $x \equiv \overline{T} - t_3$ and the substitution $\tau = t - t_3$ to obtain

$$\int_0^{\overline{T} - t_3} D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau \equiv \int_0^{x} D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau.$$

Total demand must be met by total supply, which is the output of the clean energy sector and extractions from deposit 2:

$$\int_0^{x} D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau = x\overline{q}_3 + \int_0^{x} q_2(\tau) d\tau \text{ (recall } \tau = t - t_3).$$

It follows that if $S_{20}$ is just equal to a threshold value $S_{20}^{\max}(\infty)$ defined by

$$S_{20}^{\max}(\infty) \equiv \int_0^{x} D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau - \frac{\overline{q}_3}{r} \ln \left[ \frac{\overline{p} - c_2}{c_3 - c_2} \right],$$

then $t_3$ is indeed the time at which deposit 2 begins to be extracted (and sold at price $p(t_3) = c_3$ at that moment), and the time at which deposit 1 has just been exhausted.

Can we determine time $t_3$ in this case? Analogous to the above, since over the time interval $[0, t_3)$ deposit 1 is being exploited, the Hotelling rule applied to deposit 1 must hold with equality for all $t \leq t_3$ :

$$(p(t) - c_1)e^{-rt} = p(0) - c_1 = (c_3 - c_1)e^{-rt_3}.$$

Rearranging gives us the price path between $t = 0$ and $t = t_3$ and, under the consideration that total demand must be met by total supply, we obtain

$$\int_0^{t_3} D\left[c_1 + (c_3 - c_1)\, e^{-r(t_3 - t)}\right] dt = S_{10}.$$

This equation determines $t_3$ and hence $p(0)$ as functions of $S_{10}$ (given the assumption that $S_{20} = S_{20}^{\max}(\infty)$). We summarize the results for this razor's edge case in the following proposition.

**Proposition: (Razor's edge case)** *If the size of deposit 2 is equal to the threshold value* $S_{20}^{\max}$ *defined by*

$$S_{20}^{\max} \equiv \int_0^{x} D\left[c_2 + (c_3 - c_2)\, e^{r\tau}\right] d\tau - \frac{\overline{q}_3}{r} \ln\left[\frac{\overline{p} - c_2}{c_3 - c_2}\right],$$

*with*

$$x \equiv \frac{1}{r} \ln\left[\frac{\overline{p} - c_2}{c_3 - c_2}\right],$$

*then the equilibrium time path of extraction is continuous and consists of* ***three*** *phases:*

*Phase 1 (the time interval* $[0, t_3)$*): The whole market is supplied from deposit 1 only:* $Q = q_1$*. This deposit will be exhausted at time* $t_3$*, where* $t_3$ *is the solution of*

$$\int_0^{t_3} D\left[c_1 + (c_3 - c_1)\, e^{-r(t_3 - t)}\right] dt = S_{10}.$$

*At time* $t_3$*, the price of energy is* $p(t_3) = c_3$*.*

*Phase 2 (the time interval* $\left[t_3, \overline{T}\right)$*): The whole market is supplied from both the high cost deposit (deposit 2) and the clean energy sector:* $Q = q_2 + \overline{q}_3$ *where* $q_2(t) > 0$ *for all* $t$ *in* $\left[t_3, \overline{T}\right)$*. The length of this phase is equal to* $x$*. At time* $\overline{T}$*, the price of energy is* $\overline{p}$*, and deposit 2 is exhausted.*

*Phase 3: After time* $\overline{T}$*, the whole energy market is satisfied by the clean energy sector:* $Q = \overline{q}_3$*.*

# C Appendix: Derivations of comparative results

## C.1 Effect of a subsidy for renewable energy

The effect of a change in $c_3$ on the endogenous variables $(y, z, T)$ can be computed from the following matrix equation

$$\begin{bmatrix} F_y & F_z & F_T \\ G_y & G_z & G_T \\ H_y & H_z & H_T \end{bmatrix} \begin{bmatrix} dy \\ dz \\ dT \end{bmatrix} = \begin{bmatrix} -F_{c3} \\ -G_{c_3} \\ -H_{c_3} \end{bmatrix} dc_3 \tag{C.9}$$

where

$$F_y = -r\left(\overline{p} - c_2\right) \int_0^y D'[p(\tau)] e^{r(\tau - y)} d\tau > 0$$

