2008s-04

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Série Scientifique Scientific Series

## Montréal Janvier 2008

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ISSN 1198-8177

Partenaire financier Développement économique, Innovation et Exportation Ouébec \* \*

# **Regulating Man-Made Sedimentation** in Riverways<sup>\*</sup>

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## Résumé / Abstract

La sédimentation du lit des voies navigables, dont les conséquences peuvent être coûteuses pour les populations riveraines, est souvent provoquée par l'érosion résultant des pratiques agricoles. Nous proposons un remède basé sur l'imposition d'une taxe « spatiale », variant selon la position de chaque exploitant par rapport au cours d'eau.

Mots clés : Externalités agricoles, érosion des sols, taxes spatiales

Sedimentation in river beds, which results in social disamenities, is caused by soil erosion in farmed ecosystems surrounding those rivers. This paper introduces a "spatial" corrective tax on soil erosion. We find that the optimal tax rule will depart from the classical Pigouvian method according to farmers' location in the ecosystem.

Keywords: Farming externalities, soil erosion, spatial taxes

Codes JEL : H23,Q57

<sup>&</sup>lt;sup>\*</sup> We are grateful to Justin Leroux, and seminar audiences at HEC Montréal, McGill University, and at the CEA 41st Annual Meetings for helpful comments.

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# 1. Introduction

Sedimentation in river beds is caused by soil erosion in the ecosystems surrounding those rivers. It is known that farming activities including deforestation in the vicinity of rivers plays an important role in aggravating this natural phenomenon. For instance, a recently published article<sup>1</sup> in The Economist magazine reported the findings of scientists at the Smithsonian Tropical Research Institute in Panama who studied soil erosion around the Panama riverway.

"Deforestation allows more sediment and nutrients to flow into the canal [river]. Sediment clogs the channel directly. Nutrients do so indirectly, by stimulating the growth of waterweeds. Both phenomena require regular, and expensive, dredging."

Other scientific evidence, on the negative role of farming around rivers, suggests that planting annual crops rather than perennial plants like forests increases the chances of soil erosion. According to Bockstael and Irwin (2000), farming is intimately tied to clearing and excavation, which contribute to soil erosion and sedimentation. This externality is also translated into decreased water stocks in the ecosystem because clearing destroys the forest cover. The ecological services, thus, provided by the natural riverway ecosystem are erosion prevention and water retention because the presence of the natural flora helps smooth water supply over time into the river basin.

<sup>&</sup>lt;sup>1</sup>"Environmental economics, Are you being served?". The Economist print edition, Apr 21st 2005

The objective of this paper is to investigate the mechanisms that govern the actions of farmers operating around a typical riverway that is used for hydroelectric power generation and residential water retention. Also, we seek to adapt corrective regulatory policy instruments to the spatial context of this externality problem, while policy implications will be explored to analyze the properties of the optimal taxation rule that we derive under both competitive and cooperative (monopoly) market structures.

Silt build-up behind river dams is an important environmental problem. Around the world, 261 rivers constitute internationally shared basins<sup>2</sup>. Currently, there are several hundred rivers in the world that are dammed, among which 37 are major rivers. Most of those dammed and farmed river ecosystems are farmed not only downstream, vis-à-vis the dam, but also upstream causing serious soil erosion and damaging the rivers' water sources. In addition, it is well known that silt build-up in dam reservoirs reduces the effectiveness of dams for electricity generation and both the quantity and quality of retained water.

The paper unfolds as follows. Section 2 discusses relevant spatial aspects in the literature. Section 3 presents the model. Section 4 introduces spatial erosion taxes. Section 5 discusses the cooperative case, which is relevant since it is often observed that farmers group into farming cooperatives to coordinate production efforts. Section 6 contains concluding remarks.

 $<sup>^2&</sup>quot;$ International River Basins of the World". Transboundary Freshwater Dispute Database, http://www.transboundarywaters.orst.edu/

## 2. Relation to spatial economics literature

In a typical river ecosystem spatial aspects of potential environmental externalities are very important. The effect of ecological nuisances in the immediate vicinity of riverbanks is more important that those in the hinterland, and the distance from where the externality is generated to where it is consumed is paramount. Spatial economics is well rooted in the economics tradition going back to the early attempt by Hotelling's (1929) linear city model. However, more recently with the development of urban and regional economics, and geographical economics, economic phenomena in their spatial dimension became an integral part of a large and growing literature. More recently, with the advent of spatial data and GIS in environmental and resource economics, new advances are being made in the field of spatial econometrics.

