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# **Recycling With Endogeneous Consumer Participation**

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# Recycling With Endogeneous Consumer Participation

*Soham Baksi<sup>\*</sup>, Ngo Van Long<sup>†</sup>*

## **Résumé / Abstract**

On montre que le coût de triage et les effets de réseau déterminent le taux de participation des consommateurs au processus de recyclage. La firme dominante tient compte des activités de recyclage en choisissant le prix du produit primaire. Les effets de réseau peuvent créer la multiplicité d'équilibres stationnaires. Le gouvernement peut améliorer le bien-être social par son influence sur le choix d'équilibres.

**Mots clés** : effet de réseau, firme dominante, multiplicité d'équilibres, recyclage

*We show that the cost of sorting and the network effects jointly determine the rate of participation of consumers in the process of recycling. The dominant producer of virgin material takes into account the recycling activities when it makes its pricing decision. The network effects can create multiplicity of steady-state equilibria. The government can improve welfare by influencing equilibrium selection.*

**Keywords:** *dominant firm, multiple equilibria, network effect, recycling*

**Codes JEL** : D4, L1, Q2

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## 1. Introduction

The growing environmental consciousness within society has resulted in the increase in the recycling rate over time in many countries. The percentage of municipal solid waste recycled in the US, for example, has increased from 6.4%, or 5.6 million tons of material, in 1960 to 30.6%, or 72.3 million tons, in 2003.<sup>1</sup> Recycling is important not only because it promotes conservation of natural resources through reduced use of virgin (i.e. non-recycled) materials, but also because it reduces the amount of waste that has to be disposed off through landfilling or incineration.

The process of recycling involves two essential steps: the generation and supply of scrap material by consumers, and the transformation of that scrap into secondary material by recycling firms. Thus, how much recycling finally takes place depends on decisions that are made by both consumers and recycling firms. However, the extant literature on the “recycling problem” has typically focussed on the role of the recycling firms and not the consumers (see, for examples, Gaskin, 1974; Swan, 1980; Martin, 1982; Suslow, 1986; Tirole, 1988; Grant, 1999; Gaudet and Long, 2003).<sup>2</sup> Motivated by the celebrated Alcoa case of 1945, most of these papers examine the restraints imposed by the existence of a competitive recycling sector on a monopoly virgin producer’s ability to markup price over cost.<sup>3</sup> All these papers, however, assume total participation by all (homogeneous) consumers in the recycling process. Consequently, in these models, the entire output of a recyclable product bought by consumers in one period

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<sup>1</sup>Source: <http://www.epa.gov/epaoswer/non-hw/muncpl/facts.htm>. For similar data on some other OECD countries, see [http://maps.grida.no/go/graphic/recycling\\_rates\\_for\\_selected\\_oecd\\_countries](http://maps.grida.no/go/graphic/recycling_rates_for_selected_oecd_countries)

<sup>2</sup>Grant (1999, p. 59) defines the recycling problem as “the optimal pricing problem facing the dominant producer of a recyclable good”.

<sup>3</sup>In the Alcoa case, Justice Learned Hand ruled that the Aluminum Company of America constituted an illegal monopoly as its share of the virgin aluminum market exceeded 90 percent. Alcoa’s argument that its share of the total (i.e. virgin plus secondary) market for aluminum be considered was rejected.

becomes available to the recycling firms as scrap in the next period.

The above assumption about consumer participation is not a realistic one for many households. In order to recycle their household garbage, consumers must first sort recyclables from non-recyclables. Next, the recyclables have to be taken to the curbside or to the nearest drop-off depot. In either case, participation in recycling is costly to the consumers in terms of their time and effort. The magnitude of this intrinsic cost (or disutility) of recycling varies across consumers. More environmentally friendly consumers are likely to have a lower cost of recycling. Given this heterogeneity, it is natural to expect that some consumers will take part in the recycling process whilst others will not. In other words, the entire output of a recyclable material purchased by consumers in one period will not be available as scrap to the recycling firms for reprocessing in a subsequent period, as some part of it (i.e. the part which was purchased by non-recycling consumers) will be disposed off as waste. Our paper presents a model of partial recycling when consumers are heterogeneous in terms of their recycling cost.

Although the intrinsic cost of recycling for each consumer will be idiosyncratic to that consumer, it is also likely to be influenced by social norms. In particular, the incentive for, or (indirect) social pressure on, a consumer to recycle will likely increase as more consumers around him start recycling. We assume the existence of such a “network effect” which modifies each consumer’s recycling cost.<sup>4</sup> This is in the spirit of Akerlof and Kranton (2005), who argue that in many social situations (dis)utility functions are not fixed (as traditionally assumed in neoclassical economics) but situation dependent. The existence of this network effect leads to the possibility of multiple equi-

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<sup>4</sup>That an individual’s recycling decision can be influenced by others is, for example, acknowledged by the Manitoba Product Stewardship Corporation (the provincial body in charge of administering the recycling program in Manitoba). In its “Community Recycling Handbook” (2003, Section 5-3), MPSC lists the following among “factors that motivate people to recycle”: (i) setting a good example, (ii) the right thing to do. See <http://www.mpsc.com/main.asp?contentID=25> for details.

libria arising in our model. An implication of our model is that a welfare-maximizing government might have a role to play in equilibrium selection.