$$G_y = r\left(\overline{p} - c_2\right) e^{-ry} > 0$$

$$G_z = r(c_3 - c_1) e^{rz} > 0$$

$$G_{c_3} = e^{rz} > 0$$

$$H_y = \int_0^T D'[p(t)] \left[-r\left(\overline{p} - c_2\right) e^{-ry} e^{r(t-T)}\right] dt > 0$$

$$H_z = -D(\overline{p}) < 0$$

$$H_T = D[p(T)] + \int_0^T D'[p(t)] \left[-r e^{r(t-T)}\right] \left(c_2 + \left(\overline{p} - c_2\right) e^{-ry} - c_1\right) dt > 0$$

$$F_z, F_T, F_{c_3}, G_T, H_{c_3} = 0.$$

Let $J$ denote the determinant of the $3 \times 3$ matrix on the left-hand side of Equation (C.9). Calculation shows that

$$J = F_y G_z H_T > 0. \tag{C.10}$$

Then, using Cramer's rule, we obtain the effect of an increase in $c_3$ on the variables $y, z$, and $T$ :

$$\frac{dy}{dc_3} = 0 \tag{C.11}$$

$$\frac{dz}{dc_3} = \frac{-e^{rz}}{J} \left[F_y H_T\right] < 0 \tag{C.12}$$

$$\frac{dT}{dc_3} = \frac{e^{rz}}{J} \left[F_y H_z\right] < 0. \tag{C.13}$$

Thus, we see from Equations (C.11)-(C.13) that an increase in the clean energy producer's unit cost, $c_3$, has no effect on the length of time over which deposit 2 is extracted ($dy/dc_3 = 0$), but will shorten the life of the low-cost deposit 1 ($dT/dc_3 < 0$) and will also shorten the interval of time over which both $q_1$ and $q_3$ are positive ($dz/dc_3 < 0$). The initial price $p(0)$ will be higher, as can be derived from Equation (21):

$$\frac{dp(0)}{dc_3} = -r\left(c_2 + (\overline{p} - c_2)\, e^{-ry} - c_1\right) e^{-rT} \frac{dT}{dc_3} > 0. \tag{C.14}$$

Since $\overline{p}$ and $y$ are not affected by the increase in $c_3$, we can deduce that the price at which the high cost deposits begins to be extracted will be unaffected, see Equation (17):

$$\frac{dp_2}{dc_3} = 0.$$

The effect of an increase in $c_3$ on $t_3$ (i.e., on the time interval over which all energy is supplied from deposit 1 alone) can also be computed. Since $t_3 + z = T$,

$$\frac{dt_3}{dc_3} = \frac{dT}{dc_3} - \frac{dz}{dc_3} = \frac{e^{rz} F_y}{J} \left[H_z + H_T\right] > 0. \tag{C.15}$$

## C.2 Effect of an increase in capacity

The effect of a change in $\overline{q}_3$ on the endogenous variables $(y, z, T)$, which is identical to a change in $\overline{p}$ since $D(\overline{p}) = \overline{q}_3$, can be computed, analogously to the previous section, from the following matrix equation as

$$\begin{bmatrix} F_y & F_z & F_T \\ G_y & G_z & G_T \\ H_y & H_z & H_T \end{bmatrix} \begin{bmatrix} dy \\ dz \\ dT \end{bmatrix} = \begin{bmatrix} -F_{\overline{p}} \\ -G_{\overline{p}} \\ -H_{\overline{p}} \end{bmatrix} d\overline{p}$$

where

$$F_{\overline{p}} = -yD'\left(\overline{p}\right) + \int_0^y D'[p(\tau)]e^{r(\tau - y)} d\tau \gtrless 0$$

$$G_{\overline{p}} = -e^{-ry} < 0$$

$$H_{\overline{p}} = -zD'\left(\overline{p}\right) + \int_0^T D'[p(t)]e^{r(t-T-y)} dt \gtrless 0$$

and the determinant $J$ has been determined in Equation (C.10).