More specifically, regarding environmental externalities, Tietenberg (1974) was among the first economists who emphasized the need for policy instruments like taxes and standards to vary geographically in order to take into account regional variations in both pollution emanations and impacts, and thus restore efficiency. Analytical models in the tradition of Von Thunen geographic model where also developed over the years. Among the fundamental models, using market-based incentives to deal with negative externalities, we find that of Hochman et al. (1977). Other relevant spatial studies dealing with spatial variability include Henderson's (1977) model on air pollution in the context of a circular city with firms imposing a negative spatial externality on consumers. In the model, a flat tax rate is imposed on firms' emissions functions, which are differentiated with respect to distance from the city center. Hochman and Ofek (1979), dealing with a fairly similar setting, build on his model and propose a simpler spatially differentiated Pigouvian taxation method in the context of a linear city. Also, they prove that setting zoning regulation can achieve the same efficiency results as taxation because it creates pollution rights that land owners can use to impose additional rents equivalent to the amount of the pollution tax.

In this paper, where we have a rural rather than an urban setting, we consider a spatial model of ecological externalities in the tradition of Henderson (1977), Hochman and Ofek (1979), and Chakravorty et al. (1995) where space is modeled explicitly using the production or cost functions. We also seek to understand the spatial implications of our model on the Pigouvian taxation rule and on adjustments made on it when market power is present in the externality producing industry as proposed by Barnett (1980).

## 3. The model

Consider a simple economy with two sets of agents: farmers located in the ecosystem around the river, and consumers located in a city by the river dam. The ecosystem has a rectangular shape where farmers are on both banks of the river. However since we consider that both banks are symmetrical, we focus only on the right hand side bank (Figure 3.1). We consider a Cartesian space, where at the origin (0, 0) a dam blocks the river for the purposes of hydroelectric power production and residential water retention. The total length of the river is L, where y represents the vertical distance from the dam; while the



Figure 3.1: The river ecosystem

maximal width of the ecosystem is  $\ell$ , where x represents the horizontal dimension .i.e. the distance away from the riverbank. We normalize the ecosystem surface area to unity i.e.  $L = \frac{1}{\ell}$ . The model, thus, considers that farmers are geographically differentiated, while consumers are not since they are all located at the origin (0,0). Consumers suffer a social disamenity caused by sedimentation at the river dam. Sedimentation is the result of soil erosion caused by clearing and excavation activities carried out by farmers as part of their production efforts.

#### 3.1. Agents and soil erosion

### 3.1.1. The farmers

A farmer located at (x, y) produces an output denoted by z(x, y) and contributes, through clearing and excavation, to soil erosion via an erosion function f that measures the contribution of this farmer to total soil erosion measured at location (0, y) i.e. the riverbank at location y. We consider that the ecosystem is homogenous in the vertical dimension with respect to both contributions to erosion and production costs. The contribution function then satisfies  $\frac{df}{dy} = 0$ , also  $\frac{df}{dx} < 0$  meaning the contribution is decreasing in the distance away from the river. Let the erosion contribution function be denoted by f(z, x)which is increasing and convex in z and satisfying  $f_x < 0$  and  $f_{zx} < 0$  meaning that the marginal contribution function is decreasing in x. We assume that farmers have identical production processes that produce a homogeneous good. Also, the negative environmental externality they generate does not affect them directly. Let C(z, x), the individual cost function, be strictly increasing and convex in both z and x with  $C_{zx} > 0$  because access to the river water source becomes harder when the farmer is located farther away from the river<sup>3</sup>. Also, the convexity of the cost function in x is explained by the fact that higher grounds (farther) have lower soil fertility than lower grounds (closer).

Total contribution to soil erosion measured at distance y from the river dam is simply the accumulation of farmers contribution located along an orthogonal line to the river at

<sup>&</sup>lt;sup>3</sup>The convexity of the cost function in x, also, captures transportation costs of the farming product to the consumers located at the city node (0,0). Land transportation costs are substantially higher than the river's, which are also less heterogeneous. Therefore, we implicitly normalize transportation costs via the river, along the vertical distance y, to zero.

point y

$$E^y = \int_0^\ell f(z, x) dx$$

notice that  $E^y$  is the same for any y. And total cumulated sediment deposits<sup>4</sup> caused by erosion and measured at the river dam are

$$S = \int_{0}^{L} e^{-\delta y} E^{y} dy = \int_{0}^{L} \int_{0}^{\ell} e^{-\delta y} f(z, x) dx dy$$
(1)

Since sedimentation occurs over the whole length of the river, not all sediments reach the dam reservoir. We capture this loss phenomenon by assuming that sediments are deposited in the river bed at an exponential distance rate where  $\delta$  is a positive sedimentation dispersion parameter.