What fraction of consumers will participate in recycling depends on consumers' recycling cost and the reward they receive from recycling. Following Swan's (1980, p. 86) "more sophisticated model", we assume that consumers are able to sell the scrap to recycling firms for a price (once they have sorted their household garbage). However, unlike Swan, we assume that sorting is costly, and the sorting cost varies across consumers, so that, in general, not all consumers participate. The scrap price is determined endogenously by the recycling market. Such markets for scrap are common in many developing countries such as China and India.<sup>5</sup>

Further details about our model are provided in Section 2. Much of the model setup, with the notable exception of Section 2.2 which deals with consumers, follows Swan (1980). The steady state is derived in Section 2.3. We find that multiple equilibria arise only when the values of the network effect and the scrap price satisfy certain conditions. Moreover, the long-run price charged by the virgin producer is found to depend on both the endogenous fraction of consumers who recycle, which the virgin producer can manipulate by its pricing strategy, as well as the fraction of secondary aluminum recovered per unit scrap by the recycling firms. Section 3 examines the impact of an increase in the network effect on the price of virgin. The sign of this impact can be positive or negative, depending on the fraction of consumers that participates in recycling. Section 4 provides a numerical example in support of our analytical results. The last section concludes.

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<sup>5</sup>In the US, certain items such as old printer ink cartridges can be sold for cash. See <http://www.freerecycling.com> for details.

## 2. The model

### 2.1. Recycling firms

Recycling firms are perfectly competitive. Each unit of scrapped aluminum can be transformed into  $\gamma$  unit of virgin-equivalent (secondary) aluminum at the cost  $z(\gamma)$  per unit of scrap, where  $\gamma \in [0, 1]$  is a decision variable. The function  $z(\gamma)$  is assumed to be convex and increasing in  $\gamma$ , and independent of the number of units of scrap to be recycled.

The price of a unit of virgin as well as secondary aluminum in period  $t$  is  $p_t$ . The recycling firm chooses  $\gamma_t$  to maximize

$$p_t \gamma_t - z(\gamma_t)$$

subject to  $0 \leq \gamma_t \leq 1$ . This gives the optimal  $\gamma_t$  as a function of  $p_t$ . Write  $\gamma_t = \gamma(p_t)$ . At an interior solution, we have

$$p_t - z'(\gamma_t) = 0$$

with

$$\frac{d\gamma_t}{dp_t} = \frac{1}{z''} > 0. \quad (1)$$

Let  $\phi_t$  denote the price per unit of scrap that recyclers pay to consumers who make the scrap available by sorting. The profit per unit of scrap is

$$\pi_t = p_t \gamma(p_t) - z(\gamma(p_t)) - \phi_t.$$

Assume that competition by recyclers drives profit  $\pi_t$  to zero. This implies that  $\phi_t$  is uniquely determined by

$$\phi_t = p_t \gamma(p_t) - z(\gamma(p_t)) \equiv \phi(p_t). \quad (2)$$

It is easy to show that  $\phi'(p_t) > 0$ , and  $0 < \phi(p_t) \leq p_t$ .

## 2.2. Consumers

A consumer that buys, in period  $t$ ,  $q_t$  units of virgin or secondary aluminum derives the gross utility level

$$u(q_t) = aq_t - \frac{1}{2}q_t^2. \quad (3)$$

In the following period, he has at his disposal  $s_{t+1}$  units of scrap, where  $s_{t+1} = q_t$ . He can sell each unit of scrap at the market price  $\phi_{t+1}$ , provided that he sorts the scrap from the household garbage.

Different consumers have different sorting costs. Consumers are indexed by a parameter  $\theta$ . Assume  $\theta$  is uniformly distributed over the unit interval  $[0, 1]$ . The population is normalized at unity. The per-unit sorting cost of consumer  $\theta$  at time  $t$  is

$$\theta(1 - \alpha f_t)$$

where  $\alpha \in [0, 1]$  is the “network effect” parameter, and  $f_t$  is the fraction of the population who undertake sorting in period  $t$ . We may interpret  $\theta$  as an intrinsic cost of sorting (e.g. it reflects the nuisance of having to spend time sorting), and  $\alpha f_t$  is the proportional reduction in the intrinsic cost due to network effect (or, cultural effects).

How many units the consumer of type  $\theta$  chooses to purchase in period  $t$  depends on two things: (i) the price he must pay,  $p_t$ , and (ii) the return per unit he gets next period by sorting and selling the scrap. This return per unit is denoted by  $R_{t+1}(\theta)$  where

$$R_{t+1}(\theta) \equiv \max[\phi_{t+1} - \theta(1 - \alpha f_{t+1}), 0] \leq \phi_{t+1} \leq p_{t+1}.$$

Given  $\phi_{t+1} \geq 0$  and  $f_{t+1}$ , all the consumers whose  $\theta$  are low enough so that  $R_{t+1} > 0$  will undertake sorting, and the remaining consumers will not sort. How do we determine the threshold value of  $\theta$  (denote this as  $x_{t+1}$ ) that separates sorters from non-sorters at time  $t+1$ ? Since  $\theta$  is uniformly distributed, its threshold value  $x_{t+1}$  also determines the fraction of population that sort at time  $t + 1$ , i.e.

$$x_{t+1} = f_{t+1}.$$

Given  $\phi_{t+1} \geq 0$ , consider the quadratic equation

$$x_{t+1}(1 - \alpha x_{t+1}) = \phi_{t+1} \geq 0 \quad (4)$$

Either this equation has no real roots  $\hat{x} \in [0, 1]$ , or it has one or two real roots  $\hat{x} \in [0, 1]$ . When there are two real roots, they are denoted by  $x_{t+1}^L$  and  $x_{t+1}^H$ , where

$$x_{t+1}^L = \frac{1}{2\alpha} \left[ 1 - \sqrt{1 - 4\alpha\phi_{t+1}} \right] \equiv x^L(\phi_{t+1}), \quad (5)$$

$$x_{t+1}^H = \frac{1}{2\alpha} \left[ 1 + \sqrt{1 - 4\alpha\phi_{t+1}} \right] \equiv x^H(\phi_{t+1}). \quad (6)$$

Define

$$g(x) \equiv x(1 - \alpha x).$$

Observe that  $g(x)$  is a strictly concave function, with

$$g(0) = g\left(\frac{1}{\alpha}\right) = 0,$$

$$g(1) = 1 - \alpha$$

and  $g(x)$  attains its maximum value (called maximum sorting cost) at  $x = 1/(2\alpha)$  with

$$g\left(\frac{1}{2\alpha}\right) = \frac{1}{4\alpha} \geq 1 - \alpha,$$

where the inequality follows from the fact that  $\alpha(1 - \alpha)$  attains its maximum at  $\alpha = 1/2$  (the strict inequality holds if  $\alpha \neq 1/2$ ).