The comparative static results are ambiguous:

$$\frac{dy}{d\overline{p}} = \frac{-F_{\overline{p}}}{J}\left[G_z H_T\right] \text{ has the sign of } -F_{\overline{p}}$$

$$\frac{dz}{d\overline{p}} = \frac{1}{J}\left\{F_{\overline{p}} G_y H_T - G_{\overline{p}} F_y H_T\right\} \gtrless 0$$

$$\frac{dT}{d\overline{p}} = \frac{1}{J}\left\{F_y\left[e^{-ry}D(\overline{p}) - H_{\overline{p}} G_z\right] - F_{\overline{p}}\left[-D(\overline{p})G_y - H_y G_z\right]\right\} \gtrless 0.$$

The effect on the life of the aggregate resource stock is also ambiguous:

$$\frac{d(T+y)}{d\overline{p}} = \frac{1}{J}\left\{F_y\left[e^{-ry}D(\overline{p}) - H_{\overline{p}} G_z\right] - F_{\overline{p}}\left[G_z H_T - D(\overline{p})G_y - H_y G_z\right]\right\} \gtrless 0. \quad \text{(C.16)}$$

However, the effects on the price path are unambiguous (see Equation (21)). First, an increase in capacity (a fall in $\overline{p}$) necessarily leads to a lower initial price:

$$\frac{dp(0)}{d\overline{p}} > 0. \quad \text{(C.17)}$$

Second, a fall in $\overline{p}$ lowers the price at which deposit $S_2$ begins to be exploited:

$$\frac{dp(T)}{d\overline{p}} > 0. \quad \text{(C.18)}$$

For the linear demand function, the calculations are as follows:
Assuming that demand is linear with the functional form

$$D[p(t)] = A - p(t). \quad \text{(C.19)}$$

Using Equation (C.19) in Equation (15) yields

$$\int_0^y \left[A - \left(c_2 + (\overline{p} - c_2)\, e^{r(\tau - y)}\right)\right] d\tau = y\,(A - \overline{p}) + S_{20}.$$

Differentiating totally, we obtain after some rearrangement,

$$\frac{dy}{d\overline{p}} = -\frac{S_2}{\left(1 - e^{-ry}\right)\left(\overline{p} - c_2\right)^2} < 0. \quad \text{(C.20)}$$

Thus, an expansion in capacity $\overline{q}_3$, which leads to a fall in $\overline{p}$, lengthens the life of deposit 2. Moreover, from Equations (18) and (C.20), we can derive the effect of an increase in $\overline{p}$ on

$z$ as

$$\frac{dz}{d\overline{p}} = \frac{1}{r}\left(\frac{1}{c_2 - c_1 + (\overline{p} - c_2)\, e^{-ry}}\right)\left[e^{-ry} - r\,(\overline{p} - c_2)\, e^{-ry}\frac{dy}{d\overline{p}}\right] > 0. \qquad \text{(C.21)}$$

Thus, a fall in $\overline{p}$ shortens the phase during which both $q_1$ and $q_3$ are supplied to the market. To find the effect of an increase in $\overline{p}$ on $T$, insert the linear demand function (C.19) into Equation (22), leading to

$$\int_0^T \left[A - c_1 - \left(c_2 + (\overline{p} - c_2)\, e^{-ry} - c_1\right) e^{r(t-T)}\right] dt = S_{10} + z\,(A - \overline{p}),$$

where $y$ and $z$ are both functions of $\overline{p}$, with derivatives given by Equations (C.20) and (C.21).

Rearranging terms and totally differentiating leads to

$$\left[A - c_1 - \left(c_2 + (\overline{p} - c_2)\, e^{-ry} - c_1\right) e^{-rT}\right] \frac{dT}{d\overline{p}}$$

$$= \left\{-\left(\frac{1 - e^{-rT}}{r}\right) r\,(\overline{p} - c_2)\, e^{-ry}\frac{dy}{d\overline{p}} + (A - \overline{p})\frac{dz}{d\overline{p}} + \left(\frac{1 - e^{-rT}}{r}\right) e^{-ry}\right\} - z. \qquad \text{(C.22)}$$

Consider the right-hand side (RHS) of Equation (C.22). The sum of the terms inside the curly brackets {...} is positive. However, because $z$ is positive, the sign of the RHS seems ambiguous. On the left-hand side, the expression inside the square brackets [...] is ambiguous, though it is positive if $A$ is sufficiently large.

The effect of an increase in $\overline{p}$ on the life of the aggregate resource stock, $y + T$, is also ambiguous. The results shown in Equations (C.20), (C.21), and (C.22) are summarized in Proposition 3.

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