#### 3.1.2. The consumers

Consumers are located at point (0,0), which represents a city node. They consume the farming good z and suffer from a social disamenity a(S) caused by sediment deposits S at the river dam. S decreases the storage capacity of the reservoir that is used to generate hydroelectric power, increases the costs of operations of the river dam including dredging

<sup>&</sup>lt;sup>4</sup>The spatial sedimentation process presented here can also help understand the mercury pollution process resulting from industrial activities around riverways. Diluted mercury accumulates in the soil surrounding rivers. Then it is transported, similarly to eroded soils, via streaming resulting from precipitations into rivers basins. The dangers of which are highlighted in an article in the Smithsonian newsletter. "Carried back to ground level by rain, the mercury eventually ends up in aquatic sediments. There, bacteria transform mercury into an organic and more toxic form, methylmercury, that is readily absorbed by small animals, such as plankton and worms. As those little creatures are eaten by bigger ones, methylmercury works its way up the food chain". Ultimately diluted mercury also reaches the water supplies of urban areas.

http://www.si.edu/opa/insideresearch/articles/V14\_Mercury.html

and maintenance costs, and reduces the quality of water used for residential consumption. This disamenity is reflected in more expensive utility bills like water and electricity bills. Let the level of social disamenity a increase linearly with the amount of sediments, i.e., a = vS with some positive coefficient v. Note, that social disamenity is also affected by natural sedimentation, which is here normalized to zero. Consumers' preferences are captured by the total inverse demand for farming good z,  $p(\mathbf{Z})$  with  $p'(\mathbf{Z}) < 0$ , where<sup>5</sup>

$$\mathbf{Z} = \int_0^L \int_0^\ell z(x, y) dx dy \tag{2}$$

We now solve for optimal erosion taxes given the setup of our model, which is a competitive one. Then, we turn our attention to the cooperative case in section 5.

## 4. Optimal erosion taxes

#### 4.1. Unregulated farming

In the absence of environmental taxation each farmer maximizes the following objective

$$\Pi(z) = pz - C(z, x)$$

and the resulting first-order condition indicates that the marginal cost is set equal to

<sup>&</sup>lt;sup>5</sup>Since the surface area is normalized to unity, we have  $\frac{\partial \mathbf{Z}}{\partial z} = 1$ 



Figure 4.1: The effect of taxation on marginal costs

the market price

$$p(\mathbf{Z}) = C_z(z, x)$$

Since the marginal cost is increasing in the distance from the riverbank i.e.  $C_{zx} > 0$ , then a more distant farmer has a higher marginal cost and thus a lower output z (Figure  $4.1)^6$ . This establishes that  $z_x < 0$  (Figure 4.2) i.e. decreasing land productivity.

#### 4.2. The social planner's problem

Consider a benevolent and informed regulator who maximizes jointly the net consumer surplus of consumers minus social disamenity vS, and the profits of farmers. Ignoring redistribution and income transfer issues, and replacing S by its value from (1), the tax

 $<sup>^{6}\</sup>mathrm{The}$  depicted graphs reflect quadratic cost and erosion contribution functions satisfying all of the model's assumptions.



Figure 4.2: The effect of taxation on production distribution

level will now be set in order to maximize the social welfare objective

$$W = \int_0^{\mathbf{Z}^t} p(u) du - \int_0^L \int_0^\ell C(z^t, x) dx dy - v \int_0^L \int_0^\ell e^{-\delta y} f(z^t, x) dx dy$$
(3)

We denote the variables affected by the tax by superscript t.

The necessary and sufficient first-order condition for an optimal erosion tax is given by

$$W'(t) = \int_0^L \int_0^\ell \left( p(\mathbf{Z}) \frac{dz}{dt} - C_z \frac{dz}{dt} - v e^{-\delta y} f_z \frac{dz}{dt} \right) dx dy = 0$$
(4)

Equality (4) holds only when

$$p(\mathbf{Z})\frac{dz}{dt} - C_z \frac{dz}{dt} - v e^{-\delta y} f_z \frac{dz}{dt} = 0$$
(5)

which holds for every point  $(x, y) \in [0, \ell] \times [0, L]$ .