In the determination of the threshold value  $x_{t+1}$ , the following two mutually exclusive cases can arise depending on the values of  $\alpha$  (the network effect parameter) and  $\phi_{t+1}$  (the scrap price).

Case A:  $\alpha > 1/2$ .

(i) if  $\phi_{t+1} > 1/(4\alpha)$  then equation (4) has no real root. The price of scrap exceeds the maximum sorting cost, so all consumers will sort, i.e.,  $x_{t+1} = 1$

(ii) if  $\phi_{t+1} = 1/(4\alpha)$  then  $x_{t+1} = 1/(2\alpha)$ , or  $x_{t+1} = 1$  (because  $\theta(1 - \alpha) \leq (1 - \alpha) < 1/(4\alpha) = \phi_{t+1}$ ).

(iii) if  $1 - \alpha \leq \phi_{t+1} < 1/(4\alpha)$  then  $x_{t+1} = x_{t+1}^L$  or  $x_{t+1}^H$  or 1

(iv) if  $\phi_{t+1} < 1 - \alpha$  then  $x_{t+1} = x_{t+1}^L$

Case B:  $\alpha \leq 1/2$

(i) if  $\phi_{t+1} \geq 1 - \alpha$  then  $x_{t+1} = 1$

(ii) if  $\phi_{t+1} < 1 - \alpha$  then  $x_{t+1} = x_{t+1}^L$

Thus we can state the following proposition.

**Proposition 1:** *In the determination of the threshold value ( $x_{t+1}$ ) that separates sorters from non-sorters at time  $t+1$ , multiple equilibria arise if and only if the network effect parameter is high (specifically  $\alpha > 1/2$ ) and the scrap price is intermediate (specifically  $1 - \alpha \leq \phi_{t+1} \leq 1/(4\alpha)$ ).*

Only consumers with  $\theta$  less than the threshold value  $x_{t+1}$  sort and sell their recyclables at  $t+1$ ; these consumers receive a positive return  $R_{t+1}$ . The remaining consumers (i.e. those with  $\theta$  greater than or equal to  $x_{t+1}$ ) receive zero return at time  $t+1$ . Let  $\delta \in (0, 1)$  be the discount factor. Then the *net price*,  $p_t^N$ , that consumer  $\theta$  pays for a unit of virgin aluminum at time  $t$  equals

$$p_t^N(\theta) = p_t - \delta R_{t+1}(\theta) < p_t \text{ if } \theta < x_{t+1}(\phi_{t+1}) \equiv x_{t+1}(\phi(p_{t+1})),$$

$$p_t^N(\theta) = p_t \text{ if } \theta \geq x_{t+1}(\phi_{t+1}) \equiv x_{t+1}(\phi(p_{t+1})), \quad (7)$$

At time  $t$ , the consumer  $\theta$  must know both  $p_t$  and  $p_{t+1}$  to compute  $p_t^N(\theta)$  before deciding on the quantity  $q_t(\theta)$  to be bought. The quantity  $q_t(\theta)$  must satisfy the first order condition

$$u'(q_t(\theta)) = p_t^N(\theta)$$

which, using (3), gives

$$q_t(\theta) = a - p_t^N(\theta) \quad (8)$$

where we assume that the choke price,  $a$ , is greater than the net price,  $p_t^N(\theta)$ .

The market demand for aluminum is then

$$\begin{aligned} Q_t &\equiv \int_0^1 q_t(\theta) d\theta = a - p_t + \delta \int_0^1 R_{t+1}(\theta) d\theta, \\ Q_t &= a - p_t + \delta \int_0^{x_{t+1}(\phi_{t+1})} [\phi_{t+1} - \theta(1 - \alpha f_{t+1})] d\theta, \\ Q_t &= a - p_t + \delta \phi_{t+1} x_{t+1} - \delta(1 - \alpha x_{t+1}) \int_0^{x_{t+1}} \theta d\theta, \\ Q_t &= a - p_t + \delta x_{t+1} \left[ \phi_{t+1} - \frac{x_{t+1}(1 - \alpha x_{t+1})}{2} \right]. \end{aligned}$$

Making use of (4)

$$Q_t = a - p_t + \frac{\delta}{2} x_{t+1} \phi_{t+1} \equiv Q_t(p_t, p_{t+1}) \quad (9)$$

Total demand of aluminum by the consumers who do not sort and recycle in  $t + 1$  (i.e. those with  $\theta \geq x_{t+1}$ ) is

$$Q_t^{NR} \equiv (1 - x_{t+1})(a - p_t)$$

where  $NR$  stands for “non-recycler”. Therefore, the amount of scrap that consumers supply to recyclers in period  $t + 1$  is

$$S_{t+1} = Q_t - Q_t^{NR} = x_{t+1} \left\{ a - p_t + \delta \phi_{t+1} - \delta(1 - \alpha x_{t+1}) \frac{x_{t+1}}{2} \right\}.$$

Making use of (4)

$$S_{t+1} = x_{t+1} \left\{ a - p_t + \delta \frac{\phi_{t+1}}{2} \right\} \equiv S_{t+1}(p_t, p_{t+1}) \quad (10)$$

and this yields  $\gamma(p_{t+1})S_{t+1}$  units of secondary (virgin-equivalent) aluminum in  $t + 1$ .

### 2.3. Virgin Producer and the Steady State

Primary or virgin aluminum is produced by a monopolist. The amount of virgin produced by the monopolist in period  $t$  is  $Q_t - \gamma_t S_t$ . Let  $c$  denote the constant marginal cost of virgin production. The monopolist's profit in period  $t$  is

$$\Pi_t = (p_t - c) [Q_t(p_t, p_{t+1}) - \gamma(p_t)S_t(p_{t-1}, p_t)]$$

where  $Q_t$  is given by (9) and  $S_{t+1}$  is given by (10).

The monopolist must choose a sequence of prices  $\{p_t\}$  that maximizes the present value of his profits over time

$$V = \max \sum_{t=1}^{\infty} \delta^{t-1} \Pi_t$$

The first order condition with respect to  $p_t$  is

$$(p_{t-1} - c) \frac{\partial Q_{t-1}}{\partial p_t} + \delta [Q_t - \gamma_t S_t] + \delta (p_t - c) \left[ \frac{\partial Q_t}{\partial p_t} - \gamma_t \frac{\partial S_t}{\partial p_t} - S_t \frac{\partial \gamma_t}{\partial p_t} \right] - \delta^2 (p_{t+1} - c) \gamma_{t+1} \frac{\partial S_{t+1}}{\partial p_t} = 0 \quad (11)$$

where

$$\begin{aligned} \frac{\partial Q_t}{\partial p_t} &= -1, \\ \frac{\partial Q_t}{\partial p_{t+1}} &= \frac{\delta}{2} \left[ \phi_{t+1} \frac{dx_{t+1}}{dp_{t+1}} + x_{t+1} \frac{d\phi_{t+1}}{dp_{t+1}} \right], \\ \frac{\partial S_{t+1}}{\partial p_t} &= -x_{t+1}, \\ \frac{\partial S_{t+1}}{\partial p_{t+1}} &= \frac{\delta}{2} \left[ \phi_{t+1} \frac{dx_{t+1}}{dp_{t+1}} + x_{t+1} \frac{d\phi_{t+1}}{dp_{t+1}} \right] + (a - p_t) \frac{dx_{t+1}}{dp_{t+1}}. \end{aligned}$$

Let the steady state values of the variables be denoted by an asterisk (\*). Then at steady state the first order condition (11) becomes

$$(p^* - c) \frac{1}{2} \left[ \phi \frac{dx}{dp} + x \frac{d\phi}{dp} \right] + [Q^* - \gamma^* S^*]$$

$$-(p^* - c) \left[ 1 + \frac{\delta\gamma^*}{2} \left( \phi \frac{dx}{dp} + x \frac{d\phi}{dp} \right) + S^* \frac{\partial\gamma^*}{\partial p} + \gamma^*(a - p^*) \frac{dx}{dp} \right] + \delta(p^* - c)\gamma^*x^* = 0.$$

Collecting terms, and using (9) and (10), the steady state first order condition can be rewritten as

$$-(p^* - c) \left[ 1 - \frac{1}{2} \left( \phi \frac{dx}{dp} + x \frac{d\phi}{dp} \right) (1 - \delta\gamma^*) - \delta\gamma^*x^* + S^* \frac{\partial\gamma^*}{\partial p} + \gamma^*(a - p^*) \frac{dx}{dp} \right] + (a - p^*)(1 - \gamma^*x^*) + \frac{\delta}{2} (1 - \gamma^*)x^*\phi^* = 0. \quad (12)$$

Note that in the special case where there is no recycling,  $x = 0$ ,  $\gamma = 0$ ,  $S = 0$  and then eq (12) becomes

$$Q^* = p^* - c,$$

which is the familiar marginal revenue ( $p^* + Q^*p'$ ) equals marginal cost ( $c$ ) condition.

**Proposition 2:** *The long-run price of aluminum will be close to the competitive price if and only if both the fraction of consumers that sort, as well as the fraction of virgin-equivalent aluminum recovered per unit of scrap by the recycling firms, are close to one.*

Proof: From (12),  $p^* = c$  iff  $x^* = 1$  and  $\gamma^* = 1$ .

Because the existing literature (e.g. Swan, 1980; Tirole, 1988) assumes that the entire demand in one period is available as scrap to the recycling firms in the next period, it finds that  $p^* = c$  iff  $\gamma^* = 1$ . However, when we allow for heterogeneous consumers to endogenously choose whether to participate in recycling or not, the additional condition  $x^* = 1$  has to hold before the primary producer loses all his monopoly power. This suggests that the primary producer will have

an incentive to discourage not only the recycling firms from reprocessing the scrap, but also the consumers from sorting their garbage for scrap, in order to preserve his monopoly power.

Recall that the recycling firms choose their recycling effort so as to equate their marginal cost ( $\frac{\partial z}{\partial \gamma}$ ) to the price of virgin ( $p$ ). As long as the monopolist is able to increase this price above his marginal cost ( $c$ ), the marginal cost of producing secondary (virgin-equivalent) aluminum will exceed the marginal cost of producing virgin. This creates a productive inefficiency as too much recycling is undertaken by the recycling firms, which get the wrong price signal. Hollander and Lasserre (1988) cite this as a reason why the monopolist will have an incentive to integrate into the secondary market.