A farmer located at (x, y) will react to this tax on his contribution to erosion by maximizing the profit function

$$\Pi(z) = pz - C(z, x) - tf(z, x)$$

the following first-order condition must hold where each farmer sets his full marginal cost equal to the market price

$$p\left(\mathbf{Z}\right) = C_z + tf_z \tag{6}$$

Substituting (6) into (5) yields the general formula for the optimal tax rule

$$t(y) = v e^{-\delta y} \tag{7}$$

## 4.3. Results

It is easy to observe that the optimal tax rate is set equal to the vertical-distance adjusted marginal social disamenity  $ve^{-\delta y}$ . This follows the proposition of Pigou (1920) which states that the regulator should set the tax equal to the marginal social damage, however with a difference that the marginal social disamenity is adjusted by a distance parameter of y that reflects the spatial nature of the soil erosion externality.

**Proposition 1.** The optimal tax rule indicates that the regulator must decrease the optimal tax level when the farming unit's distance y increases.

This taxation rule appears to be myopic since it only differentiates with respect to the distance from the dam but does not takes into account farmers' distance from the riverbank. As such farmers located at the same location y receive the same tax treatment t(y). However, due to the geographic nature of the farming problem, this tax treatment implicitly considers the impact of the location x. When optimal taxation is applied the first-order condition becomes expression (6) which is rewritten

$$p(\mathbf{Z}) = C_z(z, x) + t(y)f_z(z, x)$$

Because the tax rate t(y) is the same for closer and more distant farmers, the degree of adjustment of the full marginal cost depends on the marginal contribution to soil erosion  $f_z$  which is decreasing in x. Therefore, a more distant farmer has a lower full marginal cost adjustment. This indicates that his output is less affected by the corrective tax. The implications for the production distribution are illustrated in Figure 3. Taxation changes the distribution of production z by making it flatter. The most productive farmers are penalized the most. A possible effect is to cause the most distant and as such the least productive farmers to exit the market altogether where the tax forces them to produce nothing. Another possible scenario is that the production distribution rotates downwards without causing any exist from the market. This will be the case only when the contribution function  $f_z$ tends asymptotically to zero as we approach the border  $\ell$  of the ecosystem, beyond which farmers don't operate by assumption.(Figure 4.2)

## 5. Cooperative action and erosion taxes

Suppose that a cooperative of producers operates in the river ecosystem so that all farming units (x, y) are under its control. A cooperative action consists in maximizing the total profit of all farming units or farmers subject to an erosion tax on their total contribution to erosion. We write

$$\Pi(\mathbf{Z}) = p(\mathbf{Z}) \mathbf{Z} - \int_0^{\mathbf{L}} \int_0^{\ell} C(z, x) dx dy - t \int_0^{\mathbf{L}} \int_0^{\ell} f(z, x) dx dy$$

Maximizing with respect to z after replacing Z by (2). The following first-order condition must hold<sup>7</sup>

$$\int_{0}^{L} \int_{0}^{\ell} \left[ p'(\mathbf{Z})z + p(\mathbf{Z}) - C_{z} - tf_{z} \right] = 0$$
(8)

<sup>7</sup>We rewrite  $\Pi(\mathbf{Z}) = \int_0^{\mathbf{L}} \int_0^{\mathbf{l}} [p(\mathbf{Z}) z(x, y) dx dy - C(z, x) dx dy - tf(z, x)] dx dy$ 

Equality (8) holds only when

$$p'(\mathbf{Z})z + p(\mathbf{Z}) - C_z - tf_z = 0$$
(9)

which holds for every point  $(x, y) \in [0, \ell] \times [0, L]$ .

Substituting (9) into (5) yields the general formula for the optimal -per unit of farmingtax rule

$$t(x,y) = ve^{-\delta y} + \frac{p'(\mathbf{Z})z\frac{dz}{dt}}{f_z\frac{dz}{dt}}$$
(10)

(10) defines a location tax. Note that the second term on the right-hand side of (10) is negative.