Suppose both virgin and secondary production came under the monopolist's control (the "Pure Monopolist" case in Swan, 1980). The pure monopolist's combined profit in period  $t$  from secondary ( $\pi_t^m$ ) and virgin production ( $\Pi_t$ ) would be

$$\pi_t^m + \Pi_t = [(p_t \gamma_t - z_t - \phi_t) S_t] + [(p_t - c) (Q_t - \gamma_t S_t)] = (c \gamma_t - z_t - \phi_t) S_t + (p_t - c) Q_t,$$

where  $Q_t$  and  $S_{t+1}$  are given by (9) and (10). Now, in order to maximize his combined profit, the pure monopolist will recycle scrap only up to the point where  $\frac{\partial z}{\partial \gamma} = c$ . The optimal  $\gamma$  and hence  $z(\gamma)$ , therefore, are functions of  $c$  (and not  $p_t$ , as earlier), and are invariant over time. We denote them by  $\gamma^*$  and  $z^*$ . Also,  $\phi_t$  is not a function of  $p_t$  anymore. Instead, the pure monopolist chooses a sequence of  $\{p_t, \phi_t\}$  so as to maximize the present value of his combined profits over time,  $\sum_{t=1}^{\infty} \delta^{t-1} (\pi_t^m + \Pi_t)$ . The FOCs with respect to  $p_t$  and  $\phi_t$  are respectively,

$$Q_t - (p_t - c) - \delta x_{t+1} (c \gamma^* - z^* - \phi_{t+1}) = 0,$$

$$\frac{\delta x_t}{2} (c \gamma^* - z^* - \phi_t) + \left( a - p_{t-1} + \frac{\delta \phi_t}{2} \right) (c \gamma^* - z^* - \phi_t) \frac{\partial x_t}{\partial \phi_t} - x_t \left( a - p_{t-1} + \frac{\delta \phi_t}{2} \right) = 0.$$

The value of  $\frac{\partial x_t}{\partial \phi_t}$  is given by (14) and (18) below, and can be either positive or negative. Thus the pure monopolist chooses an optimal scrap price, which determines the fraction of the population that sort. This result is in sharp contrast to Swan's conclusion for the pure monopoly case, "the price of scrap  $\phi$ , and by implication the purchase price  $p$ , are immaterial to the monopoly and have no bearing on the solution. A higher purchase price which is exactly offset by increased payments for scrap metal is of no consequence since the monopolist directly controls the recycling activity" (Swan, 1980, p.91). The difference between Swan's result and ours stems from the fact that Swan's consumers have zero cost of sorting. Hence any increase in price of scrap only increases the purchase price  $p$ , without affecting the quantity of scrap sold by consumers in Swan's model. In contrast, consumers are heterogeneous in our model, and an increase in scrap price increases the amount of scrap sold by consumers.

### 3. Comparative Statics

What would be the effect of an increase in the network effect parameter,  $\alpha$ , on the steady state price and the monopoly power enjoyed by the primary producer? An increase in  $\alpha$  would imply a larger proportional decrease in each consumer's recycling cost, for any given fraction of the population that recycle. The effect on the steady state price can be found using the comparative statics methodology. Define the left-hand side of equation (12) as  $F(p, \alpha)$ . Then provided the partial derivative of  $F$  with respect to  $p$ , i.e.  $F_p$ , is not zero, we can express  $p$  as a function of  $\alpha$ , and

$$\frac{dp}{d\alpha} = -\frac{F_\alpha}{F_p}.$$

Whether the steady state  $p^*$  would increase or decrease as a result of a change in  $\alpha$  would depend on the threshold value of  $\theta$  that separates sorters from non-sorters. In what follows, we focus only on the two possible interior equilibria for the threshold value of  $\theta$ ,  $x^L$  and  $x^H$ ,

as defined in (5) and (6).

Note that, using (2) and the envelope theorem, we get

$$\frac{d\phi}{dp} = \gamma \quad (13)$$

When the threshold value of  $\theta$  is low (i.e.  $x = x^L$ ), we have the following:

$$\frac{dx^L}{d\phi} = \frac{1}{(1 - 4\alpha\phi)^{1/2}} > 0, \quad (14)$$

$$\frac{dx^L}{dp} = \frac{dx^L}{d\phi} \frac{d\phi}{dp} = \frac{\gamma}{(1 - 4\alpha\phi)^{1/2}} > 0, \quad (15)$$

and

$$\phi \frac{dx^L}{dp} + x^L \frac{d\phi}{dp} = \gamma \left[ \frac{\phi}{(1 - 4\alpha\phi)^{1/2}} + x^L \right] > 0. \quad (16)$$

Using (15) and (16), eq. (12) takes the following form for  $x = x^L$ :

$$F(p, \alpha) = - \left\{ (a - p)(1 - \gamma x^L) + \frac{\delta}{2}(1 - \gamma)x^L\phi \right\} \\ + (p - c) \left[ 1 - \frac{1}{2} \left( \phi \frac{\gamma}{(1 - 4\alpha\phi)^{1/2}} + x^L\gamma \right) (1 - \delta\gamma) - \delta\gamma x^L + S \frac{\partial\gamma}{\partial p} + \gamma(a - p) \frac{\gamma}{(1 - 4\alpha\phi)^{1/2}} \right] = 0,$$

where

$$S = (a - p)x^L + \frac{\delta}{2}x^L\phi,$$

$$x^L = \alpha^{-1} \left[ \frac{1}{2} - \frac{1}{2} (1 - 4\alpha\phi(p))^{1/2} \right] = x^L(\alpha, p),$$

and

$$\frac{\partial x^L}{\partial \alpha} = \frac{\left\{ \phi - x^L (1 - 4\alpha\phi(p))^{1/2} \right\}}{\alpha (1 - 4\alpha\phi(p))^{1/2}}.$$

It follows that

$$\begin{aligned}
F_\alpha = \frac{\partial F(p, \alpha)}{\partial \alpha} &= (p-c) \left[ -\frac{\gamma}{2}(1 - \delta\gamma) - \delta\gamma + \frac{\partial \gamma}{\partial p} \left( a - p + \frac{\delta\phi}{2} \right) \right] \frac{\partial x^L}{\partial \alpha} \\
&\quad - \left\{ \frac{\delta}{2}(1 - \gamma)\phi - (a - p)\gamma \right\} \frac{\partial x^L}{\partial \alpha} \\
&\quad + \gamma\phi(p - c) \{2\gamma(a - p) - \phi(1 - \delta\gamma)\} (1 - 4\alpha\phi(p))^{-3/2}. \quad (17)
\end{aligned}$$

Thus the sign of  $F_\alpha$  depends on the numerical values of the parameters.

Concerning  $F_p$ , we can show (see Appendix) that, under the assumption that the steady state is stable in the saddlepoint sense,  $F_p$  is negative. We state this as Lemma A:

**Lemma A:** *Under the assumption that the steady state is stable in the saddlepoint sense,  $F_p$  is negative.*

**Proof:** See the Appendix

Thus we obtain:

**Proposition 3:** *If  $x = x^L$ , then an increase in the network effect parameter  $\alpha$  will lead to a decrease in the steady state price  $p^*$  if and only if the sign of the expression  $F_\alpha$  (as given by (17)) is negative.*

On the other hand, when  $x = x^H$ , we have

$$\frac{dx^H}{d\phi} = -\frac{1}{(1 - 4\alpha\phi)^{1/2}} < 0, \quad (18)$$

$$\frac{dx^H}{dp} = \frac{dx^H}{d\phi} \frac{d\phi}{dp} = -\frac{\gamma}{(1 - 4\alpha\phi)^{1/2}} < 0, \quad (19)$$

and

$$\phi \frac{dx^H}{dp} + x^H \frac{d\phi}{dp} = \gamma \left[ -\frac{\phi}{(1 - 4\alpha\phi)^{1/2}} + x^H \right]. \quad (20)$$

Using (19) and (20), eq. (12) takes the following form for  $x = x^H$ :

$$F(p, \alpha) = - \left\{ (a - p)(1 - \gamma x^H) + \frac{\delta}{2}(1 - \gamma)x^H \phi \right\} \\ + (p - c) \left[ 1 - \frac{1}{2} \left( -\frac{\phi \gamma}{(1 - 4\alpha \phi)^{1/2}} + x^H \gamma \right) (1 - \delta \gamma) - \delta \gamma x^H + S \frac{\partial \gamma}{\partial p} - \gamma(a - p) \frac{\gamma}{(1 - 4\alpha \phi)^{1/2}} \right] = 0.$$

Now,

$$\frac{\partial x^H}{\partial \alpha} = - \frac{\left\{ \phi + x^H (1 - 4\alpha \phi(p))^{1/2} \right\}}{\alpha (1 - 4\alpha \phi(p))^{1/2}} < 0,$$

and

$$F_\alpha = (p - c) \left[ -\frac{\gamma}{2}(1 - \delta \gamma) - \delta \gamma + \frac{\partial \gamma}{\partial p} \left( a - p + \frac{\delta \phi}{2} \right) \right] \frac{\partial x^H}{\partial \alpha} \\ - \left\{ \frac{\delta}{2}(1 - \gamma)\phi - (a - p)\gamma \right\} \frac{\partial x^H}{\partial \alpha} \\ - \gamma \phi (p - c) \{ 2\gamma(a - p) - \phi(1 - \delta \gamma) \} (1 - 4\alpha \phi(p))^{-3/2}. \quad (21)$$

Thus we obtain:

**Proposition 4:** *If  $x = x^H$ , then an increase in the network effect parameter  $\alpha$  will lead to a decrease in the steady state price  $p^*$  if and only if the sign of the expression  $F_\alpha$  (as given by (21)) is negative.*

#### 4. Numerical Analysis

In this section we provide a numerical example to illustrate some of our analytical results. Suppose the parameters of our model take the following values:

$$a = 1.1, c = 0, \delta = 0.01, \alpha = 0.9, \text{ and } z = \frac{1}{3}\gamma^3$$

We find that there are two steady states. Variables in the first steady state are denoted by a single asterisk (\*) while those in the second steady state are denoted by a double asterisk (\*\*). At the first steady state,  $p^* = 0.414$  and  $x^* = x^L(p^*) = 0.221$ . (At  $p^* = 0.414$ , if we use  $x = x^H(p^*)$  to substitute into  $F(., \alpha)$  then  $F(p^*, \alpha) \neq 0$ , i.e., if consumers switch from  $x^L$  to  $x^H$  then  $p^*$  ceases to be a steady state: the virgin producer would be happy to maintain price at  $p^* = 0.414$  only if consumers expect that the fraction of consumers that participate in recycling is  $x^L$ .)

At the second steady state,  $p^{**} = 0.316$ . The corresponding steady-state scrap price is  $\phi^{**} = 0.118$ , and  $x^{**} = x^H(p^{**}) = 0.976$ . (At  $p^{**} = 0.316$ , if we use  $x = x^L(p^{**})$  to substitute into  $F(., \alpha)$  then  $F(p^{**}, \alpha) \neq 0$ , i.e., if consumers switch from  $x^L$  to  $x^H$  then  $p^{**}$  ceases to be a steady state: the virgin producer would be happy to maintain price at  $p^{**} = 0.319$  only if consumers expect that the fraction of consumers that participate in recycling is  $x^H$ .)

At the first steady state, the total quantity sold in each period is  $Q^* = 0.687$ , out of which  $S^* = 0.152$  is sorted and sold by consumers as scrap at a price of  $\phi^* = 0.177$ . The recycling firms transform 64.3 percent (i.e.  $\gamma^* = 0.643$ ) of this scrap into secondary at a cost of  $z^* = 8.8637 \times 10^{-2}$ . Saddlepoint stability is satisfied as  $F_p(p^*, \alpha) = -3.97$ . Since  $F_\alpha(p^*, \alpha) = -0.196$ ,  $\frac{dp^*}{d\alpha} < 0$  i.e. the steady state price *decreases* when the network effect parameter increases.