The second term on the right-hand side of (10) is an adjustment that takes into account the welfare of both farmers and consumers. As a matter of fact, this result is a refinement of the Pigouvian proposition.<sup>8,9</sup> But in our case, the negative adjustment

<sup>9</sup>Expression (10) can be rewritten with the demand elasticity  $\varepsilon$ 

$$t = v e^{-\delta y} - \frac{\frac{p(\mathbf{Z})}{|\varepsilon|} \frac{z}{\mathbf{Z}} \frac{dz}{dt}}{f_z \frac{dz}{dt}}$$

<sup>&</sup>lt;sup>8</sup>In the literature on Pigouvian taxation, Buchanan (1969) and Barnett (1980) where the first ones to introduce a downward adjustment term. They conclude that when polluters operate in an imperfectly competitive framework, the optimal corrective tax must be set lower than the marginal social cost of damage, because of the trade off that results between the regulator's wish to provide incentives for abatement and the requirement to avoid a greater reduction in total output. While, David & Sinclair-Desgagné (2005) introduce an upward adjustment term within an upstream-downstream industry framework. They find that imperfect competition in the eco-industry (upstream) results in abatement prices larger than the marginal cost of abatement; optimal taxes must then be raised in order to make polluters (downstream) reduce their emissions sufficiently.

As noted by Barnett (1980), If the demand for good z becomes less elastic, the size of the downward adjustment increases. This property protects the consumers from excessively high prices resulting from environmental taxation.

term we derive is spatially differentiated to take into account the impact of the location of various contributors to the negative environmental externality.

Given the maximization program of any farming unit, we verify that at the maximum  $z_y = 0$  and  $z_x < 0$ . Using this result, comparative statics on the optimal tax rule yield

$$\frac{\partial t}{\partial y} = -\delta v e^{-\delta y} \tag{11}$$

it is obvious that  $\frac{\partial t}{\partial y} < 0.^{10}$ 

**Proposition 2.** When farmers behave cooperatively, holding everything else constant, the regulator must decrease the optimal tax level when the farming unit's distance y increases as in the competitive case discussed in proposition 1.

This result suggests that, in both the competitive and cooperative cases, the optimal tax rule has a built-in incentive to encourage the farmers/cooperative to shift part of the production farther away (upstream) from farming units located close to the river dam.

Comparative statics, also, yield

$$\frac{\partial t}{\partial x} = p'(\mathbf{Z}) \left( \frac{z_x \left( f_z - z f_{zz} \right) - z f_{zx}}{\left( f_z \right)^2} \right)$$
(12)

 $p'(\mathbf{Z})$  is negative. The sign of (12) depends on the sign of  $A^{11}$ . Simple manipulations

 $<sup>{}^{10}\</sup>frac{\partial t}{\partial y} = -\delta v e^{-\delta y} + p'(\mathbf{Z}) \left(\frac{f_z - zf_{zz}}{(f_z)^2}\right) z_y$  ${}^{11}A = z_x \left(f_z - zf_{zz}\right) - zf_{zx}$ 

yield the following result

$$A \stackrel{\leq}{\leq} 0 \Leftrightarrow \frac{z_x}{z} (1 - \eta) \stackrel{\leq}{\leq} \frac{f_{zx}}{f_z} \Leftrightarrow \frac{\partial t}{\partial x} \stackrel{\geq}{\geq} 0 \tag{13}$$

where  $\eta = \frac{f_{zz}}{f_z} z$  is the elasticity of marginal contribution to erosion with respect to z. This result can be summarized by the following.

**Proposition 3.** When farmers behave cooperatively, holding everything else constant, the regulator must increase (decrease) the optimal tax level when the farming unit's distance x increases if and only if the adjusted rate of change of output z in the x dimension  $\left(\frac{z_x}{z}(1-\eta)\right)$  is smaller (larger) than the rate of change of the marginal contribution to erosion  $f_z$  in the x dimension  $\left(\frac{f_{zx}}{f_z}\right)$ 

The negative adjustment term in the optimal tax rule implies that there is a corrective incentive that protects consumers from excessively high prices -when the tax is implemented- and ensures that consumers' surplus is not excessively adversely affected by the tax. This adjustment appears in the optimal tax rule due to the presence of market power as first suggested by Barnett (1980). By cooperating, farmers create this market power.

There are two possible ranges for the elasticity of marginal contribution to erosion  $\eta$ .