On the other hand, at the second steady state 97.6 percent of the population sort. The price of virgin aluminum is lower at  $p^{**} = 0.316$ , while total quantity sold in each period is higher at  $Q^{**} = 0.784$ . Of the latter,  $S^{**} = 0.766$  is sorted and sold by consumers as scrap at a price of  $\phi^{**} = 0.118$ . The recycling firms transform only 56.2 percent (i.e.  $\gamma^{**} = 0.562$ ) of this scrap into secondary at a cost of  $z^{**} = 5.9272 \times 10^{-2}$ . Saddlepoint stability is again satisfied as  $F_p(p^{**}, \alpha) = -0.807$ . Since  $F_\alpha(p^{**}, \alpha) = 0.756$ ,  $\frac{dp^{**}}{d\alpha} > 0$  i.e. the steady state price now *increases* when the network effect parameter increases.

The fact that both steady states are stable in the saddlepoint sense can be explained as follows. The first steady state is stable in the sad-

point sense conditional on the hypothesis that consumers expect  $x = x^L$  at  $p^*$  and in some neighborhood of  $p^*$ . If for some reason the expectation is switched to  $x = x^H$ , the virgin producer will no longer be happy at  $p^*$ , he will choose a different price path  $p_t$ , which eventually converges to  $p^{**}$  (assuming that the switch to  $x^H$  is permanent).

Which of the above two steady states is socially more desirable? To answer this we have to compute social welfare under each steady state. Gross utility ( $U$ ) of all consumers can be expressed as

$$U = \int_0^1 u(q(\theta)) d\theta = \int_0^1 aq(\theta) d\theta - \frac{1}{2} \int_0^1 (q(\theta))^2 d\theta,$$

which, using (7) and (8), becomes

$$U = \frac{1}{2} (a^2 - p^2) + \delta\phi x \left( p - \frac{1}{2}\delta\phi \right) + \frac{1}{2} (\delta x^2 (1 - \alpha x)) \left( \delta\phi - \frac{1}{3}\delta x (1 - \alpha x) - p \right).$$

The total recycling cost ( $E$ ) of the consumers who participate in recycling is given by

$$E = \int_0^x \theta(1 - \alpha x)q(\theta)d\theta = (1 - \alpha x) \left( (a - p + \delta\phi) \frac{x^2}{2} - \delta(1 - \alpha x) \frac{x^3}{3} \right).$$

Total cost of manufacturing virgin and secondary is

$$c(Q - \gamma S) + zS.$$

Any aluminum that is not recycled by consumers ends up going to landfills or incinerators as waste. Landfilling and incineration have well-known environmental costs such as ground water contamination and air pollution, resulting in decreased value of surrounding land. Suppose the environmental damage from disposing  $(Q - S)$  units of aluminum as waste is given by  $D = \frac{\varepsilon}{2} (Q - S)^2$ , where  $\varepsilon > 0$  is the environmental damage parameter. Then, defining social welfare ( $W$ ) as gross utility minus total cost of recycling by consumers and cost of

manufacturing virgin and secondary by producers minus environmental damage from waste disposal, we have

$$W = U - E - c(Q - \gamma S) - zS - \frac{\varepsilon}{2}(Q - S)^2. \quad (22)$$

In what follows, we set  $\varepsilon = 1$ .

Using our numerical example, social welfare in the low steady state comes out to be  $W = 0.350$ . The monopoly primary producer's profit per period is  $\Pi = (p - c)(Q - \gamma S) = 0.243$ . Recycling firms earn zero profit. Environmental damage is  $D = 0.143$ . Consumer surplus ( $CS$ ) is  $CS = U - E - pQ + \phi S = U - E - c(Q - \gamma S) - zS - \Pi = 0.249$ .

On the other hand, in the high steady state the corresponding values are  $W = 0.464$ ,  $\Pi = 0.112$ ,  $D = 0.00016$  and  $CS = 0.353$ . Thus, while the monopolist makes less profit in the high steady state, consumers are better off.

Notice that social welfare excluding environmental damage (i.e.  $\Pi + CS$ ) is smaller in the high steady state where more consumers participate in the recycling effort. However, since most of the metal is recycled in the high steady state, environmental damage in this state is much smaller than that in the low steady state. As a result, social welfare turns out to be greater in the high steady state. Of course, if the environmental damage parameter  $\varepsilon$  was sufficiently smaller than that assumed in our numerical example, social welfare in the low steady state could turn out to be greater than that in the high steady state. More recycling would not necessarily be more desirable in that case.

Our analysis suggests a role for the government: by switching consumers' expectation from  $x^L$  to  $x^H$ , the government can steer the economy away from the low equilibrium.

## 5. Conclusion

This paper relaxes the assumption made by the existing literature on the recycling problem that homogeneous consumers always participate in recycling. When heterogeneous consumers differ in terms of

their intrinsic recycling cost, we show that all consumers may not always participate in the recycling process. Consequently, the long-run price charged by the virgin producer will depend on both the fraction of consumers who recycle as well as the fraction of secondary aluminum recovered per unit scrap by the recycling firms.

As discussed in the introduction, consumers' recycling behaviour can sometimes be influenced by social norms and pressures. This can lead to the existence of a network effect with respect to the consumers' recycling cost. We show that existence of such an effect can lead to the emergence of multiple equilibria in certain cases. As a result, an increase in the scrap price can either increase or decrease the fraction of consumers that recycle. Moreover, an increase in the network effect can have a positive or negative impact on the monopolist's price of virgin, as illustrated by our numerical example. When there are multiple steady state equilibria, our analysis suggests that the government might be able to influence equilibrium selection by switching consumers' expectation.

**APPENDIX: Proof of Lemma A.**

(**Lemma A:** Under the assumption that the steady state is stable in the saddlepoint sense,  $F_p$  is negative.)

The dynamics of the system is given by the Euler's equation (11), which is a second-order difference equation in  $p$ . This equation gives us the steady state  $p^*$  when we set  $p_{t+1} = p_t = p_{t-1}$ .

To determine the stability of the steady state  $p^*$ , we must linearize the equation (11) around the steady-state  $p^*$ , and evaluate the characteristic roots.

Equation (11) is of the non-linear form

$$G(p_{t+1}, p_t, p_{t-1}, \alpha) = 0$$

Linearization around the steady state  $p^*$  gives

$$(p_{t+1} - p^*) \frac{\partial G}{\partial p_{t+1}} + (p_t - p^*) \frac{\partial G}{\partial p_t} + (p_{t-1} - p^*) \frac{\partial G}{\partial p_{t-1}} = 0 \quad (23)$$

In simpler notation:

$$A(p_{t+1} - p^*) + B(p_t - p^*) + C(p_{t-1} - p^*) = 0$$

where

$$A \equiv \frac{\partial G(p_{t+1}, p_t, p_{t-1}, \alpha)}{\partial p_{t+1}} \text{ evaluated at } (p_{t+1}, p_t, p_{t-1}) = (p^*, p^*, p^*)$$

$$B \equiv \frac{\partial G(p_{t+1}, p_t, p_{t-1}, \alpha)}{\partial p_t} \text{ evaluated at } (p_{t+1}, p_t, p_{t-1}) = (p^*, p^*, p^*)$$

$$C \equiv \frac{\partial G(p_{t+1}, p_t, p_{t-1}, \alpha)}{\partial p_{t-1}} \text{ evaluated at } (p_{t+1}, p_t, p_{t-1}) = (p^*, p^*, p^*)$$

Note that the second order condition (obtained by differentiating (11) with respect to  $p_t$ ) implies that  $B < 0$ .

Now

$$G \equiv \frac{1}{2} (p_{t-1} - c) \left( \phi_t \frac{dx_t}{dp_t} + x_t \frac{d\phi_t}{dp_t} \right) + \left[ a - p_t + \frac{\delta}{2} x_{t+1} \phi_{t+1} - \gamma_t x_t \left( a - p_{t-1} + \frac{\delta \phi_t}{2} \right) \right]$$

$$\begin{aligned}
& + (p_t - c) \left[ -1 - \gamma_t \left\{ \frac{\delta}{2} \left( \phi_t \frac{dx_t}{dp_t} + x_t \frac{d\phi_t}{dp_t} \right) + (a - p_{t-1}) \frac{dx_t}{dp_t} \right\} - x_t \left( a - p_{t-1} + \frac{\delta\phi_t}{2} \right) \frac{\partial\gamma_t}{\partial p_t} \right] \\
& \quad + \delta (p_{t+1} - c) \gamma_{t+1} x_{t+1} \tag{24}
\end{aligned}$$

From this, we get

$$A = \frac{\delta}{2} \left[ x \frac{\partial\phi}{\partial p} + \phi \frac{dx}{dp} \right] + \delta\gamma x + \delta(p - c) \frac{d(\gamma x)}{dp},$$

$$B = -2(1 + \gamma S_p + S\gamma_p) - (p - c)(2S_p\gamma_p + \gamma S_{pp} + S\gamma_{pp}) + \frac{\gamma(p - c)}{2} \frac{d}{dp}(\gamma x),$$

where  $B$  must be negative, by the second order condition, and

$$C = \frac{1}{2}(\phi x_p + x\phi_p) + \gamma x + (p - c) \frac{d(\gamma x)}{dp}$$

Evaluated at  $(p_{t+1}, p_t, p_{t-1}) = (p^*, p^*, p^*)$  we have

$$A = \delta C$$

Equation (23) gives rise to the characteristic equation

$$A\lambda^2 + B\lambda + C = 0$$

ie

$$\lambda^2 + \left( \frac{B}{A} \right) \lambda + \frac{C}{A} = 0$$

The two roots are

$$\lambda_{1,2} = -\frac{1}{2} \left( \frac{B}{A} \right) \pm \frac{1}{2} \sqrt{\left( \frac{B}{A} \right)^2 - 4 \left( \frac{C}{A} \right)} \tag{25}$$

By “regular saddlepoint stability”, we mean that both roots are real and positive, with one root exceeding unity and the other root

is inside the interval  $(0,1)$ . The two roots are real and distinct if  $B^2 > 4AC$ . Assume this condition holds. Since, from (25),

$$\lambda_1\lambda_2 = C/A = \delta > 0$$

we conclude that the roots are either both positive, or both negative.

Also, from (25),

$$\lambda_1 + \lambda_2 = -\frac{B}{A}$$

both roots are positive if and only if  $-B/A > 0$ . Since  $B < 0$  by the second order condition, it follows that  $A > 0$  is a necessary condition for regular saddlepoint stability. Since  $A = \delta C$ ,  $A > 0$  implies  $C > 0$  as well.

Recall that the condition for regular saddlepoint stability is that the larger root exceeds 1 and the smaller root is inside the interval  $(0,1)$ . Given that  $A > 0$ ,  $C > 0$  and  $B < 0$ , this condition is satisfied if and only if

$$0 > A + B + C$$

It follows that regular saddlepoint stability implies  $A + B + C < 0$ . But clearly, by definition,

$$F_p(p^*, \alpha) = G_{p_{t+1}}(p_{t+1}, p_t, p_{t-1}, \alpha) + G_{p_t}(p_{t+1}, p_t, p_{t-1}, \alpha) + G_{p_{t-1}}(p_{t+1}, p_t, p_{t-1}, \alpha)$$

where  $(p_{t+1}, p_t, p_{t-1})$  is evaluated at  $(p^*, p^*, p^*)$ . So regular saddlepoint stability implies  $F_p < 0$ .

This completes the proof of the Lemma.

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