In the inelastic range  $(0 < \eta < 1)$  the trade-off highlighted in proposition 3 holds. This means that when the marginal contribution to erosion is not too responsive to an increase in output z, which is a proxy for soil erosion effort, the regulator is confronted with a tradeoff between productivity and contribution to erosion. The Second-best considerations highlighted by Barnett (1980) no longer depend solely on the presence of market power in the polluting industry. Rather, our space augmented tax rule trades-off two problems: erosion externality and underprovision by monopoly.

When output z(x, y) drops by a larger amount than the marginal contribution to erosion for a similar increase in the distance from the river x, the optimal tax is set to increase with the distance x. Therefore, the tax rule provides reduced incentives for the cooperative of farmers to react to environmental taxation by shifting production from lower ground into higher grounds where higher production costs prevail translating into higher prices for consumers. Opposite incentives are present when the marginal contribution to erosion drops by a larger amount than production as x increases. For this reason the optimal tax is set to decrease with the distance x. Due to the presence of social disamenities caused by soil erosion, the positive impact on social welfare of shifting production away from the riverbank outweighs the negative impact this has on the selling price.

Logically, this last result also holds for the elastic range  $(\eta \ge 1)$  when the marginal contribution to erosion is very responsive to increases in z, which cause a more than proportional increase in contributions to the externality. In such case, society is severely affected by the externality. We have from (13) that

$$\frac{z_x}{z}\left(1-\eta\right) > \frac{f_{zx}}{f_z} \Leftrightarrow \frac{\partial t}{\partial x} < 0$$

The tax rate is set to decrease in the distance x unambiguously.

# 6. Concluding remarks

We have shown that the structure of the farming output market matters, and that the optimal tax rule varies accordingly. We have also shown that the spatial aspect of the soil erosion problem introduces downward adjustments on the tax rule under both competition and cooperative monopolization scenarios. Moreover, since the optimal tax rule provides productivity related incentives it has implications on possible zoning regulation. The tax rule under competition suggests that zoning needs to create a buffer zone free of farming activities near the river dam, thus pushing farmers upstream. This is what we call y - zoning. Under the cooperative scenario, it is precisely the productivity-contribution (to erosion) trade-off outlined by the optimal tax rule that determines which type of zoning is better, y - zoning or x - zoning i.e. pushing farmers away from the riverbank.<sup>12</sup> Zoning could be strict i.e. complete ban on all farming activities, or it could be partial where farmers offset their destructive behavior by planting soil preserving plants. In the former case, the regulator has implicit preferences for the auto generation of the original natural forest cover that prevents soil erosion on the riverbanks.

A possible application of the model we develop could be in the context of industrial standards regulation like the car industry for instance. In this case, the x - dimension can be seen as the engines technology dimension. While the y - dimension simply be-

 $\frac{12 \, \underline{z_x}}{z} \, (1-\eta) \stackrel{<}{\underset{}{\underset{}{\stackrel{}{\rightarrow}}}} \frac{f_{\underline{zx}}}{f_z} \Leftrightarrow \frac{\partial t}{\partial x} \stackrel{\geq}{\underset{}{\underset{}{\rightarrow}}} 0 : \stackrel{+}{\underset{}{\xrightarrow{}}} \Rightarrow \begin{array}{c} y - zoning \\ x - zoning \end{array}$ 



Figure 6.1: Environmental technology regulation

comes the time dimension of the problem, where  $\delta$  could be defined as the inherent rate of improvement in engine efficiency over time (Fig. 6.1). This rate can also be made heterogeneous; while the contribution function could be simply defined as the contribution to social nuisance of each car model at any given time t. And the cost function retains all its properties namely rising marginal costs in the technology efficiency parameter.

The impact of a more complex dynamic setup on optimal taxation and anti soil erosion regulation in general remains to be explored. Pursuing this path, can also help understand "spatially" a number of hybrid differential game models of managerial decision making like the Cattle Ranching problem<sup>13</sup> for example. In that model, the objective is maximized

<sup>&</sup>lt;sup>13</sup>The Cattle Ranching Problem is discussed in Sethi and Thompson (2000) p. 318

over two dimensions, time, which could be replaced by y, and another dimension, age of an animal in this case, which replaces x, with  $\ell$  as the age of maturity of the animal. This indicates that there is potentially another side to the static story in our model. As such, our spatial setup may help make the dynamics in those models more transparent.<sup>14</sup>.

<sup>&</sup>lt;sup>14</sup>What seems to be an important lead to follow through is the idea of spatial independence of the x and y dimensions. It remains to be seen what would be the implications this assumption on the strategies chosen by be it open loop or feedback.

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